

Application of Extended Kalman filter and Modified Gain Extended Kalman Filter in Underwater Active Target Tracking

(Tracking from a stationary observer)

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Abstract: For underwater measurements, the noise is very high, turning rate and speed of the platform is low when compared with the missiles in air. In the present paper, Adaptive filters, Extended Kalman filter (EKF) and Modified Gain Extended Kalman Filter (MGEKF), are applied to underwater active target tracking using angles only measurements. Monte-Carlo simulated results for several typical scenarios with Range, Speed and Coarse Errors in tracking using MATLAB software are presented. From the results it is concluded that this MGEKF algorithm is suitable for underwater active target tracking applications.

Keywords: EKF,MGEKF,turning rate.

I. INTRODUCTION

An observer, in underwater environment, monitors noisy SONAR bearings and elevations from a radiating target, these raw bearing and elevation measurements are used to get range, course, bearing, elevation and speed of the target. Input bearing and elevation measurements are nonlinear, making whole process nonlinear. The modified gain extended kalman filter (MGEKF) developed by Song and Speyer [1], was the successful contribution for angles only passive target tracking applications in air. This MGEKF algorithm was further improved by P.J. Galkowski and M.A. Eslam [2]. In this paper, this improved MGEKF and Extended Kalman Filter (EKF) algorithms are explored for underwater applications, specifically in Active target tracking, when observer is observing from stationary location. As the noise in the measurements is very high, turning rate of the platforms is low and speed of the platforms is also low when compared with the missiles in air, performance of the EKF and MGEKF for such type of environment can be observed here. Next sections will deal with mathematical modeling of Range, Bearing and Elevation measurements, implementation of the EKF and MGEKF Filter for several typical scenarios and results obtained in simulation are presented.

II. MATHEMETICAL MODELLING

Let observer be at the origin and initially the target be at point P, as shown in Fig -1.

The initial observation parameters Range, bearing and elevation are obtained are noise corrupted.

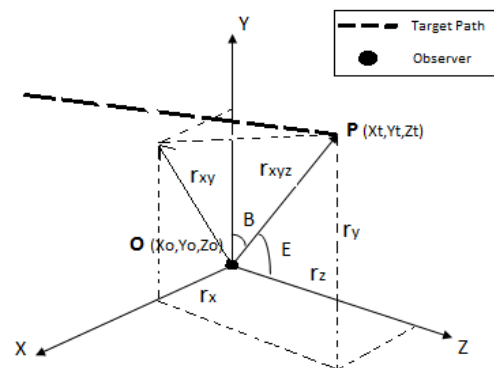


Fig.1. A typical target observer geometry

The measurement vector, Z, is written as

$$z = \begin{bmatrix} R_m \\ B_m \\ \phi_m \end{bmatrix} = \begin{bmatrix} r + \sigma_r \\ \tan^{-1} \frac{r_x}{r_y} + \sigma_B \\ \tan^{-1} \frac{r_{xy}}{r_z} + \sigma_\phi \end{bmatrix} \quad (1)$$

Where σ_r , σ_B and σ_ϕ are zero mean, uncorrelated normally distributed errors in the Range (R_m), bearing (B_m) and elevation (ϕ_m) measurements respectively.

Let the state vector be

$$X_s = \begin{bmatrix} \dot{x} & \dot{y} & \dot{z} & r_x & r_y & r_z \end{bmatrix}^T \quad (2)$$

Where $\dot{x}, \dot{y}, \dot{z}$ and r_x, r_y, r_z are velocities and range measurements of target in x, y and z directions respectively.

