

Multi-scale Block Compressed Sensing image Reconstruction using Smoothed Projected Landweber

C. Manohar¹, S. Swarnalatha²

M.Tech Student, Department of E.C.E, S.V.U College of Engineering, Tirupati, Andhra Pradesh, India¹.

Associate Professor, Department of E.C.E, S.V.U College of Engineering, Tirupati, Andhra Pradesh, India²

Abstract: Compressed sensing is new technique for an efficient data acquisition. In this paper, we proposed, a multi-scale variant of block compressed sensing of images coupled with Smoothed Projected Landweber Reconstruction. In essence, block-based compressed sampling is deployed independently with each subband of each decomposition level of a wavelet transform of an image. The corresponding multi-scale reconstruction interleaves Landweber steps on the individual blocks with a smoothing filter in the spatial domain of the image and thresholding within a sparsity transform. Experimental results shows that the proposed multi-scale reconstruction outperform over original block compressed sensing with Smoothed Projected Landweber.

Keywords: Compressed sensing, bivariate shrinkage, smoothing filter.

I. INTRODUCTION

Compressed sensing has emerged as a new framework for signal acquisition and sensor design that enables a potentially large reduction in the sampling and computation costs for sensing signals that have a sparse or compressible representation. While the Nyquist–Shannon sampling theorem states that a certain minimum number of samples is required in order to perfectly capture an arbitrary band limited signal, when the signal is sparse in a known basis we can vastly reduce the number of measurements that need to be stored. Consequently, when sensing sparse signals we might be able to do better than suggested by classical results. This is the fundamental idea behind CS: rather than first sampling at a high rate and then compressing the sampled data, we would like to find ways to *directly* sense the data in a compressed form – i.e., at a lower sampling rate. The field of CS grew out of the work of Candès, Romberg, and Tao and of Donoho, who showed that a finite-dimensional signal having a sparse or compressible representation can be recovered from a small set of linear, non-adaptive measurements [1–2]. There has been significant interest in the paradigm of compressed sensing for the sampling and reconstruction of image data. One of the primary challenges for compressed sensing on image data is the large computational cost associated with reconstruction for multidimensional signals. One prominent technique to mitigating such computational burdens is to limit CS sampling to relatively small blocks (e.g., [3, 4]). Block based CS image reconstruction with smoothed Projected Landweber algorithm (BCS-SPL) [4] deployed in the domain of discrete wavelet transform (DWT), typically provide much faster reconstruction than techniques based on full-image CS sampling.

In this paper, we proposed a multi-scale algorithm that deploys existed block based CS image reconstruction [4] in the domain of a wavelet transform. In detail, block-

based compressed sampling is deployed independently with each subband of each decomposition level of a wavelet transform of an image. The corresponding multi-scale reconstruction interleaves iterative thresholding on the individual blocks with a smoothing filter. Experimental results for image demonstrate that this proposed multi-scale reconstructions usually provide significant gain in reconstruction quality over existed algorithm.

II. BACKGROUND

Suppose we want to recover real-valued signal $x \in \mathbb{R}^N$ from M measurements such that $M \ll N$; i.e., $y = \Phi x$, where $y \in \mathbb{R}^M$, and Φ is a $M \times N$ measurement matrix with sampling rate, being $S = M/N$. Because the number of unknowns is much larger than the number of observations, recovery every x from its corresponding M measurements is impossible in general; however CS theory holds that, if x is sufficiently sparse in some domain Ψ then exact recovery of x is recoverable from y by the optimization.

$$\hat{x} = \Psi x \rightarrow (1)$$

$$\hat{x} = \arg \min_x \|\hat{x}\|_1, \text{ such that } y = \Phi \Psi^{-1} \hat{x} \rightarrow (2)$$

Where the measurement matrix Φ is a random matrix; here, we further assume that Φ is orthonormal such that $\Phi \Phi^T = I$ and Ψ^{-1} is the inverse transform.

