

Robust Ramp Metering Design using Sliding Mode Control of a Hybrid Dynamical Model with Functional Uncertainties

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Abstract: This paper presents an application of sliding mode control based concept to real time freeway ramp metering. A robust hybrid feedback control design for controlling the traffic flow on ramp and entering a freeway to reduce traffic congestion has been designed. This model uses Godunov based hybrid switching with sliding mode control to produce robust stability for the system in presence of parametric uncertainties and functional disturbances. The main obstacle encountered in real time application using sliding mode control is chattering which is suppressed by introducing a boundary layer around the switching surface and using a continuous control function within the boundary layer. Simulations have been performed that show the effectiveness of the proposed novel approach and confirm that sliding mode is reachable in finite time.

Keywords: Sliding mode, freeway, ramp metering, hybrid dynamics, chattering, switching surface.

I. INTRODUCTION

An increasingly important area in the field of intelligent transportation systems is freeway traffic control, which has become feasible owing to the freeway infrastructure development in metropolitan areas in both developed and developing countries. The dramatic increase in the vehicular traffic worldwide has led to the congestions on the freeways resulting in increased environmental pollution, reduced traffic safety and thereby economic loss to the country.

Ramp metering is a way to improve traffic flow by regulating the ramp inflow to a freeway. By effectively controlling the ramp inflow, the traffic density on the mainline freeway can be kept below critical level to provide a congestion free freeway. For this type of operation, many factors have to be considered such as the inflow at the mainline, the queue holding capacity of the ramp, availability of sensors and the arterial system connected to the ramp system.

The ramp flow problem has been studied for more than forty five years. Some of the early work is documented in references [1], [2], [3] and [4]. This work was related to merge control and ramp metering control design based on demand -capacity relationships. Some early deployment studies were also performed at various sites such as Chicago and Houston. Reference [5] provides a current overall overview of ramp metering. References [6], [7] and [8] show the work that used optimization techniques for solving optimal ramp control problems. Some evaluation studies and simulation

based evaluation methods are described in [9] and [10]. Some researchers have designed feedback control laws for ramp metering [11] and [12]. These laws are designed after performing linearization of the dynamics about the nominal equilibrium state. Recently, ramp meters have been deployed in many places internationally, such as in France [13], Germany [14], U.S.A [15], Italy [16], U.K. [17] and New Zealand [18]. Model formulations of distributed model and lumped model are discussed in [19] and [20].

II. BACKGROUND

Feedback control is a very powerful technique for ramp control since it is traffic responsive and has the least computation cost, hence is a real-time control strategy. However, until now, mainly linear control design has been studied, which is powerful but provides only local results. For global results, nonlinear techniques become necessary. The topology of a ramp metering system is shown in Fig. 1. The designer of the controller needs to address issues such as controllability and observability of the traffic system, actuation and sensing, robustness, and stability of the closed loop system.

Earlier ramp metering models using lumped parameters for feedback control design do not produce the rarefaction behaviour of the traffic. It means that when the traffic density is at jam density, the outflow of traffic from the section becomes zero, thereby meaning that traffic would never come out of the jam. Moreover, these models do not satisfy the entropy conditions required by the distributed

parameter hybrid dynamic model. However, this problem has been overcome by Godunov based model for feedback ramp metering [21]. This model gives the feedback control design for ramp metering which provides asymptotic behaviour for the closed loop system and reproduces rarefaction behaviour of the traffic. This model is also entropy consistent and uses Godunov based switching ordinary differential equations to produce a hybrid dynamic model.

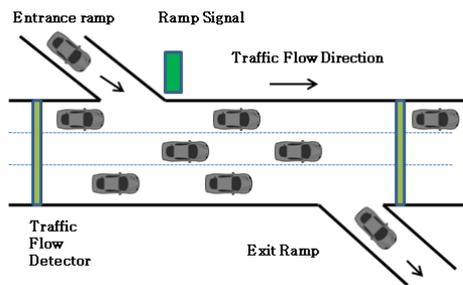


Fig. 1. Topology of Ramp Metering System

However, there are uncertainties in the dynamics of the non linear control systems which result in modelling inaccuracies. Robust control deals with uncertainties in its approach to controller design. Robust control methods are designed to function properly so long as uncertain parameters or disturbances are within some typically compact set. Robust methods aim to achieve robust performance and stability in the presence of bounded modelling errors.