The measurement matrix H is given by

$$H = \begin{bmatrix} 0 & 0 & 0 & \frac{\hat{r}_x}{\hat{r}} & \frac{\hat{r}_y}{\hat{r}} & \frac{\hat{r}_z}{\hat{r}} \\ 0 & 0 & 0 & \cos \hat{B} & -\sin \hat{B} & 0 \\ 0 & 0 & 0 & \frac{\hat{r}_{xy} \sin \hat{B}}{\hat{r}} & \frac{\hat{r}_{xy} \cos \hat{B}}{\hat{r}} & \frac{\sin \hat{\phi}}{\hat{r}} \end{bmatrix} \quad (3)$$

III. EXTENDED KALMAN FILTER

The following table explains the structure of Continuous-Discrete Extended Kalman Filter [5]

TABLE-1

Model	$\dot{\hat{x}}(t) = f(x(t), u(t), t) + G(t)w(t), w(t) \sim N(0, Q(t))$ $\hat{y}_k = h(\hat{x}_k) + v_k, v_k \sim N(0, R_k)$
Initialize	$\hat{x}(t_0) = \hat{x}_0$ $P_0 = E\{\hat{x}(t_0)\hat{x}^T(t_0)\}$
Gain	$K_k = P_k^- H_k^T (\hat{x}_k^-) [H_k(\hat{x}_k^-) P_k^- H_k^T (\hat{x}_k^-) + R_k]^{-1}$ $H_k(\hat{x}_k^-) \equiv \frac{\partial h}{\partial x} \Big _{\hat{x}_k^-}$
Update	$\hat{x}_k^+ = \hat{x}_k^- + K_k [\hat{y}_k - h(\hat{x}_k^-)]$ $P_k^+ = [I - K_k H_k(\hat{x}_k^-)] P_k^-$
Propagation	$\dot{\hat{x}}(t) = f(\hat{x}(t), u(t), t)$ $\dot{P}(t) = F(\hat{x}(t), t) P(t) + P(t) F^T(\hat{x}(t), t) + G(t) Q(t) G^T(t)$ $F(\hat{x}(t), t) \equiv \frac{\partial f}{\partial x} \Big _{\hat{x}(t)}$

IV. MODIFIED GAIN EXTENDED KALMAN FILTER

The above mentioned improved algorithm is implemented using MGEKF.

Song and Speyer define a modified gain extended Kalman filter (MGEKF) by the following set of equations [2]:

Time update (Prediction equations)

$$\hat{X}_{i+1}^- = \phi_i \hat{X}_i^+ \quad (4)$$

$$P_{i+1}^- = \phi_i P_i^+ \phi_i^T + Q_i \quad (5)$$

Measurements update (correction)

$$k_{i+1} = P_{i+1}^- h_{i+1} (h_{i+1} P_{i+1}^- h_{i+1} + R)^{-1} \quad (6)$$

$$\hat{X}_{i+1}^+ = \hat{X}_{i+1}^- + k_{i+1} (z_{i+1} - h_{i+1}(\hat{X}_{i+1}^-)) \quad (7)$$

$$P_{i+1}^+ = (I - k_{i+1} g(z_{i+1}, \hat{X}_{i+1}^-)) P_{i+1}^- \cdot (I - k_{i+1} g(z_{i+1}, \hat{X}_{i+1}^-))^T + (k_{i+1} R k_{i+1}^T) \quad (8)$$

Where

ϕ_i = transition matrix at time i.

\hat{X}_i^- = state estimate at time i before update.

\hat{X}_i^+ = state estimate at time i after update.

k_i = filter gain at time i.

z_i = measurement at time i.

$h_i(\hat{X}_i^-)$ = predicted measurement based on state estimate at time i before update.

P_i^- = state covariance matrix at time i before update.

P_i^+ = state covariance matrix at time i after update.

Q_i = process noise matrix at time i.

R = measurement noise covariance matrix.

$$h_i = \frac{\partial h_i(X_i)}{\partial X_i} \Big|_{X_i = \hat{X}_i^-}$$

$g(z_i, \hat{X}_i^-)$ is a function such that

$$h_i(X) - h_i(\hat{X}_i^-) = g(z_i, \hat{X}_i^-)(X_i - \hat{X}_i^-)$$

I = Identity Matrix.