Recently CS reconstruction techniques based on projections have been proposed [5]. Algorithms of this class form \hat{x} by successively projecting and thresholding: for example, the reconstruction in [5] starts from some initial approximation $\hat{x}^{(0)}$ and forms the approximation at iteration $i + 1$ as

$$\tilde{x}^{(i)} = \check{x}^{(i)} + \frac{1}{\gamma} \Psi \Phi^T (y - \Phi \Psi^{-1} \check{x}^{(i)}) \rightarrow (3)$$

$$\check{x}^{(i)} = \begin{cases} \tilde{x}^{(i)} & |\tilde{x}^{(i)}| \geq \tau^{(i)}, \\ 0 & \text{else.} \end{cases} \rightarrow (4)$$

Here γ is a scaling factor ([5] uses the largest eigenvalue of $\Phi^T \Phi$) while $\tau^{(i)}$ is a threshold set appropriately at each iteration. It is straightforward to see that this procedure is like a Projected Landweber (PL) algorithm [6]. The next section explores Block based CS and wiener filtering into the Projected Landweber to search for compressed sensing reconstruction of image.

III. BLOCK BASED CS WITH SMOOTHED PL RECONSTRUCTION

In [3] compressed sensing of 2D images was proposed. In this scheme, the sampling of image using random matrices applied on block by block basis while the recovery of image based on the PL reconstruction of (3)-(4) that incorporates a smoothing operation. The overall technique was called BCS-SPL in [4]

A. Block based CS sampling

In BCS, an image is partitioned into smaller blocks while sampling is applied on block- by-block basis. In such BCS, the global measurement matrix takes a block-diagonal structure,

$$\Phi = \begin{bmatrix} \Phi_B & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \Phi_B \end{bmatrix} \rightarrow (5)$$

wherein Φ_B independently samples blocks within the image. That is $y = \Phi_B x_i$; where x_i , a column vector with length B^2 representing block i of the image, and Φ_B is a $M_B \times B^2$ measurement matrix such that the subrate of BCS is $S = M_B/B^2$. Using block based CS sampling rather than random sampling applied to entire image x has several advantages [3]. First, the sampling operator Φ_B conveniently stored and employed because of its small size. Second, the encoder does not need to wait until the whole image is measured, but may send each block after its linear random projection. Last, an initial approximation x^0 with MMSE can be easily calculated due to compact size of Φ_B [3].

B. Smoothed Projected Landweber variant

The recovery of image based on the variant PL reconstruction that incorporates a smoothing. This operation imposes smoothness, in addition to the sparsity inherent to PL. Here wiener filter used for smoothing in order to remove blocking artifacts. A smoothing filter step Was interleaved with the PL projection of (3)-(4); thus, the approximation to the image at iteration $i + 1$, $x^{(i+1)}$, is produce from $x^{(i)}$ as:

$$\text{function } x^{(i+1)} = \text{SPL}(x^{(i)}, y, \Phi_B, \Psi, \lambda)$$

$$\hat{x}^{(i)} = \text{wiener}(x^{(i)})$$

for each block j

$$\hat{x}_j^{(i)} = \hat{x}_j^{(i)} + \Phi_B^T (y - \Phi_B \hat{x}_j^{(i)})$$

$$\tilde{x}^{(i)} = \Psi \hat{x}_j^{(i)}$$

$$\check{x}^{(i)} = \text{Threshold}(\tilde{x}^{(i)}, \lambda)$$

$$\bar{x}^{(i)} = \Psi^{-1} \check{x}^{(i)}$$

for each block j

$$x_j^{(i+1)} = \bar{x}_j^{(i)} + \Phi_B^T (y - \Phi_B \bar{x}_j^{(i)})$$

Here, $\text{wiener}(\cdot)$ lowpass-filters a grayscale image that has been degraded by constant power additive noise. This filter uses a pixelwise adaptive Wiener method on statistics estimated from a local neighbourhood of each pixel. And $\text{Threshold}(\cdot)$ is thresholding method as discussed below. In our use of SPL reconstruction, we initialize with $x^{(0)} = \Phi^T y$ and terminate when $|D^{(i+1)} - D^{(i)}| < 10^{-4}$ where $D^{(i)} = x^{(i)} - x^{(i+1)}$

