

# Multirate Signal Reconstruction Using Two Channel Orthogonal Filter Bank

Sijo Thomas<sup>1</sup>, Darsana P<sup>2</sup>

PG Scholar, Department of Electronics and Communication Engineering, Amal Jyothi College of Engineering, Kanjirappally, Kerala, India

Assistant Professor, Department of Electronics and Communication Engineering, Amal Jyothi College of Engineering, Kanjirappally, Kerala, India

**Abstract:** A multirate digital filter is a variable filter that provides variations on different parameters such as bandwidth, sampling rate and frequency. Filter banks are particularly useful for communication applications since the channel bandwidths and frequencies are easily and rapidly adjusted during their operation. This paper proposes an alias free two channel orthogonal filter bank by Parks McClellan algorithm. The synthesis filter responses of proposed filter bank can be selected as time reversed version of analysis filters. Finite impulse response filter is used to get stable and linear output at synthesis side.

**Keywords:** Analysis filters, Filter banks, Finite impulse response filter, Orthogonal filter bank, Parks McClellan algorithm, Synthesis filters.

## I. INTRODUCTION

Filter banks are an array of band pass filters that can either be used to decompose a signal into narrowband components or reconstruct a wideband signal from its narrowband components [1]. The advantage of using decimator and interpolator are that they can reduce the computations by changing sampling rates.

Two channel filter banks are classified based on selection of prototype filter. Finite impulse response (FIR) filters are used for designing prototype filter. Window methods are mainly used for designing FIR filters in image or speech processing applications. The best window method is based on Kaiser Window which allows adjustments between overshoot reduction and transition region width spreading. Kaiser filters do not allow variation on filter coefficients with respect to pass band or stop band ripples [2]. The disadvantage of the frequency sampling technique was that the frequency response produces errors at the points where it was not sampled. In order to reduce these errors, optimal filter design techniques are used.

Alias free two channel filter banks are classified based on the relation between analysis and synthesis filters and also based on transfer function of filter. Two channel filter banks are also called maximally decimated filter bank if the number of channel is equal to sampling rate factor. Maximally decimated perfect reconstruction filter banks are mainly used for secure data communication or for high quality audio since the output is very sensitive to errors at the synthesis side. Perfect reconstruction filter banks are obtained by adjusting synthesis filter response with respect to analysis filters. In a two channel quadrature mirror filter banks [3], frequency responses of low pass and high pass filters in analysis and synthesis sides are mirror image by a

factor  $\pi/2$ . In an orthogonal filter banks, the orthogonality maintains error as minimum without accumulating at synthesis side. It also provides simple approximation in filter design compared to quadrature mirror filter banks.

## II. PROPOSED FILTER BANKS

Proposed filter bank is developed by orthogonal filters which are implemented by Parks McClellan algorithm that minimize the error between actual and desired response by repeated iterations.

The input signal is splits into two frequency bands by passing through a low pass and a high pass filters and reduces sampling rates by decimation process.

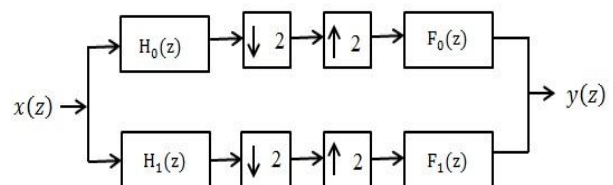


Fig1. Two channel filter bank

At synthesis side, zeros are added between samples and recombined into an original signal. Fig. 1 shows the structure of two channel orthogonal filter bank. Reconstructed output at synthesis filter section is,

$$Y(z) = \frac{1}{2} F_0(z)[X(z)H_0(z) + X(z)H_0(-z)] + \frac{1}{2} F_1(z)[X(z)H_1(z) + X(z)H_1(-z)] \quad (1)$$

Rearranging output into

$$Y(z) = T(z)X(z) + A(z)X(-z) \quad (2)$$

$T(z)$  be the transfer function of filter and  $A(z)$  be the aliasing components. For eliminating amplitude distortion and aliasing, the optimum value of transfer function  $T(z) = 2$  and aliasing function  $A(z) = 0$ .

