

# Edge Detection in Images based on Approximation Theory

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**Abstract:** Edges characterize boundaries and edge detection is one of the most difficult tasks in image processing hence it is a problem of fundamental importance in image processing. Several edge detection algorithms have been developed such as Prewitt, Sobel, LOG, etc. But, they are not able to produce ideal or optimized results. This paper presents an edge detection approach applicable to gray level images based on Approximation Theory. The performance of proposed method is compared against other methods such as Sobel and Prewitt edge detector by using various tested images. Experimental results reveal that the proposed method exhibits better performance and may efficiently be used for the detection of edges in image.

**Keywords:** Edge detection, Approximation Theory, Gray scale images.

## I. INTRODUCTION

Edge detection in image is a fundamental feature of image analysis and image recognition[1]-[4]. For object detection, it is crucial to have a good understanding of edge detection algorithms. It is one of the most commonly used operations in image analysis. An edge is the boundary between an object and the background.

Classical methods of edge detection were applied, the majority of these methods may be grouped in two categories: the gradient (Roberts, Prewitt and Sobel), and the second order derivatives (Laplace and LOG), etc[5]. The Sobel operator consists of three filters, defined as  $h_1, h_2, h_3$  [6]. In normal case, only  $h_1$  and  $h_3$  can be interpreted as finding horizontal and vertical gradients respectively. The Sobel operator is often used as a simple detector of horizontal and vertical edges. In this case only masks  $h_1$  and  $h_3$  are used. Canny edge detector method[7] uses derivative of Gaussian, in this case different values of the Gaussian Kernel parameter can be used for detecting of edges with different precision.

In this paper a new edge detection algorithm is proposed to detect edges of Gray scale images based on approximation theory. The proposed method give good results than other classical methods.

The paper is organized as follows: An introduction of Approximation Theory is given in Section 2. An overview of Sobel operator is presented in Section 3. Section 4, introduce Illustration of the Algorithm for the proposed approach applied to Gray scale images. In Section 5, some particular images will be analysed using proposed method based algorithm and moreover, a comparison with some existing

methods will be provided for these images, and conclusions is included in Section 6.

## II. INTRODUCTION TO APPROXIMATION THEORY

Approximation theory involves two general types of problems[8]. One problem arises when a function is given explicitly. The other problem in approximation theory is concerned with fitting functions to given data and finding the "best" function in a certain class to represent the data.

At times it is appropriate to assume that the data are exponentially related. This requires the approximating function to be of the form

$$y = bx^a \quad (1)$$

for some constants  $a$  and  $b$ . Let  $y_i = bx_i^a$  denote the  $i$ -th value on the approximating line and  $y_i$  be the  $i$ -th given  $y$ -value. The problem of finding the equation of the best approximation in the absolute sense requires that values of  $a$  and  $b$  be found to minimize.

$$E_2(a, b) = \sum_{i=1}^n (y_i - bx_i^a)^2 \quad (2)$$

Or

$$E_{Abs}(a, b) = \sum_{i=1}^n |y_i - bx_i^a| \quad (3)$$

The quantity (2) is called lease square approach and (3) is called the absolute error. To minimize a function of two variables, we need to set its partial derivatives to zero and simultaneously solve the resulting equations.

$$0 = \frac{\partial E_2}{\partial b} = 2 \sum_{i=1}^n (y_i - bx_i^a)(-x_i^a)$$

$$0 = \frac{\partial E_2}{\partial a} = 2 \sum_{i=1}^n (y_i - bx_i^a)(-b(\ln x_i)x_i^a)$$