A simple approach to robust control is sliding control methodology. Sliding mode control strategy helps in achieving a satisfactory level of robustness and invariant behavior by a simple method of changing the structure of control law. Robust hybrid feedback control design for ramp metering using sliding mode has been presented in [22]. Here feedback control design is presented for controlling the inflow into the freeways to reduce congestion on the highways using hybrid dynamics based on sliding control methodology. The sliding controller design provides a systematic approach to the problem of maintaining stability and consistent performance.

Our contribution in this paper is to modify the control laws using sliding mode control to minimize the estimation error in the upstream traffic flow and downstream traffic flow. Thus sliding controller design has been presented to provide a systematic approach to the problem of maintaining stability and consistent performance. A trade-off between tracking performance and functional uncertainties has been achieved. The major drawback of application of sliding mode control to real applications is control chattering. This drawback has been dealt with using the boundary layer approach wherein the design has been further modified to move to the steady state to the sliding surface in finite time by reducing chattering. Simulation results of the proposed hybrid model are also presented.

The next section presents the mathematical model for the system, followed by a section on control design, and finally the section that presents simulation results.

III. MATHEMATICAL MODEL

The LWR (Lighthill-Whitham-Richards) model [23] and [24] is a macroscopic one dimensional traffic model. According to this model, traffic behavior for a single one-way road can be described using three variables that vary in time t and space x : flow $u(x,t)$, density $\rho(x,t)$, and speed $v(x,t)$. Flow is the product of speed and the density:

$$u(x,t) = \rho(x,t) v(x,t) \quad \forall x,t \quad (1)$$

For a highway without entrance or exits, the number of vehicles between any two locations x_1 and x_2 ($x_1 < x_2$) needs to be conserved at any time t , i.e. the change in the number of vehicles between x_1 and x_2 is equal to the flow entering via x_1 minus the flow leaving via x_2 :

$$\frac{d}{dt} \int_{x_1}^{x_2} \rho(x,t) dx = u(x_1,t) - u(x_2,t) \quad (2)$$

or in differential form:

$$\frac{\partial}{\partial t} \rho(x,t) + \frac{\partial}{\partial x} u(x,t) = 0 \quad (3)$$

The two relations (1) and (3) are the basic relations that any model must satisfy. We have three variables of interest, a third relation is needed. Greenshield's model [25] uses a linear relationship between traffic density and traffic speed.

$$v(\rho) = v_f \left(1 - \frac{\rho}{\rho_{max}} \right) \quad (4)$$

Here v_f is free flow speed and ρ_{max} is the maximum traffic density. Equation (4) indicates that as the density (ρ) approaches zero, speed (v) approaches free flow speed v_f . Also, as the speed v approaches zero, the density approaches ρ_{max} which is the jam density. The maximum flow occurs when the traffic is flowing at half of free flow speed.

A space discretized model of (3) for a free way ramp metering is presented in Fig. 2

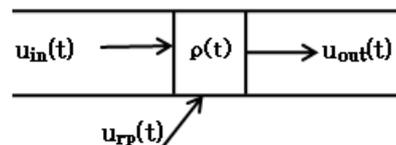


Fig. 2. Space Discretized Model for Ramp Metering

Here $u_{in}(t)$ is the inflow, $u_{out}(t)$ is the outflow and $u_{rp}(t)$ is the ramp inflow into the free way. For a unit length of the section, the ordinary differential equation model for Fig. 2 given by:

$$\frac{d\rho(t)}{dt} = u_{in}(t) + u_{rp}(t) - u_{out}(t) \quad (5)$$



The outflow traffic $u_{out}(t)$ using Greenshield's model is given by

$$u_{out}(t) = v_f \rho(t) \left(1 - \frac{\rho(t)}{\rho_{max}} \right) \quad (6)$$

Using (5) and (6), if the traffic density is equal to jam density and $u_{rp}(t)$ is zero, then the rate of increase of traffic density is non-negative. For positive inflow, the density increases according to the equation. Hence, there are two problems in this model which needs to be fixed when traffic density is equal to the jam density. Firstly, the inflow from upstream can increase the density above the jam density and secondly the outflow is zero from the section not allowing for the traffic to be dissipated to downstream.

