IJARCCE

ISSN (Online) 2278-1021
ISSN (Print) 2319 5940

International Journal of Advanced Research in Computer and Communication Engineering
ISO 3297:2007 Certified
Vol. 6, Issue 8, August 2017

Josephus Cube: A Novel Interconnection

Rayees Ahmad[1], Zahoor Ahmad Shah[2]

Assistant Lecturer, Computer Applications, Govt Degree College for Women, Anantnag, J&K India[1]

Assistant Professor, Computer Applications, Govt. Degree College for Women, Anantnag, J&K India[2]

Abstract: In this paper we present a novel interconnection topology, The Josephus Cube that satisfies greater flexibility in network sizes, more efficient embeddings, incremental scalability, reduced diameter and fault tolerance. Josephus Cube is a super graph of the N-cube and several of its variants i.e. hyper-tree and X-trees. The topology can systematically interconnect an arbitrary number of nodes; its scalability, diameter, communication performance and total number of edges compare favourably with the N-cube. It also supports a simple routing strategy that is minimal, deadlock-free and· live lock-free.

Keywords: Interconnection network, Embedding, Routing Strategy; traffic congestion; deadlock and live-lock free.

1. INTRODUCTION

Ever since the first computer was built, the need for higher and higher computation speeds has always outstripped available technology. Areas requiring great computational speed include partial differential equations such as computational fluid dynamics, weather prediction, image processing, solution of general system of equations, mathematical programming and optimization problems etc. Such problems often need huge repetitive calculations on large amount of data to give valid results and computations must be completed within a reasonable time period. One way of increasing the computational speed is by using multiple processors operating together on a single problem. The overall 'problem is spilt into parts, each of which is performed by separate processors in parallel. The concept of speeding-up the execution of a program by dividing the program into multiple fragments that can execute simultaneously, each on its own processor is known as Parallel Processing. Parallel Processing is appropriate only if one's problem has enough parallelism to make good use of multiple processors. One has to identify the portions of the program that can execute independently and simultaneously on separate processors. In practice, it is often difficult to divide a program in such a way that separate CPU can execute different potions without interfering each other. Parallel Processing is also called parallel computing. The computing platform, a parallel computer, is a specially designed computer system containing multiple processors or several independent computers interconnected in some way. Even though higher speed obtainable with parallel computers is the main motivating force for building parallel computers, there are some other features of parallel computers, which are not obvious but nevertheless important.

The JC which has almost similar properties as Hypercube enjoys a superior speed up and efficiency as compared to Hypercube. It is a super graph of the N-cube and several of its variants. The topology can systematically interconnect an arbitrary number of nodes; its scalability, diameter, communication performance and total number of edges compare favourably with the N-cube. One advantage is that in JC we can use any number of processors whereas in hypercube the number of processors is a power of 2.

2. JOSEPHUS CUBE: DEFINITION

The definition of the JC is based on Flavius Josephus problem.

Consider the following functions:

J (l)=l

J (2i) =21 (i) -1 for i ≥1, where i E :Z:+

J (2i + 1)=2J(i) + 1 for i ≥1, where i E :Z:+

A sample of the sequence J(i) for 1 ≤ I ≤ 15, in binary representation is shown in Table 1. Three characteristics of the J operator can be observed from Table 1. Firstly, there are recurring sequences of consecutive odd numbers with a maximum of 2' odd numbers in each series, where r ≥ 0. Secondly, ignoring leading Os in the binary representation of i,

ISSN (Online) 2278-1021
ISSN (Print) 2319 5940

International Journal of Advanced Research in Computer and Communication Engineering
ISO 3297:2007 Certified
Vol. 6, Issue 8, August 2017

J(i) corresponds to a one-bit left cyclic shift of i. Thirdly, with a sufficiently large number of J(i)s, a consistent recurring sequence is obtained for J(i) and the 3-bit substring l E {001, 011, 101,{θ_1,$\theta_{2,,}$..............., $\theta_{r-2l|r\geq 2}$} where θ_i $_{\varepsilon}${0,1}Ill}.

If we define N to represent the number of J(i)s in a given sequence of odd numbers ,then we have $(2^{r-1}+1) \leq N \leq 2^r$ and $2^r \leq i \leq (2^{r+1}-1)$ for $r \geq 1$.The order of a group of J(i) is represented by r. An order zero (r=0) group contains only J (1).

Table 1: Sample sequence for J (n)

n (decimal)	n (binary)	J(n)
1	1	1
2	10	01
3	11	11
4	100	001
5	101	011
6	110	101
7	111	111
8	1000	0001
9	1001	0011
10	1010	0101
11	1011	0111
12	1100	1001
13	1101	1011
14	1110	1101
15	1111	1111

Theorem: Let J (n) be the "survivor" in the Josephus problem for n processors.