The method that is commonly used when the data are suspected to be exponentially related is to consider the logarithm of the approximating equation:

$$\ln y = \ln b + a \ln x \quad (4)$$

set  $Y = \ln y$ ,  $B = \ln b$  and  $X = \ln x$  then the above equation can be written in the following form

$$Y = B + aX \quad (5)$$

a linear problem now appears, and solutions for  $\ln b$  and  $a$  can be obtained by appropriately modifying the normal equations

$$B = \ln b = \frac{\sum_{i=1}^n X_i^2 \sum_{i=1}^n Y_i - \sum_{i=1}^n X_i Y_i \sum_{i=1}^n X_i}{n(\sum_{i=1}^n X_i^2) - (\sum_{i=1}^n X_i)^2} \quad (6)$$

$$a = \frac{n \sum_{i=1}^n X_i Y_i - \sum_{i=1}^n X_i \sum_{i=1}^n Y_i}{n(\sum_{i=1}^n X_i^2) - (\sum_{i=1}^n X_i)^2} \quad (7)$$

### III. SOBEL FILTER OPERATOR

The Sobel operator performs a 2-D spatial gradient measurement on an image and so emphasizes regions of high spatial frequency that correspond to edges. Typically it is used to find the approximate absolute gradient magnitude at each point in an input grayscale image [1], [9].

In theory at least, the operator consists of a pair of  $3 \times 3$  convolution kernels as shown in Figure 1. One kernel is simply the other rotated by  $90^\circ$ . This is very similar to the Roberts Cross operator.

-1	0	1
-2	0	2
-1	0	1

Gx

1	2	1
0	0	0
-1	-2	-1

Gy

Fig.1: Sobel convolution kernels

These kernels are designed to respond maximally to edges running vertically and horizontally relative to the pixel grid, one kernel for each of the two perpendicular orientations. The kernels can be applied separately to the input image, to produce separate measurements of the gradient component in each orientation (call these  $G_x$  and  $G_y$ ). These can then be combined together to find the absolute magnitude of the gradient at each point and the orientation of that gradient. The gradient magnitude is given by:

$$|G| = \sqrt{(G_x)^2 + (G_y)^2}$$

Typically, an approximate magnitude is computed using:

$$|G| = |G_x| + |G_y|$$

which is much faster to compute.

The angle of orientation of the edge (relative to the pixel grid) giving rise to the spatial gradient is given by:

$$\theta = \tan^{-1}(G_x/G_y)$$

In this case, orientation  $\theta$  is taken to mean that the direction of maximum contrast from black to white runs from left to right on the image, and other angles are measured anti-clockwise from this.

### IV. ALGORITHM FOR EDGE DETECTION

We can represent image areas data with non-linear function (equation (1)), the edges in image can be obtained by comparing the absolute error (see section 2) with the threshold value. If the absolute error value is great than threshold value, so this point represent an edge and if it less than threshold value, so this point belonging to a homogeneous area.

Now, we can describe the Approximation Theory Algorithm to determine the edges:

#### ALGORITHM 1: EDGE DETECTION

1. **Input:** A Gray-scale image  $I$  of size  $M \times N$ .
2. Applying the Sobel masks on the image plane, then extract the values from the image ( a new image  $J$  of size  $M \times N$  )
3. Rearrange every  $3 \times 3$  value ascending (the size of mask from the image  $J$  ), So we will get data as form  $y(x)$ , where  $(x = 1, 2, \dots, n)$ . In our case  $n = 3 \times 3 = 9$
4. From the values  $y(x)$  we do the following steps:
  - I. Re-ascending the odd values.
  - II Re-descending the even values.
5. Calculate the values  $B = \ln b$  and  $a$  from the equations (6) and (7), respectively.
6. Calculate  $y(x)$  from the equation (1).
7. Now we have two vectors:
  - I. The values estimate from equation (5) (the linear form  $Y$ )
  - II The values estimate from equation (1) (approximation form  $y$ )
8. Estimate the absolute error between the vectors  $Y$  and  $y$  (step 7), as follows:
 
$$E_{Abs}(Y, y) = \sum_{i=1}^n |Y_i - y_i|$$
9. Estimate the mean of the errors
 
$$Mean_E = \frac{E_{Abs}(Y, y)}{n}$$
 if  $Mean_E > th$ , (where  $th$  is the threshold value) then can be assigned the central mask point as edge point.
10. **Output:** The edge detection image  $g$  of  $I$ .

