

# A Study on Analog and Digital Hybrid Modulation

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**Abstract:** This paper seeks to present ways to eliminate the inherent quantization noise component in digital communications, instead of conventionally making it minimal. It deals with a new concept of signaling called the Signal Code Modulation (SCM) Technique. The primary analog signal is represented by: a sample, which is quantized and encoded digitally, and an analog component, which is a function of the quantization component of the digital sample. The advantages of such a system are two sided offering advantages of both analog and digital signaling. The presence of the analog residual allows for the system performance to improve when excess channel SNR is available. The digital component provides increased SNR and makes it possible for coding to be employed to achieve near error-free transmission.

**Keywords:** Quantization, QAM, SCM, PAM, bandwidth, SNR.

## I. INTRODUCTION

Let us consider the transmission of an analog signal over a band-limited channel. This could be possible by two conventional techniques: analog transmission, and digital transmission, of which the latter uses sampling and quantization principles. Analog Modulation techniques such as Frequency and Phase Modulations provide significant noise immunity as known and provide SNR improvement proportional to the square root of modulation index, and are thus able to trade off bandwidth for SNR.

However, the SNR improvement provided by these techniques is much lower than the ideal performance as shown by the Shannon's capacity theorem.

On the other hand, Digital techniques of transmission can utilize error-correction codes that provide performance close to theoretical prediction. However, the major disadvantage of digital transmission techniques is the inherent quantization error introduced, which is imminent all the while the signal is relayed. This error causes distortion in the original signal being relayed and cannot be later recovered by any means possible.

If we quantized the sampled signal using QAM or any other method, using a fixed number of bits, a fixed digital distortion is introduced in the developmental stage itself. This distortion is present regardless of the transmission quality of the channel being used. Thus the original signal can be considered to be permanently impaired.

Communications systems are normally constructed for SNR much higher than the minimum that is required, so as to leave a margin for fading and other effects, which might occasionally reduce the SNR .

So, it is essential to design a communications system where the output SNR increases as the channel SNR increases. While, as already stated, this technique is not feasible through digital modulation, it is an inherent property in analog modulation.

Here, we introduce the concept of Signal Code Modulation (SCM) which utilizes both the analog, as well as, digital modulation techniques. The primary analog input signal is sampled at the appropriate rate and quantized. The digital samples are denoted by symbols  $D$ . The resulting  $D$  symbols are then transmitted using digital transmission techniques (like QAM) optimized for that channel. Those  $D$  symbols represent  $N$  bits per analog input sample.

The quantization residual, which is not left behind, is transmitted over the noisy channel as an analog symbol  $A$ , corresponding to the digital symbol  $D$ , as shown in the figure 1. To produce the quantization error  $A$ , the quantized data is converted back into analog form and subtracted from the original analog input signal. This symbol  $A$ , for noise immunity, is amplified by a gain of  $2^N$  (or any proportional factor that will optimize the voltage swing of the signal with that of the channel).

The SCM receiver performs the opposite operation by combining the  $D$  symbol and its corresponding residual. This would not bring about significant improvement if transmitted over a noisy channel as noise could vary the symbol  $A$  and cause bit errors in the  $D$  symbols.

However, the  $2^N$  amplitude gain of the analog components provides a noise immunity of  $2^{2N}$  to boost the SNR and provide a near ideal scheme for error-free transmission.

## II. THE SCM TECHNIQUE: AN ANALYTICAL APPROACH

Suppose we are given a band limited signal of bandwidth  $B$  Hz, which needs to be transmitted over a channel of bandwidth  $B_c$  with Gaussian noise of spectral density  $N_0$  watts per Hz. Let the transmitter have an average power of  $P$  watts. We consider that the signal is sampled at the Nyquist rate of  $2B$  samples per second, to produce a sampled signal  $x(n)$ .

Next, let the signal be quantized to produce a discrete amplitude signal of  $M=2^b$  levels. Where  $b$  is the no. of bits per sample of the digital symbol  $D$ , which is to be encoded. More explicitly, let the values of the  $2^b$  levels be,  $q_1, q_2, q_3, q_4 \dots q_M$  which are distributed over the range  $\alpha[-1, +1]$ , where  $\alpha$  is the proportionality factor determined relative to the signal. Given a sample  $x(n)$  we find the nearest level  $q_{i(n)}$ . Here,  $q_{i(n)}$  is the digital symbol and  $x_a(n) = x(n) - q_{i(n)}$  is the analog representation. The exact representation of the analog signal is given by  $x(n) = q_{i(n)} + x_a(n)$ .