C. Thresholding

As originally described in [3], $\text{SPL}(\cdot)$ used hard thresholding in the form of (4). To set a proper τ for hard thresholding, we employ the universal threshold method of [7]. Specifically, in (4),

$$\tau^{(i)} = \lambda \sigma^{(i)} \sqrt{2 \log K} \rightarrow (6)$$

where λ convergence control factor, K is the number of transform coefficients, and noise variance $\sigma^{(i)}$ is estimated using a robust median estimator,

$$\sigma^{(i)} = \frac{\text{median}(|\tilde{x}^{(i)}|)}{0.6745} \rightarrow (7)$$

hard thresholding cannot model statistical dependencies between wavelet coefficients. However, In [8], a non-Gaussian bivariate (having 2 variables) distribution was proposed for wavelet coefficients of natural images in order to characterize the dependencies between a current coefficient and its parent based on an empirical joint histogram of DWT coefficients. The corresponding bivariate shrinkage functions are derived from them using Bayesian estimation, in particular, the MAP estimator. Let ξ is specific transform coefficient its parent coefficient is ξ_p .

$$\text{Threshold}(\xi, \lambda) = \frac{\left(\sqrt{\xi^2 + \xi_p^2} - \lambda \frac{\sqrt{3\sigma^{(i)}}}{\sigma_\xi} \right)_+}{\sqrt{\xi^2 + \xi_p^2}} \cdot \xi \rightarrow (8)$$

Where $(g)_+ = 0$ for $g < 0$, $(g)_+ = g$ else; $\sigma^{(i)}$ is the median estimator of (7) and again, λ is a constant control factor. Here, σ_ξ is the marginal standard deviation of coefficient ξ estimated in a local 3×3 neighborhood surrounding ξ .

IV. MULTI-SCALE BLOCK BASED CS WITH SMOOTHED PL RECONSTRUCTION

A. Multi-scale Block based CS sampling

The measurement operator Φ for multi-scale BCS is split into two components-a multi-scale transform Ω (DWT) and a multi-scale block based sampling measurement process Φ' such that $\Phi = \Phi' \Omega$, then we have

$$y = \Phi' \Omega x \rightarrow (9)$$

Assume that Ω produces L levels of wavelet decomposition, thus Φ' consists of L different block based sampling operators, one for each level. That is let the Discrete wavelet transform of image x be

$$\tilde{x} = \Omega x \rightarrow (10)$$

subband s at level l of \tilde{x} is then divided into $B_l \times B_l$ blocks and measure using an appropriately sized Φ_l (here, $l = L$ is the highest resolution level). That is, suppose $\tilde{x}_{l,s,j}$ is a vector representing, in raster scan fashion, block j of subband s at level l , such that $1 \leq l \leq L$. Then,

$$y_{l,s,j} = \Phi_l \tilde{x}_{l,s,j} \rightarrow (11)$$

Since the different levels of wavelet decomposition have different importance to the final image reconstruction quality, here we adjust the sampling process so as to yield a different subrate, S_l at each level l . In all cases, we set the subrate of the wavelet subband to full measurement ($S_0 = 1$). Then, we let the subrate for level l be

$$S_l = W_l S' \rightarrow (12)$$

such that the overall subrate becomes

$$S = \frac{1}{4^L} S_0 \sum_{l=1}^L \frac{3}{4^{L-l+1}} W_l S' \rightarrow (13)$$

TABLE I

Wavelet domain block based CS subrates S_l at level l for target overall subrate S for DWT with $L = 3$ levels. In all cases, the subband is given full measurement ($S_0 = 1$).

Level subrate S_l			
S	S_1	S_2	S_3
0.1	1.0000	0.1600	0.0100
0.2	1.0000	0.5867	0.0367
0.3	1.0000	1.0000	0.0667
0.4	1.0000	1.0000	0.2000
0.5	1.0000	1.0000	0.3333

Given a target subrate S and a set of level weights W_l , one can easily solve (13) for S' and then we get the set of level subrates S_l via (12). However this process will produce one or more $S_l > 1$. Thus, we modify the solution to enforce $S_l \leq 1$ for all l . Specifically, after finding S' and S_1 via (13) and (12), we check if $S_1 > 1$.