J (1) = 1
J (2n) =2J (n) − 1,
J (2n + 1) =2J (n) + 1.
J (2) = 2J (1)-1=2-1=1
J (3) = 2(J (1) +1)-1 =2J (1) +2-1=2J (1) +1=2+1=3
J (4) = 2(J (2)-1) = 2J (2)-1=2-1=1
J (5) = 2(J (2) +1)-1 = 2J (2) +1 = 2+1 =3
J (6) = 2(J (3)-1) = 2j (3) -1 =2(3)-1 =5
J (7) = 2(J (3) +1) -1 = 2J (3) +1= 2(3) +1=7
J (8) = 2J (4)-1= 2(1)-1=1
J (9) = 2(J (4) +1)-1 = 2J (4) +2-1 =2(1) +1 =3
J (10) = 2(J (5)-1) = 2J (5)-1= 2(3)-1=5
J (11) = 2(J (5) +1)-1= 2J (5) +1 =2(3) +1=7
J (12) = 2J (6)-1 = 2(5)-1 = 9
J (13) = 2(J (6) +1)-1 = 2J (6) +1= 2(5) +1=11
J (14) = 2J (7)-1=2(7)-1=13
J (15) = 2(J (7) +1)-1=2J (7) +1=2(7) +1=15

There is an almost explicit expression for J (n): if 2^m is the largest power of $2 \leq n$, then

$$J(n) = 2(n - 2^m) + 1.$$

Corollary: The binary expansion of J (n) is obtained by transfer-ring the leftmost digit 1 of the binary expansion of n to the rightmost.

$$J(15) = J([1111]_{2)}) = [1111]_{2\,=}\ 8+4+2+1 = 1\,5$$

FIGURE BELOW ILLUSTRATES JC INTERCONNECTION NETWORK FOR 1< N< 8

DOI10.17148/IJARCCE.2017.6850

ISSN (Online) 2278-1021
ISSN (Print) 2319 5940

International Journal of Advanced Research in Computer and Communication Engineering
ISO 3297:2007 Certified
Vol. 6, Issue 8, August 2017