We can accomplish the transmission of this information over the noisy channel by dividing it into two channels: one for analog information and another for digital information. The analog channel bandwidth is  $B_a = \beta_a B$ , and the digital channel bandwidth being  $B_d = \beta_d B$ , where  $B_a + B_d = B_c$ , the channel bandwidth. Let  $\beta = B_c/B$ , be the bandwidth expansion factor, i.e. the ratio of the bandwidth of the channel to the bandwidth of the signal.

Similarly, the variables  $\beta_a$  and  $\beta_d$  are the ratios of  $B_a/B$  and  $B_d/B$ . Here we will assume that  $\beta_a = 1$  so that  $\beta_d = \beta - 1$ . The total power is also divided amongst the two channels with fraction  $p_a$  for the analog channel and fraction  $p_d$  for the digital one, so that  $p_a + p_d = 1$ .

The SNR of the channels is first conveniently defined where no bandwidth expansion is used

$$\gamma = \frac{P}{BN_0} \quad (1)$$

The SNR of the analog channel is given by:

$$SNR_a = \frac{p_a P}{B_a N_0} = \frac{p_a}{\beta_a} \gamma \quad (2)$$

And the SNR of the digital channel is given by:

$$SNR_d = \frac{p_d P}{B_d N_0} = \frac{p_d}{\beta_d} \gamma \quad (3)$$

Of special interest is the case where the signal power is divided in proportion to bandwidth. This is the case where the analog and digital channels have the same spectral density of the transmitted signal. Inferring that in this case:

$$SNR_a = SNR_d = \frac{\gamma}{\beta} \quad (4)$$

The objective of the communication system is to transmit the signal  $x(n)$  as accurately as possible. In other words, we want to design the system so as to maximize, the output SNR of the demodulated signal  $\hat{x}(n)$ , where the output SNR is:

$$SNR_0 = \frac{E\{(\hat{x}(n) - x(n))^2\}}{E\{x^2(n)\}} \quad (5)$$

In the following, we calculate the output SNR as the function of the channel SNR and the bandwidth expansion factor and plot the corresponding graph.

### III. MAXIMUM OUTPUT SNR

Let us consider the best possible SNR that can be obtained by bandwidth expansion, when we wish to transmit a signal of bandwidth  $B$  through a Gaussian channel of bandwidth  $\beta B$ .

It can be derived using Shannon's capacity theorem that the formula for capacity of a Gaussian Channel is given by:

$$C = \beta B \log_2 \left( 1 + \frac{\gamma}{\beta} \right) \quad (6)$$

While at the demodulator end, we have,

$$C = B \log_2 (1 + SNR_0) \quad (7)$$

The two capacities must be equal since both contain the same information. Equating the two yields

$$SNR_0 = \left( 1 + \frac{\gamma}{\beta} \right)^\beta - 1 \quad (8)$$

The figure 2 depicts the Output SNR vs.  $\gamma$  for different bandwidth expansion factors.

### IV. THE IDEAL SCM

Now let us consider the SCM technique with the mixed analog/digital link: Assume for the moment that the digital symbols are transmitted error free. Note: the analog symbol  $x_a(n)$  produced by the SCM process described above, has a smaller variance than the original symbol  $x(n)$ .

Consider the case when  $x(n)$  is a uniformly distributed random variable. Assuming that  $x(n) \in [-\alpha, +\alpha]$ . As there are  $2B$  symbols/sec and  $C$  bits/sec, we have  $b = C/2B$  bits per symbol. Now the analog sample in the range  $[-\alpha, +\alpha]$  is not transmitted in full, instead it is divided into  $M = 2^b$  equal segments and only one segment consists of the analog information. This segment is magnified to the range  $[-\alpha, +\alpha]$  and transmitted with PAM. The  $b$  bits associated with it are transmitted through the digital channel and recovered. The receiver in turn will take the analog signal, shrink it by  $2^b$  times and translate it to its original level.

Analytically, the residual  $x_a(n) \in 2^{-b}[-\alpha, +\alpha]$ . That is to say, the amplitude of  $x_a(n)$  is exactly  $g_a = 2^b$  times smaller than that of the original signal. This means that we can amplify the analog residual by  $g_a$  to give it the same size and power of the original signal.