If so, we set $S_1 = 1$, remove its corresponding term from the sum in (13), and then we solve

$$S = \frac{1}{4^L} S_0 + \frac{3}{4^L} S_1 \sum_{l=2}^L \frac{3}{4^{L-l+1}} W_l S' \rightarrow (14)$$

for S' , again using (12) to recalculate S_l for $l = 1, 2, \dots, L$. We repeat this process as needed to ensure that all $S_l \leq 1$. Here, we use level weights as,

$$W_l = 16^{L-l+1} \rightarrow (15)$$

The resulting level subrates S_l for various target subrates S for $L = 3$ levels are shown in Table I.

B. Wavelet-Domain Multi-scale reconstruction

The block based CS reconstruction algorithm couples a full-image Wiener-filter smoothing process with a sparsity enhancing thresholding process in the domain of sparsity transform Ψ . Interleaved between the smoothing and thresholding operations lie Landweber steps in the form of

$$x \leftarrow x + \Phi^T (y - \Phi x), \rightarrow (16)$$

where Φ is measurement matrix. Here we modified BCS reconstruction to accommodate the situation in which CS sampling take place within in multi-scale transform Ω as in (9). In essence, the resulting proposed multi-scale reconstruction applies a Landweber step on each block of each subband in each decomposition level separately using the appropriate block based Φ_l for the current level l .

V. RESULTS

We now evaluate the performance of the BCS-SPL and the proposed Multi-Scale reconstructions described above on a number of grayscale images of size 512×512 (see Fig. 1). Here, we use dual tree DWT [9] for multi-scale whereas original BCS-SPL uses DWT as the sparsity transform Ψ with bivariate shrinkage [8] applied within the wavelet domain to enforce sparsity. Multi-scale BCS uses a 3-level DWT with the popular 9/7 biorthogonal wavelets as the sampling domain transform Ω . At decomposition level l of Ω , blocks of size $B_l \times B_l$ are individually sampled in the DWT domain using the scrambled block discrete cosine transform (DCT) sampling operator of [10]; we use block of sizes $B_l = 16, 32$ and 64 for decomposition level $l = 1, 2$, and 3 , respectively ($l = 3$ is the highest resolution level). On the other hand BCS uses $B \times B$ block based sampling applied directly on the image data in its ambient domain; here $B = 32$.



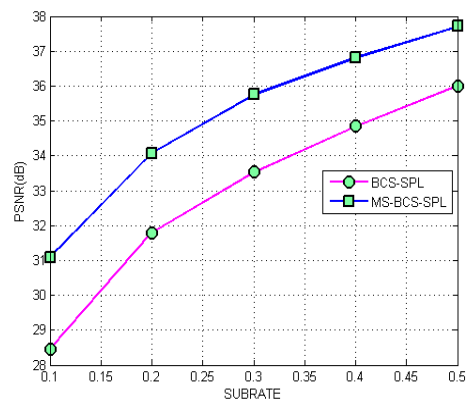
Fig. 1. The 512×512 grayscale still images used in the experiments. First row (left to right): Leena, Barbara; second row (left to right): cameraman, peppers.

The reconstruction performance of the two algorithms under consideration is presented in Table II. In most cases, the wavelet-domain measurement and multi-scale reconstructions provides a significant gain in reconstruction quality over the spatial domain measurement of BCS-SPL generally on the order of a 1- to 3-dB increase in PSNR metric. The proposed multi scale reconstruction in wavelet domain Provides significantly

superior reconstruction over original BCS-SPL presented in Fig. 2. A visual comparison of BCS-SPL and proposed method for $S=0.1$ (10%) for “peppers” image is shown in Fig. 3.