A. Optimal filter bank

The Minimum phase half band filter is used for designing low pass filter [4]. The optimal filter coefficients are obtained by Parks McClellan algorithm which uses Remez exchange algorithm. According to chebyshev approximation, the optimal linear phase FIR filter can be designed by solving following error function,

$$E(e^{j\omega}) = W(e^{j\omega})[D(e^{j\omega}) - F(e^{j\omega})] \quad (3)$$

The chebyshev approximation problem stated as minimum error function [5] is obtained by finding set of filter coefficients that minimize the maximum error over ripples and is given by equation (4) which is called Alternation theorem.

$$\|E(e^{j\omega})\| = \min_{\text{coefficients}} [\max_{\text{ripples}} |E(e^{j\omega})|] \quad (4)$$

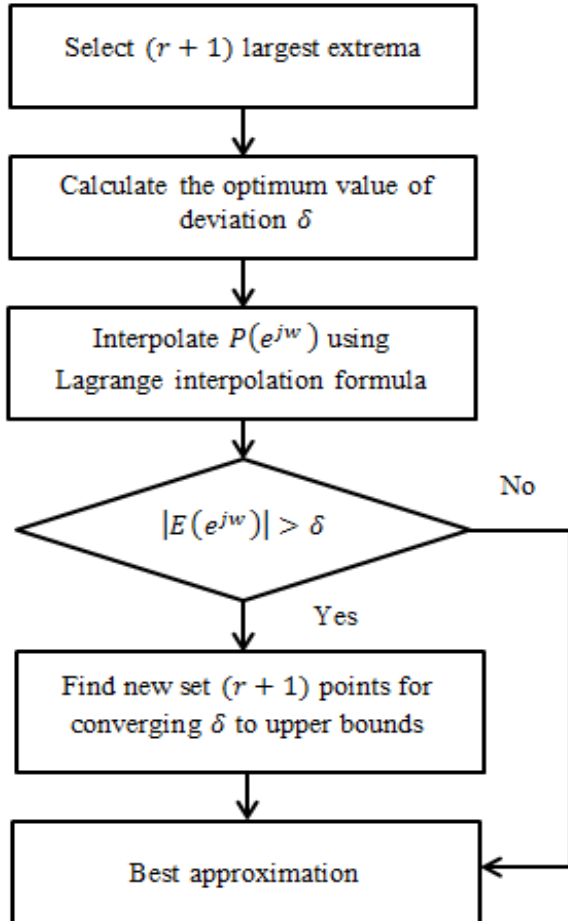


Fig. 2. Remez exchange algorithm

Remez exchange algorithm finds the solution of Alternation theorem by assuming  $(r+1)$  alternations [6]. It replaces equation (3) using deviation  $\delta$  between actual and desired response as,

$$\widehat{W}(e^{j\omega})[\widehat{D}(e^{j\omega}) - P(e^{j\omega})] = (-1)^k \delta; k = 0, 1, \dots, r \quad (5)$$

Calculate the value of deviation  $\delta$  analytically using equation (6) which is given by,

$$\delta = \frac{a_0 \widehat{D}(e^{j\omega_0}) + \dots + a_r \widehat{D}(e^{j\omega_r})}{\frac{a_0}{\widehat{W}(e^{j\omega_0})} + \frac{a_1}{\widehat{W}(e^{j\omega_1})} + \dots + (-1)^r \frac{a_r}{\widehat{W}(e^{j\omega_r})}} \quad (6)$$