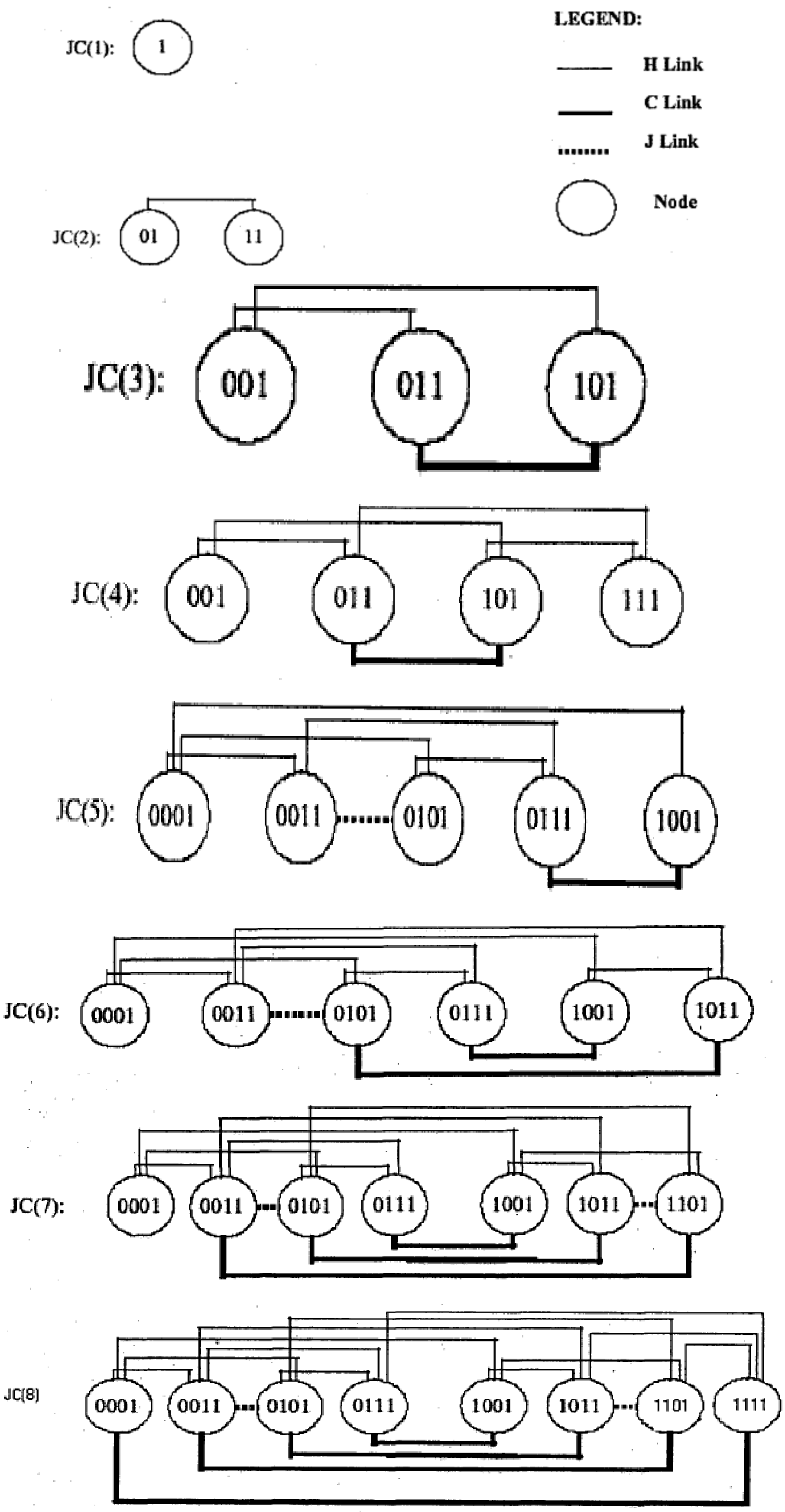


Fig. Josephus Cube Interconnection Networks

ISSN (Online) 2278-1021
ISSN (Print) 2319 5940

International Journal of Advanced Research in Computer and Communication Engineering

ISO 3297:2007 Certified

Vol. 6, Issue 8, August 2017

Formally, the JC interconnection network of size N is an undirected graph JC(N) = (V(N),E(N)), where V(N) = {J(n) | 2^r □ n □ (2^{r+1}-1) for r □ 0} and E(N) = E_h(N) □□E_j(N) □ E_c(N). The edge set, E(N), comprises three edge subsets which are mutually exclusive with each other [16]. These are E_h(N), E_j(N), and E_c(N), representing the Hamming(H), Josephus(J) and Complementary(C) edges respectively. Let r denote the order of a cluster of JC interconnection networks. For N □ 1 and r = □lgN□, let x, y □ V(N) with x □ y :

E_h (N) □ {□ x, y □| H (x, y) □ 1} ,

Where H(x, y) is the Hamming distance between x and y.

E_h (N) □ {□ x, y □| □(x) □ y} ,

Where □(x) denotes the 2's complement of x.

E_j (N) □ {□ x, y □|□ x, y □□ E_c (N)and □ x, y □□{□□□$_1$□$_2$ ...□$_r$

Where □$_i$ □ {0,1} for all 1 □ i □ (r - 2)}.

This link is known as J (Josephus) link.

3. FEATURES OF JOSEPHUS CUBE

The main features of a JC are as follows:

(i) The JC can interconnect an arbitrary number of nodes, permitting interconnection networks of varying sizes. This provides improved scalability with increase in network size.

flog $_2$Nl

(ii) The JC(N) network has a diameter of 2

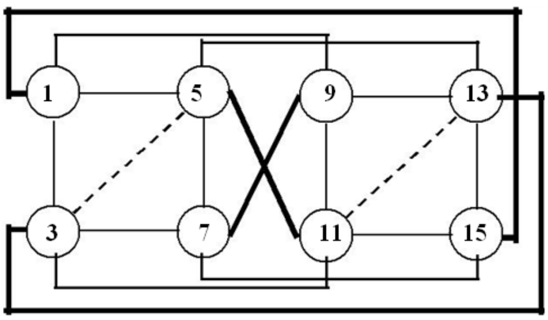
(iii) The JC optimally embeds the n-cube and several variants. Arbitrary sized linear arrays, rings, 2P x q meshes, single and double rooted binary trees can also be optimally embedded in a JC(N) network.