Calculation to get Modified gain parameter 'g':

Horizontal plane and bearing measurements is given by [4]

$$\sin(B - \hat{B}) = \frac{\cos B(r_x - \hat{r}_x) - \sin B(r_y - \hat{r}_y)}{\hat{r}_{xy}} \quad (9)$$

Elevation measurement is given by [4]

$$\sin(\phi - \hat{\phi}) = \frac{\cos \phi}{\hat{r}} \left[\frac{(r_x - \hat{r}_x) \sin \frac{(B - \hat{B})}{2} + (r_y - \hat{r}_y) \cos \frac{(B + \hat{B})}{2}}{\cos \frac{(B - \hat{B})}{2}} \right] - \frac{\sin \phi}{\hat{r}} (r_z - \hat{r}_z)$$

(10)

Using (4) and (5) from [4] And from range error for modified gain we will get modified gain as follows

$$\begin{bmatrix} \hat{r} - r \\ \hat{B} - B \\ \hat{\phi} - \phi \end{bmatrix} = \begin{bmatrix} \frac{x_i + \hat{r}_x}{2(r_x + r_y + r_z)} & \frac{y_i + \hat{r}_y}{2(r_x + r_y + r_z)} & \frac{z_i + \hat{r}_z}{2(r_x + r_y + r_z)} \\ \frac{\cos B_m}{\hat{r}_{xy} = \hat{r}_s \sin \phi_m} & \frac{-\sin B_m}{\hat{r}_{xy} = \hat{r}_s \sin \phi_m} & 0 \\ \cos \phi \sin \left(\frac{B_m + \hat{B}}{2} \right) & \cos \phi \cos \left(\frac{B_m + \hat{B}}{2} \right) & -\sin \phi \\ \hat{r} \cos \left(\frac{B_m - \hat{B}}{2} \right) & \hat{r} \cos \left(\frac{B_m - \hat{B}}{2} \right) & \hat{r} \end{bmatrix}$$

$$g = \begin{bmatrix} \frac{x_i + \hat{r}_x}{2(r_x + r_y + r_z)} & \frac{y_i + \hat{r}_y}{2(r_x + r_y + r_z)} & \frac{z_i + \hat{r}_z}{2(r_x + r_y + r_z)} \\ \frac{\cos B_m}{\hat{r}_{xy} = \hat{r}_s \sin \phi_m} & \frac{-\sin B_m}{\hat{r}_{xy} = \hat{r}_s \sin \phi_m} & 0 \\ \cos \phi \sin \left(\frac{B_m + \hat{B}}{2} \right) & \cos \phi \cos \left(\frac{B_m + \hat{B}}{2} \right) & -\sin \phi \\ \hat{r} \cos \left(\frac{B_m - \hat{B}}{2} \right) & \hat{r} \cos \left(\frac{B_m - \hat{B}}{2} \right) & \hat{r} \end{bmatrix}$$

Considering $\dot{x}, \dot{y}, \dot{z}$ also (velocities in three directions),

Modified gain 'g' is given by

$$* \begin{bmatrix} \hat{r}_x - r_x \\ \hat{r}_y - r_y \\ \hat{r}_z - r_z \end{bmatrix} \quad (11)$$

As true bearing and range are not available, it is replaced by measured bearing and range respectively in (11) and obtained (12) as follows.

$$\begin{bmatrix} \hat{r} - r \\ \hat{B} - B \\ \hat{\phi} - \phi \end{bmatrix} = \begin{bmatrix} \frac{x_i + \hat{r}_x}{2(r_x + r_y + r_z)} & \frac{y_i + \hat{r}_y}{2(r_x + r_y + r_z)} & \frac{z_i + \hat{r}_z}{2(r_x + r_y + r_z)} \\ \frac{\cos B_m}{\hat{r}_{xy} = \hat{r}_s \sin \phi_m} & \frac{-\sin B_m}{\hat{r}_{xy} = \hat{r}_s \sin \phi_m} & 0 \\ \cos \phi \sin \left(\frac{B_m + \hat{B}}{2} \right) & \cos \phi \cos \left(\frac{B_m + \hat{B}}{2} \right) & -\sin \phi \\ \hat{r} \cos \left(\frac{B_m - \hat{B}}{2} \right) & \hat{r} \cos \left(\frac{B_m - \hat{B}}{2} \right) & \hat{r} \end{bmatrix}$$