Here, we can define  $g_a$  as,

$$g_a^2 = \frac{E\{x^2(n)\}}{E\{x_a^2(n)\}} \quad (9)$$

The amplified analog signal  $g_a x_a(n)$  will be transmitted through the analog channel which has a signal to noise ratio  $SNR_a$ . At the receiver, the original signal will be reconstructed by,

$$\hat{x}(n) = \hat{q}_{i(n)} + \hat{x}_a(n) \quad (10)$$

where,  $\hat{x}(n)$  is the estimated analog symbol, and  $\hat{q}_{i(n)}$  is the estimated digital symbol, both of which are assumed to be equal to the transmitted symbol. Because the transmitted analog symbol was  $g_a x_a(n)$ , the received analog symbol will need to be divided by  $g_a$  to produce  $\hat{x}_a(n)$ . This will reduce channel noise by  $g_a$  and consequently improve the SNR experienced by the analog symbol by the factor of  $g_a^2$ .

We could conclude that in general,

$$SNR_0 = g_a^2 SNR_a \quad (11)$$

And therefore

$$SNR_0 = g_a^2 \frac{P_a \gamma}{\beta_a} \quad (12)$$

The analog gain of a uniformly distributed input is  $g_a=2^b$ , and therefore

$$SNR_0 = 2^{2b} \frac{P_a \gamma}{\beta_a} \quad (13)$$

Here we can note that, different distributions of the analog signal lead to different analog gain factors. The gain is largest for a uniformly distributed input and becomes smaller as the distribution approaches a Gaussian distribution.

## V. PROSPECTIVE APPLICATIONS

### A. Broadband Wireless Transmission

An SCM-based communications link is basically a transparent, band-limited analog pipe with near-ideal performance in noisy channels. Every analog signal could potentially use SCM because it can outperform other existing modulation schemes. However, SCM has a compelling advantage for digital communications applications as well.

For example, SCM can pass digital information by acting as a repeater of a digital channel. This application provides a wireless extension of cable modem digital information. As illustrated in Figure 8, a cable modem termination system (CMTS) transmits a 42 Mb/s 256-QAM signal in a 6 MHz cable channel shared among the cable modems located at the subscribers' premises. The return upstream path from the cable modems is a 10 Mb/s 16-SQAM signal in a 3.2 MHz cable channel. The signals are carried by a combination of fiber and coax referred to as a hybrid fiber/coax (HFC) network.

The fiber delivers a large amount of bandwidth over long distances with strong noise immunity. Coax cables distribute the signal between the fiber and each subscriber. To reach a station located beyond the reach of the existing HFC network, the cable operator installs an SCM-based point-to-multipoint wireless access system at any point on the HFC network that has line-of-sight to the unreachable station. All customers located at a particular site share the SCM radio located at that site. The subscribers simply use low-cost cable modems that connect to the SCM radio via a shared coax cable. The wireless subscribers can even share the same cable channels with purely wired

subscribers because the wireless link is transparent to the cable equipment. The significance of SCM in this application is its ability to take a 256-QAM signal and transport it over a wireless link suitable only for a lower modulation scheme, such as 16-QAM. SCM provides significant additional noise immunity, as is depicted in Figure 2 because it uses bandwidth expansion to improve the destination SNR.

There is a non-SCM alternative: the 256-QAM signal could first be demodulated back to the original data bits, then modulated as 16-QAM, transmitted over the wireless link, demodulated at the destination, and finally remodulated using 256-QAM. This alternative would be much more costly, given the amount of processing required. It would also add significant latency to the information transported because an efficient channel must perform the error correction of the original signal before transmitting it over the wireless link.

Furthermore, because SCM provides a transparent link that is not sensitive to protocol evolution or variations, it is more future-proof and versatile than specific digital standards.

### B. Superior Digital Audio Recording and Playback

A new-generation audio CD could include a digital track identical to and compatible with the existing CD tracks, and in addition, have an analog track to provide the enhanced quantization error. Such an analog track would provide audio performance that depends on the quality of the recording and of the disc player. The most discriminating audio enthusiasts could use the more sophisticated player for true analog reproduction, while the less discriminating users would enjoy the low-cost CD technology in its current format.

## VI. CONCLUSION

SCM or signal code modulation is a technique, a new method to eliminate the inherent quantization noise. Digital signals are robust and immune to external noise but the quantization technique itself produces the quantization noise from which the original signal cannot be recovered anymore. SCM can be used to deal with the problem and to remove it. Various aspects of improve the performance have also been discussed here.

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