Interpolate  $P(e^{j\omega})$  on  $r$  points to the values  $c_k$  using Lagrange interpolation formula in barycentric form which is given by following equation,

$$P(e^{j\omega}) = \frac{\sum_{k=0}^{r-1} \frac{\beta_k c_k}{x - x_k}}{\sum_{k=0}^{r-1} \frac{\beta_k}{x - x_k}} \quad (7)$$

where parameter  $\beta_k = \prod_{i=0, i \neq k}^{r-1} \frac{1}{x - x_i}$  and the coefficients  $c_k = \widehat{D}(e^{j\omega_k}) - (-1)^k \frac{a_k}{\widehat{W}(e^{j\omega_k})}$ . Next step is to evaluate the maximum value of error function. Fig. 2 represents the flow chart of Remez exchange algorithm. If error function is greater than deviation, again consider another set of extremal frequencies and repeated the above process to achieve required bound.

New points are chosen as the peaks of resulting curve, thereby forcing  $\delta$  to increase and converges to its upper bound. In the event there are more than  $(r+1)$  alternations, then largest are retained. The values of error function is less than deviation, it gives filter coefficients. The filter impulse response is obtained by evaluating  $P(e^{j\omega})$  at  $N$  equally spaced frequencies and using DFT to get the coefficients.

After designing FIR half band filter, spectral factorization is used for designing minimum phase low pass filter [7]. It is obtained by calculating roots of filter coefficients and select zeros that lie on or inside unit circle. Using this minimum phase low pass filter  $H_0(z)$ , find analysis and synthesis filter response of two channel filter bank using following equations,

$$H_1(z) = z^{-N} H_0(-z^{-1}) \quad (8)$$

$$F_0(z) = 2z^{-N} H_0(z^{-1}) \quad (9)$$

$$F_1(z) = 2z^{-N} H_1(z^{-1}) \quad (10)$$

III. PERFORMANCE ANALYSIS

To support the performance analysis, simulations were undertaken using MATLAB. Fig. 3 shows the magnitude response of analysis and synthesis filters. The synthesis filter responses are time reversed version of analysis filters.

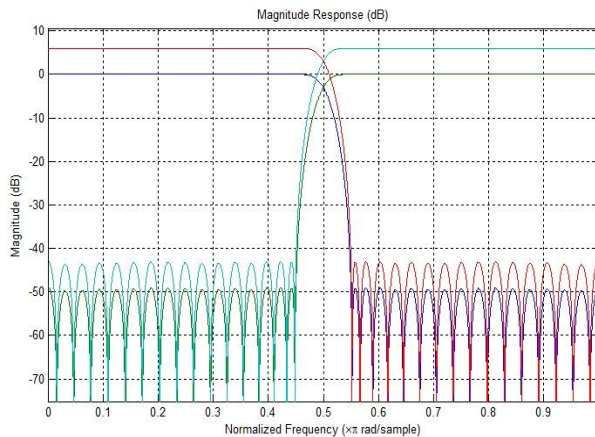


Fig. 3. Magnitude response of orthogonal filters

#### A. Reconstructed output

Consider a design example of two channel filter bank with chirp input signal. Chirp signal is a multirate signal with different frequencies that varies with time. Fig. 4 shows the spectrogram of reconstructed output in proposed filter bank with filter order 64. Spectrogram of a sequence is magnitude of Short Time Fourier Transform versus time. If values of filter order increases, it also increases the delay at the output. In general the reconstructed output of perfect reconstruction filter bank is delayed and scaled version of input signal.