Godunov's model has been used successfully to address those two problems and is based on using the characteristic information within the framework of a conservation method. Godunov proposed to solve Riemann problems forward in time rather than attempting to follow characteristics backward in time. Here initial condition is a piece wise constant function with two values ρ_{lf} and ρ_{rt} for the upstream (left) and downstream (right) densities [27]. From the junction of two densities, either a shockwave or a rarefaction wave emanates. A shockwave develops if $f'(\rho_{lf}) > f'(\rho_{rt})$. A rarefaction

develops if $f'(\rho_{lf}) < f'(\rho_{rt})$ [28]. The shockwave and rarefaction is shown in Fig. 3.

The shockwave speed, s is given by (7) in which $x_s(t)$ is the position of the shockwave as a function of time. The inflow at the junction between the two traffic densities will be a function of upstream traffic density if the shockwave speed is positive. But if the shockwave speed is negative, then the inflow at the junction will be a function of downstream traffic density.

$$s = \frac{dx_s(t)}{dt} = \frac{[f(\rho_{lf}) - f(\rho_{rg})]}{\rho_{lf} - \rho_{rg}} \quad (7)$$

Godunov based ODE model for traffic is obtained from the analysis of shockwave and rarefaction conditions. The ODE for the Godunov law obeys the conservation law and is given as

$$\frac{d\rho(t)}{dt} = u_{in}(t) - u_{out}(t) + u_{rp}(t) \quad (8)$$

The Godunov based hybrid dynamics are shown in Fig.4. Here the inflow $u_{in}(t)$ is a function of the upstream density and downstream density, where upstream and downstream are with respect to left junction and $u_{in}(t)$ is given by (9), using a new function $F(\dots)$ obtained from Godunov's method.

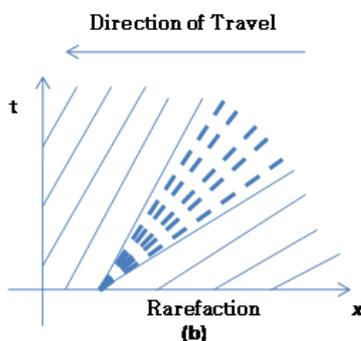
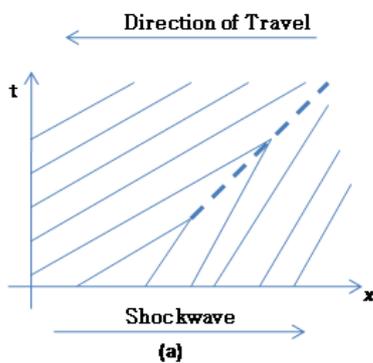


Fig. 3. Shockwave and Rarefaction Phenomenon

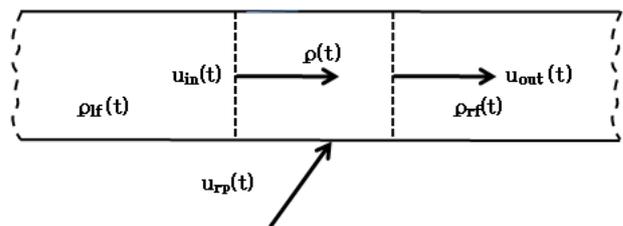


Fig. 4. Godunov based hybrid dynamics

$$u_{in}(t) = F(\rho_{lf}, \rho) \quad (9)$$

For the right junction, the outflow $u_{out}(t)$ is given as:

$$u_{out}(t) = F(\rho, \rho_{rf}) \quad (10)$$

The function $F(\rho_{lf}, \rho_{rf})$ is given by the Godunov method [27].

