

A single natural image denoising based on independent component analysis

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Abstract: Denoising of natural image is challenging problem in image processing. Image denoising problem is very simple. Independent component analysis (ICA) is higher order statistical tool for analysis of multidimensional data. Image denoising using independent component analysis assume a Gaussian noise. The noise is a Gaussian and considered image data is non-Gaussian random variable. In practical situations type of noise, type of image, amount of noise are all variable parameters. Such a parameter cannot effectively remove the noise. In this paper we use a independent component analysis to remove a noise from a single image. Natural images provide the basic knowledge for understanding.

Keywords: Adaptive PCA, Denoise, ICA, PCA.

I. INTRODUCTION

Image denoising is an essential part in an image processing field. Denoising of image is a research problem and a challenging work in image processing because image have variable parameters such as type image, type of noise and amount of noise in images. Denoising techniques such as Fourier transform and wavelet transform method both are not data adaptive methods these are localized in frequency domain and spatial domain.(ie.,[1])A data adaptive comes out from filtering approach that results in a inherent property of ICA techniques. Data adaptive is a very important part in the denoising of image.(ie.,[2]). For this method considered image are corrupted by Gaussian noise. We compare peak single to noise ratio (PSNR) of denoised and noisy image of various algorithms. The following denoised methods are discussed to understand denoising of natural image.

- 1.Principle component analysis.
2. Adaptive component analysis.
3. Independent component analysis.

II. PRINCIPAL COMPONENT ANALYSIS

Principal Component Analysis (PCA) is an exploratory tool designed by Karl Pearson in 1901 to identify unknown trends in a multidimensional data set \mathbf{X} . The algorithm was introduced to psychologists in 1933 by H. Hotelling, hence sometimes it is called Hotelling's Transform. However, today we know that implementing PCA is the equivalent of applying Singular Value Decomposition (SVD) on the covariance matrix of a data set (1.2, 1.3). By providing a tutorial on PCA using SVD, students are familiarized with both matrix decomposition techniques. When there is need to dimension reduction (reduce the no. of variables) and further analyse the relationship between different variables (quantity); we can use PCA to solve this kind of problems. Principal components are the direction of greatest

Variability (covariance) in the data, then the next orthogonal (uncorrelated) direction of the greatest variability. So first remove all the variability along the first component, and then find the next direction of greatest variability and so on.....

The mathematical calculation for this is as given below-(ie.,[3]) PCA is the evident from the two projections, that what we see depends on the direction onto which we project.

The mean can be calculate by using the formula-[3]

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (1.1)$$

The variance can be calculate by using-

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \quad (1.2) [3]$$

To find the direction \hat{u} , such that projecting n points in the d dimensions, \vec{e}^i , $i=1,2,\dots,n$, onto \hat{u} , gives the largest variance.(ie.,[3])

To construct the covariance matrix-

$$c_{ij} = \frac{1}{n} \sum_{k=1}^n (e_i^k - \bar{e}_i) (e_j^k - \bar{e}_j) \quad (1.3)$$

\hat{u} is the eigenvector of c with the largest eigenvalue:

$$C\hat{u} = \lambda_1 \hat{u} \quad (1.4) [3]$$

To show that the variance of the projections is indeed λ_1 , we first calculate our sample vectors their projections onto \hat{u} : [3]

$$e_u^k = \vec{e}^{(k)} \cdot \hat{u} = \sum_{i=1}^d e_i^{(k)} \hat{u}_i \quad (1.5)$$

The total variance of the data is,[3]

$$\sigma^2 = \sum_k (\vec{e}^{(k)} - \bar{e})^2$$

Is also given by [3] the sum of the variance of the projections on the principal components

$$\sigma^2 = \sum_{\alpha=1}^d \sigma_{\alpha}^2 = \sum_{\alpha} \lambda_{\alpha}$$

The fraction of the total variance that has been captured is:[3]

$$\sigma_{\text{captured}} = \sum_{\alpha=1}^d \frac{\sigma_{\alpha}^2}{\sigma^2} \text{(ie.,[3])}$$