$$* \begin{bmatrix} \hat{r}_x - r_x \\ \hat{r}_y - r_y \\ \hat{r}_z - r_z \end{bmatrix} \quad (12)$$

$$= g * \begin{bmatrix} \hat{r}_x - r_x \\ \hat{r}_y - r_y \\ \hat{r}_z - r_z \end{bmatrix} \quad (13)$$

Therefore g is given by

$$g = \begin{bmatrix} 0 & 0 & 0 & \frac{x_i + \hat{r}_x}{2(r_x + r_y + r_z)} & \frac{y_i + \hat{r}_y}{2(r_x + r_y + r_z)} & \frac{z_i + \hat{r}_z}{2(r_x + r_y + r_z)} \\ 0 & 0 & 0 & \frac{\cos B_m}{\hat{r}_{xy} = \hat{r}_s \sin \phi_m} & \frac{-\sin B_m}{\hat{r}_{xy} = \hat{r}_s \sin \phi_m} & 0 \\ 0 & 0 & 0 & \cos \phi \sin \left(\frac{B_m + \hat{B}}{2} \right) & \cos \phi \cos \left(\frac{B_m + \hat{B}}{2} \right) & -\sin \phi \\ 0 & 0 & 0 & \hat{r} \cos \left(\frac{B_m - \hat{B}}{2} \right) & \hat{r} \cos \left(\frac{B_m - \hat{B}}{2} \right) & \hat{r} \end{bmatrix}$$

(14)

V. IMPLEMENTATION OF THE ALGORITHMS

EKF and MGEKF for underwater active target tracking as follows. As only range, bearing and elevation measurements are available, the velocity components of the target are assumed to be each 10 m/sec which is very close to the realistic speed of the vehicles in underwater (scenario is given in table 1). The range of the day, say 15000 meters, is utilized in the calculation of initial position estimate of the target is as

$$X(0|0) = [10, 10, 10, 15000 \cdot \sin B_m(0) \cdot \sin \phi_m(0), 15000 \cdot \sin \phi_m(0) \cdot \cos B_m(0), 15000 \cdot \cos \phi_m(0)]^T$$

Where $B_m(0)$ and $\phi_m(0)$ are initial bearing and elevation measurements.

VI. SIMULATION RESULTS

All raw Range, bearings and elevation measurements are corrupted by additive zero mean Gaussian noise with a r.m.s level of 0.3 degree. Corresponding to a tactical scenario is given in table-2

TABLE -2

Parameters	Scenario-1	Scenario-2	Scenario-3
Initial Range (m)	20000	20000	2000
Initial Bearing (deg)	0.5	0.5	0.5
Initial elevation (deg)	45	45	45
RMS error in bearing and elevation (deg)	0.33	0.33	5
Target Speed (m/s)	10	20.6	10
Target Course (deg)	140	180	140

Above scenario is applied, when observer is stationary.