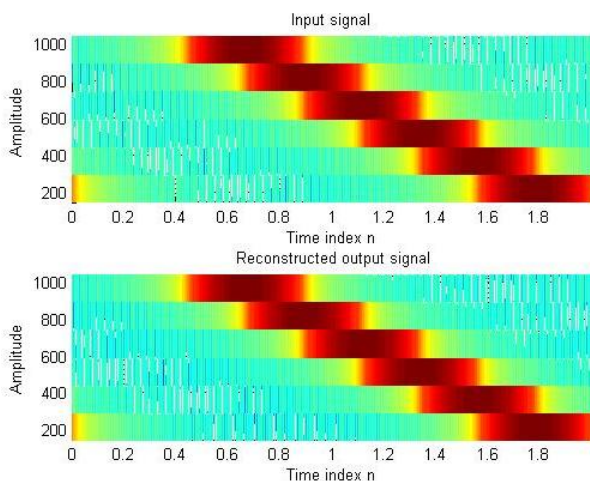


Fig. 4. Reconstructed output

#### B. Comparison

In order to evaluate the performance of proposed filter bank in terms of reconstruction error, compare it with cosine modulated filter bank using Kaiser Window [8]. Fig. 5 illustrates the comparison of proposed method with cosine modulated filter bank using error difference between input and output. Cosine modulated filter bank has continuous variation of error over large time samples, but for proposed method the variation in error occurs for few samples. So the average reconstruction error of proposed method is less compared to cosine modulated filter bank using Kaiser Window.

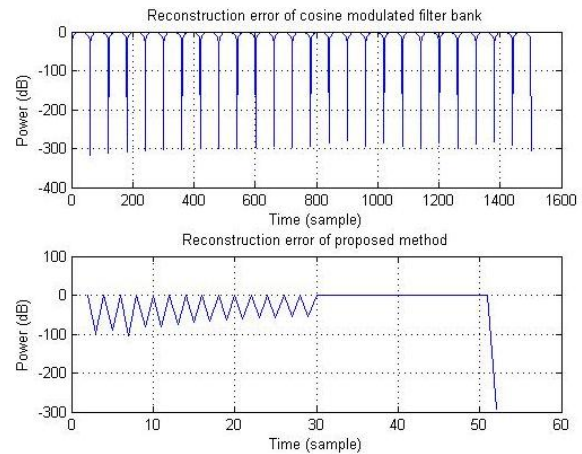


Fig. 5. Comparison with cosine modulated filter bank

### IV. CONCLUSION

In signal processing, a filter is used to remove or modify some components or features of a signal. A filter bank is an array of filters that separates the input signal into different spectral components each one carrying a single frequency subband of the original signal. Two channel filter bank is developed by orthogonal filters and these filter coefficients are adjusted by optimal Parks McClellan algorithm. Average reconstruction error of proposed method is less compared to cosine modulated filter bank using Kaiser window due to optimal filter coefficients. This approach can be extended to the case of multidimensional filters and for filters with higher order.

### REFERENCES

- [1] P. P. Vaidyanathan, "Multirate systems and filter banks," Prentice-Hall, Englewood Cliffs, NJ, 1993.
- [2] S. Venkataraman and B. C. Levy, "A comparison of design methods for 2-D fir orthogonal perfect reconstruction filter banks," IEEE Transaction on Circuits and Systems, vol. 42, 1995, pp. 520-536.
- [3] Chimin Tsai, "Design of quadrature mirror filter banks using complex half band filters," IEEE. International Conference on Multimedia and Signal Processing, vol. 1, 2011, pp. 229-231.
- [4] J. H. Gilchrist, "Design of minimum phase filter banks," IEEE Transaction on Signal processing, vol. 124, 1977, pp. 665-665.
- [5] L. R. Rabiner, J. H. McClellan and T.W. Parks, "FIR digital filter design techniques using weighted chebyshev approximation," in Proc. IEEE, vol. 63, 1975, pp. 595-610.
- [6] D. J. Shpak and A. Antoniou, "A generalized Remez method for the design of FIR digital filters," IEEE Transaction on Circuits and Systems, vol. 37, 1990, pp. 161-174.
- [7] Martin Vetterli and Didier Le Gall, "Perfect reconstruction FIR filter banks: some properties and factorizations," IEEE Transaction on Acoustics, Speech and Signal processing, vol. 37, 1989, pp. 1057-1071.
- [8] Yuan-Pei Lin and P. P. Vaidyanathan, "A kaiser window approach for the design of prototype filters of cosine modulated filter banks," IEEE Signal Processing Letters, vol. 5, 1998, pp. 132-134.