III INDEPENDENT COMPONENT ANALYSIS

ICA is nothing but the independent component analysis, as its name implies it is the method for finding underlying factors or components from multi dimensional statistical data. What distinguishes ICA from other methods is that it looks for components that are both statistically independent and non-Gaussian. Blind source separation or ICA is the identification and separation of mixtures of sources with little prior information. Multiple sources human being can focus on the interested source/sources, but hard to separate by signal processing techniques, then we can use independent component analysis to solve this problem. A speech source signal s_1 is represented as $s_1 = (s_{11}, s_{21}, \dots, s_{N1})$, where s_1 adopts amplitudes s_{11} , then s_{21} , and so on; superscripts specify time and subscripts specify signal identity (e.g., speaker identity). We will be considering how to mix and unmix a set of two or more signals, and we define a specific set of two time varying speech signals s_1 and s_2 in order to provide a concrete example. Now, the amplitudes of both signals can be written as a vector variable \mathbf{s} , which can be rewritten in one of several mathematically equivalent forms: (ie., [5]).

$$\mathbf{S} = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} \quad (3.1)$$

$$= \begin{pmatrix} (s_1^1, s_1^2, \dots, s_1^N) \\ (s_2^1, s_2^2, \dots, s_2^N) \end{pmatrix} \quad (3.2)$$

The Mixing and Unmixing Matrices (ie., [5]).

The set of mixtures defines a vector variable $\mathbf{x} = (x_1, x_2)^T$, and the transformation from \mathbf{s} to \mathbf{x} defines a mixing matrix \mathbf{A} :

$$\mathbf{X} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} s_1^1, s_1^2, \dots, s_1^N \\ s_2^1, s_2^2, \dots, s_2^N \end{pmatrix} \quad (3.3)$$

The mapping from \mathbf{x} to $\mathbf{s} = (s_1, s_2)^T$ defines an optimal unmixing matrix $\mathbf{W}^* = (\mathbf{w}_1, \mathbf{w}_2)^T$ with (row) weight vectors $\mathbf{w}_1 = (\alpha, \beta)$ and $\mathbf{w}_2 = (\gamma, \delta)$

$$\mathbf{S} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} x_1^1, x_1^2, \dots, x_1^N \\ x_2^1, x_2^2, \dots, x_2^N \end{pmatrix} \quad (3.4)$$

$$= (\mathbf{w}_1, \mathbf{w}_2)^T (\mathbf{x}_1, \mathbf{x}_2) \quad (3.5)$$

$$= \mathbf{W}^* \mathbf{x} \quad (3.6)$$

It can be seen that \mathbf{W}^* reverses, or inverts, the effects of \mathbf{A} , and indeed, \mathbf{W}^* could be estimated from the matrix inverse $\mathbf{W}^* = \mathbf{A}^{-1}$, if \mathbf{A} were known. However, as we are ultimately concerned with finding \mathbf{W}^* when \mathbf{A} is not known, we cannot, therefore, use \mathbf{A}^{-1} to estimate \mathbf{W}^* . For arbitrary values of the unmixing coefficients, the unmixing matrix is suboptimal and is denoted \mathbf{W} . In this case, the signals extracted by \mathbf{W} are not necessarily source signals, and are denoted $\mathbf{y} = \mathbf{W}\mathbf{x}$ (ie., [5]). Consider a (mixture) vector variable \mathbf{x} with joint pdf $p_{\mathbf{x}}$, and a (source) vector variable \mathbf{s} with joint pdf $p_{\mathbf{s}}$, such that $\mathbf{s} = \mathbf{W}^*\mathbf{x}$, where \mathbf{W}^* is the optimal unmixing matrix. As noted above, the number of source signals and mixtures must be equal, which ensures that \mathbf{W}^* is square. In general, the relation between the joint pdfs of \mathbf{x} and \mathbf{s} is [5]