Scenario-1:

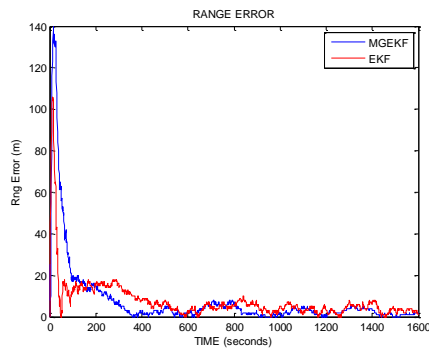


Fig.2. Range error

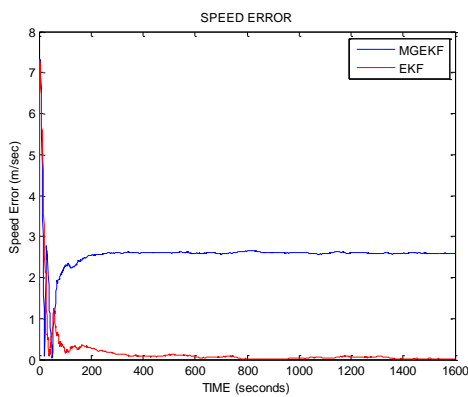


Fig.3. speed error

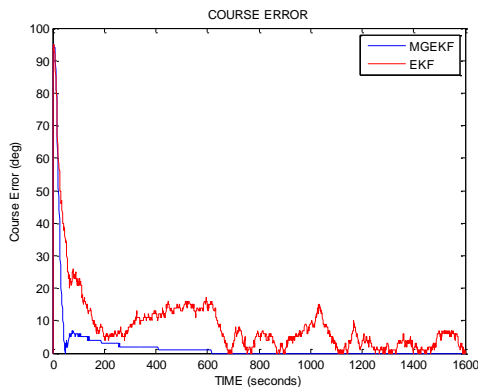


Fig.4. course error

From the above results the required accuracy obtained for stationary observer is, 50 seconds onwards for both EKF and MGEKF, in range, velocity and course measurements. However MGEKF providing greater accuracy than EKF.As for the speed in MGEKF, it is almost 20%of its true speed, which is acceptable in underwater scenario. So this MGEKF algorithm seems to be very much useful for underwater active target tracking when observer is stationary, for a moving target.

Scenario-2:

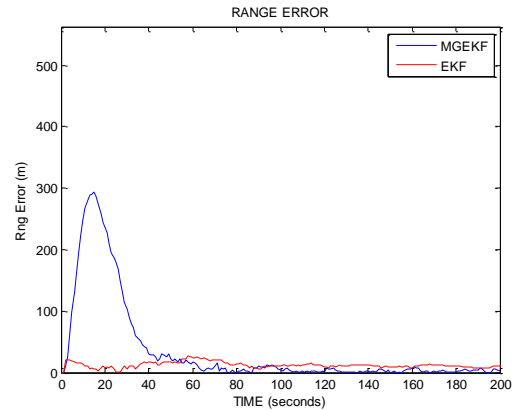


Fig.5. Range error

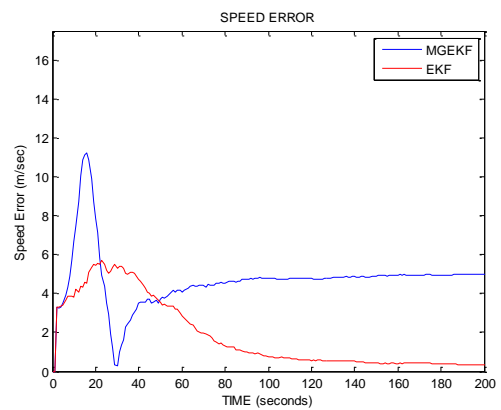


Fig.6. speed error

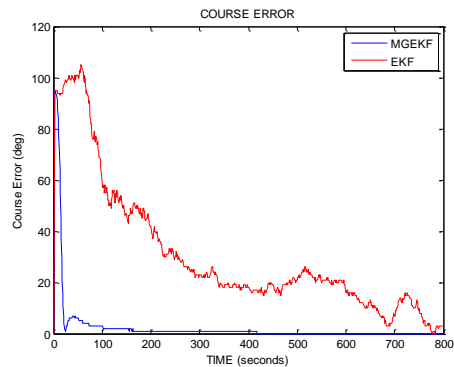


Fig.7. course error

This scenario is when target is attained its practical maximum speed in underwater, From the above results the required accuracy is 30 seconds onwards for MGEKF, where as for EKF, even range accuracy comes earlier but speed and course will take more time to get minimized error compared to MGEKF.