### V. EXPERIMENTAL RESULTS

To demonstrate the efficiency of the proposed approach, the algorithm is tested over a number of different Gray-scale images and compared with traditional operators. The images detected by LOG, Sobel, Prewitt and the proposed method, respectively. All the concerned experiments were implemented on Intel® Core™ i3 2.10GHz with 4 GB RAM using MATLAB R2007b.

Some selected results of edge detections for these test images using the classical methods and proposed scheme are shown in Fig.2(a)-(c). From the results; it has again been

observed that the performance of the proposed method works well as compare to the performance of the previous methods (with default parameters in MATLAB).

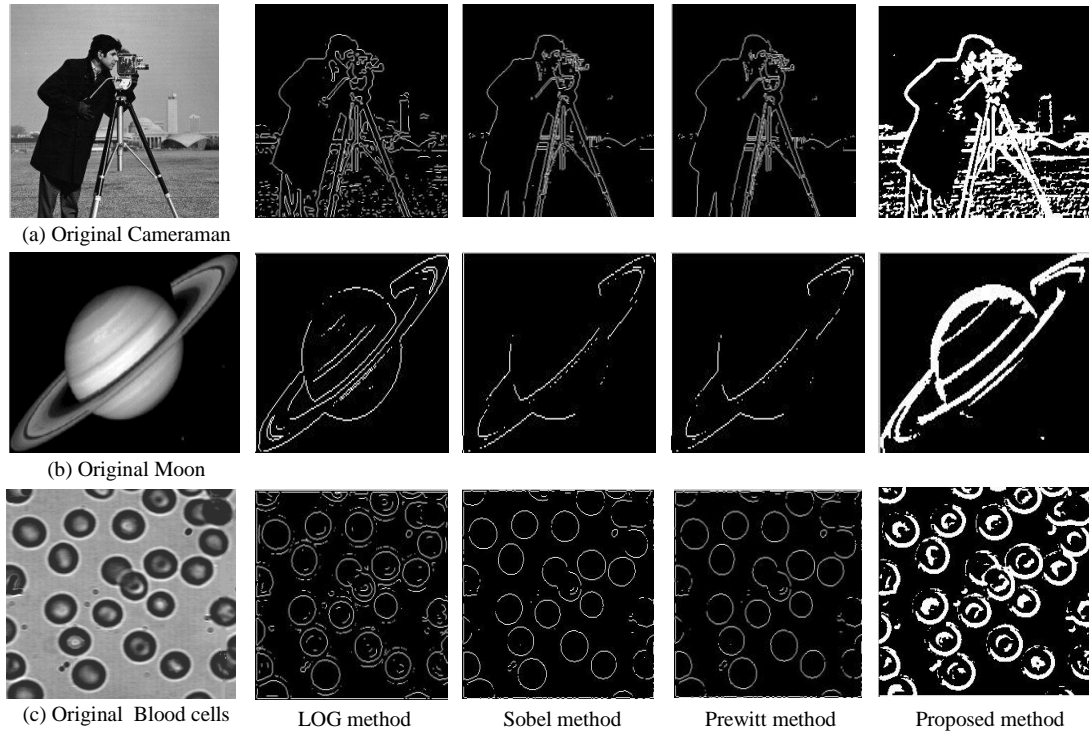


Fig. 2: Performance of Proposed Edge Detector for various images

## VI. CONCLUSION

An efficient approach using Approximation Theory for detection of edges in Gray-scale images is presented in this paper. The proposed method is compared with traditional edge detectors. On the basis of visual perception and edge counts of edge maps of various Gray-scale images it is proved that our Algorithm is able to detect highest edge pixels in images. Also it gives smooth and thin edges without distorting the shape of images. Another benefit comes from easy implementation of this method

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