$$p_{\mathbf{x}}(\mathbf{x}) = p_{\mathbf{s}}(\mathbf{s}) |\mathbf{W}^*| \quad (3.7)$$

where $|\mathbf{W}^*| = |\partial \mathbf{s} / \partial \mathbf{x}|$ is the Jacobian of \mathbf{s} with respect to \mathbf{x} . Equation (3.7) defines the likelihood of the observed mixtures \mathbf{x} , which is the probability of \mathbf{x} given \mathbf{W}^* . For any non optimal unmixing matrix \mathbf{W} , the extracted signals are $\mathbf{y} = \mathbf{W}\mathbf{x}$. Making the dependence on \mathbf{W} explicit, the likelihood $p_{\mathbf{x}}(\mathbf{x}|\mathbf{W})$ of the signal mixtures \mathbf{x} given \mathbf{W} is [5]

$$p_{\mathbf{x}}(\mathbf{x}|\mathbf{W}) = p_{\mathbf{s}}(\mathbf{W}\mathbf{x}) |\mathbf{W}| \quad (3.8)$$

We would naturally expect $p_{\mathbf{x}}(\mathbf{x}|\mathbf{W})$ to be maximal if $\mathbf{W} = \mathbf{W}^*$. Thus, (3.8) can be used to evaluate the quality of any putative unmixing matrix \mathbf{W} in order to find that particular \mathbf{W} that maximizes $p_{\mathbf{x}}(\mathbf{x}|\mathbf{W})$. By convention, (3.8) defines a likelihood function $L(\mathbf{W})$ of \mathbf{W} , and its logarithm defines the log likelihood function $\ln L(\mathbf{W})$. If the M source signals are mutually independent, so that the joint pdf is the product of its M marginal pdfs, then can be written (ie., [5]).

$$\ln L(\mathbf{W}) = \sum_i^M \sum_t^N \ln p_s(\mathbf{W}_i^T \mathbf{x}^t) + N \ln |\mathbf{W}| \quad (3.9)$$

Note that the likelihood $L(\mathbf{W})$ is the joint pdf $p_{\mathbf{x}}(\mathbf{x}|\mathbf{W})$ for \mathbf{x} , but using MLE, it is treated as if it were a function of the parameter \mathbf{W} . If we substitute a commonly used leptokurtotic model joint pdf for the source signals $p_{\mathbf{s}}(\mathbf{y}) = (1 - \tanh(\mathbf{y}/2))$, then $\ln L(\mathbf{W}) = \sum_i^M \sum_t^N \ln \left(\frac{1 - \tanh(\mathbf{W}_i^T \mathbf{x}^t)}{2} \right) + N \ln |\mathbf{W}|$ (3.10)

The matrix \mathbf{W} that maximizes this function is the maximum likelihood estimate of the optimal unmixing matrix \mathbf{W}^* . Equation (3.9) provides a measure of similarity between the joint pdf of the extracted. (ie., [5]).

IV RESULTS

The figure which is added by noise and the output image which is by passing ICA, PCA, Adaptive PCA and the final graph is as follows:-



Figure(1) The original + noisy figure Fig(2) Image obtained after adaptive PCA signals $\mathbf{y} = \mathbf{W}\mathbf{x}$ and the joint model pdf of the source signals \mathbf{s} . Having such a measure permits us to use standard optimization methods to iteratively update the unmixing matrix in order to maximize this measure of similarity.

BIOGRAPHIES



Vipul Patil received the B.E. degree in 2010 in Electronics and communication Engineering. Currently he is doing M.E. (Electronics and communication) from ShriGulabraoDeokar college of Engineering, Jalgaon. His interest in image processing, medical engineering.



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Fig(3) Image obtained after ICA
Fig(4) Image obtained after PCA

Methods	Signal to noise ratio in db
Original image+noisy image	8.504725
Local PCA	30.5415
Fast ICA	31.9723
Adaptive PCA	23.9884

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