(iv) If d(x) is the degree of a node x in a JC(N) then

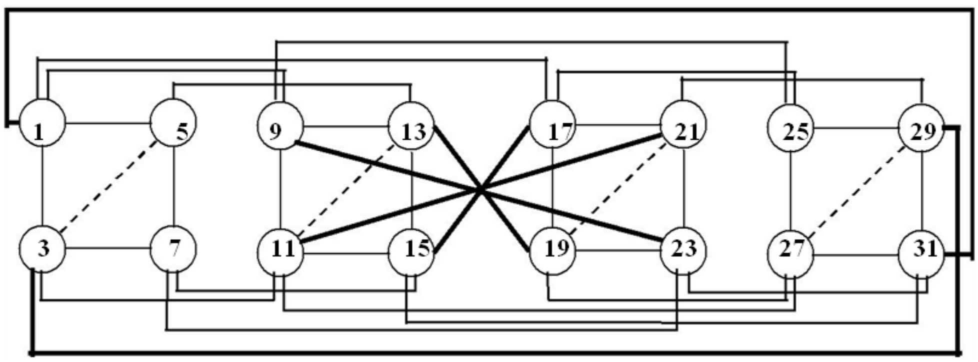
(v) The total number of links for any n, (n = 2k), k ~ 1 is given by

n log 2 n 3n 0(1)
_.....:::2.::..... _+ -4 = n og2 n

Any JC(N) can be decomposed into two separate networks- a complete (r··l)-cube, HC(r-1) = (V(r-1),E(r-1)), and an incomplete K-node (r-1)-cube, PC(K) = (Vpc(K), Epc(K)), where K = N- 2r-l for all r>O. When N = 2r, JC(N) can be decomposed into two complete (r~ 1)-cubes.



Josephus cube with n= 8 or JC (8)



Josephus cube with n= 16 or JC (16)

ISSN (Online) 2278-1021
ISSN (Print) 2319 5940

International Journal of Advanced Research in Computer and Communication Engineering
ISO 3297:2007 Certified
Vol. 6, Issue 8, August 2017

4. CONCLUSION

In this brief correspondence we have presented a novel interconnection topology, The Josephus Cube that satisfies greater flexibility in network sizes, more efficient embeddings, incremental scalability, reduced diameter and fault tolerance. Josephus Cube is a super graph of the N-cube and several of its variants i.e. hyper-tree and X-trees. Embedding techniques used for both hyper tree and X-trees are simple and may be recursively applied. It also supports a simple routing strategy that is minimal, deadlock-free and lives lock-free. (Interconnection networks have routing algorithms to find a path between nodes. Some routing algorithms are adaptive in that they choose alternative paths through the network depending upon certain criteria, notably local traffic conditions. Deadlock occurs when packets cannot be forwarded to the next node because they are blocked h)' other packets waiting to be forwarded and these packets are blocked in a similar way such that none of the packets can move.

REFERENCES

[1] Ammon, J., Hypercube Connectivity within cc NUMA architecture Silicon Graphics, 201LN. Shoreline Blvd. Ms 565, Mountain View, CA 91343.
[2] Rakov, M., Multidimensional Hyperstar and Hypercube Interconnection Methods and Structure, US Patent Application No. 091140 175, filed September 1999. www.wikipedia.com
[3] Parhami, B., Swapped Interconnection Networks: Topological, Performance, and Robustness Attributes, J.Parallel and Distributed Computing, to appear.
[4] Lob 2000] P.K.K. Loh, Hso W. J.,"The Josephus Cube: A novel interconnection network," Parallel Computing, 26,427-453, Elsevier Science Publishers B.V.,(2000).
[5] L.N. Bhuyan and D.P. Agrawal, "Generalized Hypercube and Hyperbus Structures for a Computer Network," IEEE Trans. Computers, vol. 33, no. 4, pp. 323-333, Apr. 1984.
[6] Behrooz Parhami, and Mikhail Rakov "Perfect Difference Network and Related Interconnection Structures for Parallel and Distributed Systems", IEEE transaction on Parallel and Distributed Systems, Vol. 16, No 8, August 2005.
[7] Katare R.K., Chaudhari N.S., "Study Of Topological Property Of Interconnection Networks And Its Mapping To Sparse Matrix Model" International Journal of Computer Science and Applications, Technomathematics Research Foundation Vol. 6, No. 1, pp. 26 – 39, October 2009.
[8] Katare, R.K., Chaudhari, N.S., "A comparative study of Hypercube and perfect difference network for Parallel and Distributed system and its application to sparse linear system." Varahmihir Journal Of Computer And Information Sciences Sandipani Academy, Ujjain,(M P) India, Vol. 2, pp. 13-30, 2007.
[9] LOH, PETER K. K. and HSU, W. J., "The Josephus Cube: A Novel Interconnection Networks", Parallel Computing, Vol. 26, No. 4, March 2000, pp 427-453
[10] Chandra 2003) Ramesh Chandra and C. Siva Ram Murthy, " A faster algorithm for solving linear algebraic equations on the star graph," Journal of Parallel and Distributed Computing, 63 pp.465-480 (2003).
[11] Flynn 96]M.J. Flynn and K.W.Rudd, " Parallel Architectures," ACM Computing Surveys, Vol. 28, No.1 (March),pp. 67-70,(1996).