Scenario -3:

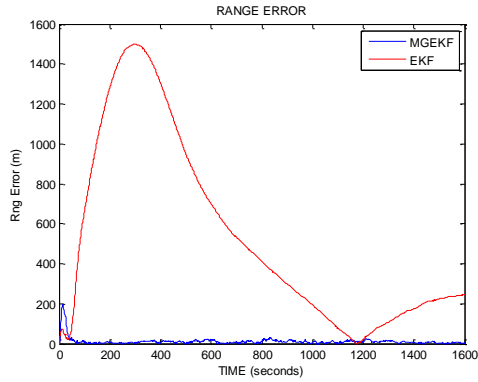


Fig.8. Range error

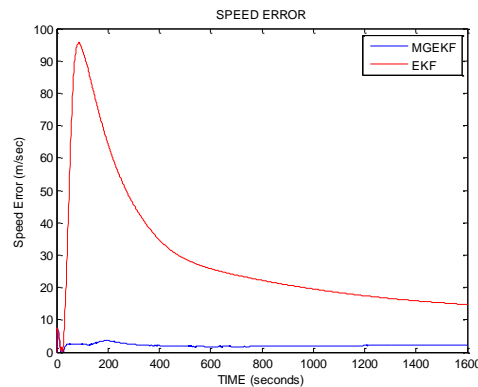


Fig.9. speed error

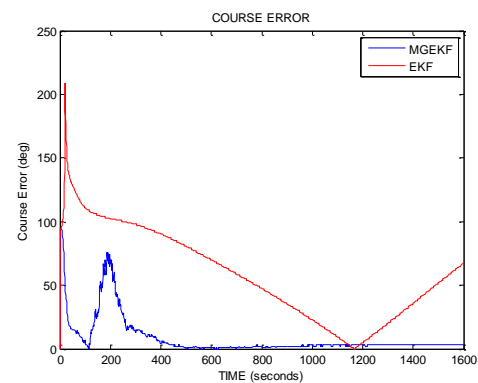


Fig.10. course error

In this scenario RMS error is increased to 5 degrees. From the results EKF never converges, whereas MGEKF is converged for active target tracking and required accuracy will be 100 seconds onwards due to course error will take time to minimize to zero.

VII. CONCLUSION

The performance of the EKF and MGEKF algorithms are presented for a normal scenario and it is observed that the solution with required accuracy is obtained approximately at 50 seconds (less than a minute) for stationary observer, and from other two scenarios, The monte-carlo simulation results confirm that MGEKF is suitable for Active target tracking in underwater compared to EKF.

REFERENCES

- [1]. T. L. Song and J. L. Speyer, "A stochastic analysis of a modified gain extended Kalman filter with applications to estimation with bearings only measurements", *IEEE Trans. Autom. Control*, Vol.AC-30, No. 10, pp 940-949, Oct. 1985.
- [2]. P. J. Galkowski and M. A. Islam, "An alternative derivation of the modified gain function of Song and Speyer", *IEEE Trans. Autom. Control*, Vol. AC-36, No. 11, pp 1323-1326, Nov. 1991.
- [3]. S.KoteswaraRao, "Modified gain extended Kalman filter with application to bearings-only passive manoeuvring target tracking" *IEE Proc.-Radar Sonar Navig.*, Vol. 152, No. 4, August 2005
- [4]. S.KoteswaraRao, "Modified gain extended Kalman filter with application to Angles only Underwater Passive Target Tracking" *Proceedings of ICSP '98*, pp 1439-1442.
- [5]. Yuan Huang and Taek Lyul Song, "Iterated Modified Gain Extended Kalman Filter with Applications to Bearings Only Tracking" *Journal of Automation and Control Engineering* Vol. 3, No. 6, December 2014.