$$F(\rho_{lf}, \rho_{rf}) = f(\rho^*(\rho_{lf}, \rho_{rg})) \quad (11)$$

where ρ^* is the flow dictating density and is obtained from the four cases [27]:



- Case 1 $f'(\rho_{lf}), f'(\rho_{rg}) \geq 0 \Rightarrow \rho^* = \rho_{lf}$
- Case 2 $f'(\rho_{lf}), f'(\rho_{rg}) \leq 0 \Rightarrow \rho^* = \rho_{rg}$
- Case 3 $f'(\rho_{lf}) \geq 0 \geq f'(\rho_{rg}) \Rightarrow \rho^* = \rho_{lf}$,
 if $s > 0$, otherwise $\rho^* = \rho_{rg}$
- Case 4 $f'(\rho_{lf}) < 0 < f'(\rho_{rg}) \Rightarrow \rho^* = \rho_s$

Here ρ_s is obtained as a solution to $f'(\rho_s) = 0$, where, ρ_s is neither ρ_{lf} nor ρ_{rg} but is some intermediate value satisfying Godunov dynamics.

Godunov method calculates the steady state traffic density of a free way segment on the basis of densities on both sides of the junction viz. upstream(left) and downstream(right) i.e. the know boundary conditions. But there are functional uncertainties. Robust control is one of the best approaches to deal with uncertainties in the model.

A. Sliding Control Methodology

Sliding mode control (SMC) [29] is a powerful non linear robust control technique. The sliding mode design is widely used due to the finite time convergence, robustness with respect to uncertainties and the possibility of uncertainty estimation. However, full state information is required in controller design which is basically a drawback, since in most practical applications, only the output measurement is available. To solve this problem, a feedback based sliding mode control will be designed which provides systematic approach to achieve the stability and consistent performance.

In this methodology, the n^{th} order differential equation is replaced its equivalent first order equations and then in presence of arbitrary parametric inaccuracies, the perfect performance shall be achieved. SMC utilizes discontinuous control laws to drive the system state trajectory onto a specified surface in the state space, the so called sliding or switching surface, and to keep the system state on this manifold for all the subsequent times. Consider the first order dynamical system,

$$\dot{x}(t) = f(x(t)) + u(t) \tag{12}$$

where the $x(t)$ is the output of interest and $u(t)$ is the control input. In (12) the dynamics of function $f(x(t))$ is not exactly known, but the extent of the imprecision on $f(x(t))$ is upper bounded by a known continuous function of $x(t)$.

So the time varying surface $s(t)$ is written as

$$s(t) = x(t) - x_d(t) \tag{13}$$

which is also the tracking error. Here $x_d(t)$ is the desired output. The problem of keeping the scalar $s(t)$ at zero can be achieved by choosing the control law $u(t)$ of (12) such that outside of $s(t)$,

$$\frac{1}{2} \frac{d}{dt} s^2 \leq -\eta |s| \tag{14}$$

where η is a strictly positive constant. Equation (14) states that the squared distance to the surface s^2 decreases along all system trajectories pointing towards surface $s(t)$ as shown in Fig. 5.