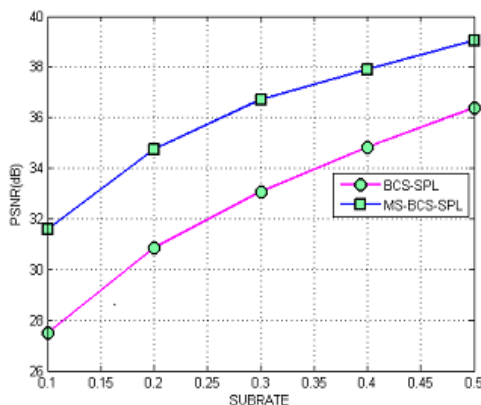
TABLE II: Image reconstruction PSNR (dB)

		Subrate				
		0.1	0.2	0.3	0.4	0.5
Leena						
BCS-SPL	Original	27.4852	30.8597	33.0795	34.8265	36.3958
	MS	31.5803	34.7452	36.7059	37.8948	39.0376
Barbara						
BCS-SPL	Original	22.1681	23.4436	24.8224	26.2487	27.864
	MS	23.9127	25.1443	26.0663	27.2872	28.8578
Cameraman						
BCS-SPL	Original	25.4501	29.928	33.1538	35.8589	38.1891
	MS	31.2834	36.8677	40.1676	43.1069	45.137
Peppers						
BCS-SPL	Original	28.45	31.8011	33.5358	34.8438	36.0099
	MS	31.0956	34.081	35.76	36.8127	37.7086

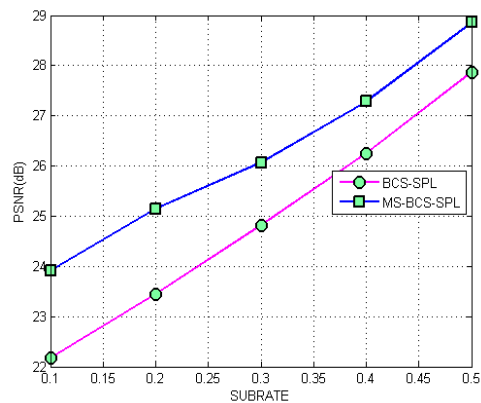


2(d)

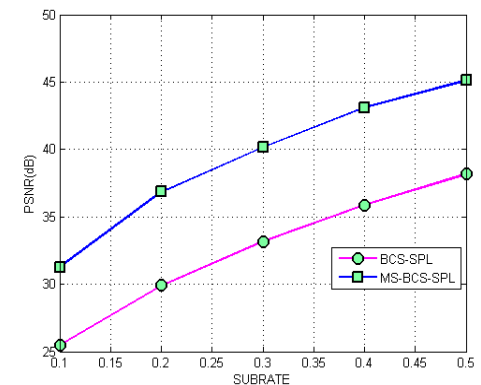
Fig. 2. Comparison of BCS-SPL and MS-BCS-SPL based on reconstruction performance (PSNR). (a) Leena (b) Barbara (c) Cameraman (d) Peppers



2(a)



2(b)



2(c)



3(a)

3(b)

Fig. 3. Reconstructed peppers image for subrate= 0.1. (a) BCS-SPL (28.45), (b) MS-BCS-SPL (31.0956).

VI. CONCLUSION

In this paper, we formed multi-scale variant reconstructions by deploying block based CS sampling within the domain of a wavelet transform. The corresponding reconstructions applies the Landweber step to each block in each decomposition level independently. The resulting method achieves a significant reconstruction performance over the original BCS-SPL. Overall, the multi-scale reconstruction algorithm effectively retains the fast execution speed associated with block based measurement while rivaling the quality of CS reconstructions which employ full image sampling.

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BIOGRAPHIES



C. Manohar completed his B.Tech in Electronics and Communication Engineering at Rajeev Gandhi College of Engineering and Technology College, Nandyal, in 2013. He is pursuing his M.tech degree with Specialization

Signal Processing (SP) at sri Venkateswara University College of Engineering, Tirupati,. His areas of interest include image and video processing.



Mrs S. Swarnalatha received her Bachelor's Degree in Electronics and Communication Engineering from JNTUCEA (JNTU) in 2000, received Master's Degree in Digital Electronics and Communication Systems from

JNTUCEA (JNTU) in 2004 and Pursuing Ph.D. from sri Venkateswara University College of Engineering, Tirupati in the Image Processing Domain. Worked as Lecturer at JNTUCEA, Anantapur. For 4 Years, as Assistant and Associate Professor in the Department Of ECE, at MITS, Madanapalle, as Associate Professor at CMRIT, Hyderabad and Presently Working as Associate Professor sri Venkateswara University College of Engineering, Tirupati