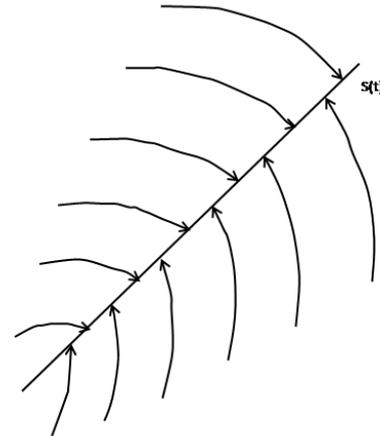


Fig. 5. Sliding Condition

Here, $s(t)$ is referred to as sliding surface and the system behavior is called sliding mode. Thus a stable feedback control $u(t)$ in (12) is designed such that s^2 remains a Lyapunov like function of the closed loop system in spite of functional uncertainties. The designing of controller is a two step process. First, a feedback control law $u(t)$ is selected to verify sliding condition given in (14). The control law has to be discontinuous across $s(t)$. But in real life applications, the control signal cannot switch at infinite frequency and is necessarily imperfect, thereby leading to chattering as shown in Fig 6. Chattering is dangerous high frequency vibration of the controlled system and is undesirable in practice. Therefore, in second step the discontinuous control law $u(t)$ is suitably smoothed to achieve an optimal tradeoff between control bandwidth and tracking precision. Thus in totality both functional uncertainties and robustness is achieved.

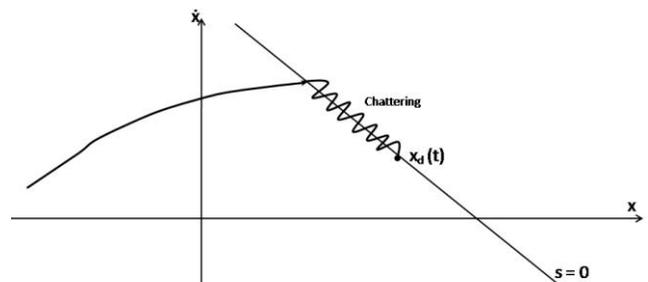


Fig. 6. Chattering Effect

Now (14) can be rewritten as

$$s\dot{s} \leq -\eta |s| \tag{15}$$

From (13), \dot{s} is given as

$$\dot{s}(t) = \dot{x}(t) - \dot{x}_d(t) \tag{16}$$

Substituting the value of $\dot{x}(t)$ from (12), we have

$$\dot{s}(t) = f(x(t)) + u(t) - \dot{x}_d(t) \quad (17)$$

Therefore,

$$s\dot{s} = [f(x(t)) - \dot{x}_d(t) + u(t)]s \quad (18)$$

As the dynamics of $f(x(t))$ is not known, but it can be estimated as $\hat{f}(x(t))$. The estimation error on f is assumed to be bounded by some known function

$$F = F(x, \dot{x}) : |\hat{f}(x(t)) - f(x(t))| \leq F \quad (19)$$

In order to have the system track $x(t) \cong x_d(t)$, the sliding surface $s(t)$ is defined to be zero. Therefore, the best approximation $\hat{u}(t)$ of a continuous control law that would achieve $\dot{s}(t) = 0$ is given by:

$$\hat{u}(t) = -\hat{f}(x(t)) + \dot{x}_d(t) \quad (20)$$

In order to satisfy the sliding condition (14) in spite of uncertainties on the dynamics $f(x(t))$, a term discontinues across the surface $s(t)$ is added to $\hat{u}(t)$:

$$u(t) = \hat{u}(t) - k \operatorname{sgn}(s(t)) \quad (21)$$

where sgn is the sign function and is defined as

$$\operatorname{sgn}(s(t)) = \begin{cases} +1 & \text{if } s(t) \geq 0 \\ -1 & \text{if } s(t) < 0 \end{cases} \quad (22)$$

and k is the control discontinuity across the surface. From (18) and (21) we have

$$s\dot{s} = [f(x(t)) - \dot{x}_d(t) + \hat{u}(t) - k \operatorname{sgn}(s(t))]s \quad (23)$$

Substituting $\hat{u}(t)$ from (20) in (23), we have

$$s\dot{s} = [f(x(t)) - \hat{f}(x(t)) - k \operatorname{sgn}(s(t))]s \quad (24)$$

or

$$s\dot{s} = Fs - k|s| \quad (25)$$

Assuming, $k = F + \eta$, (25) is reduced to (15), which proves the sliding condition for the control function. The control discontinuity k across the surface $s(t)$ increases with the extent of parametric uncertainty. The control algorithm causes chattering, which can be removed for the controller to be stable. This can be achieved by smoothing out the control discontinuity in a thin boundary layer neighbouring the switching surface by introducing the saturation function. Saturation function is continuous approximation of sign function [26] and is defined as:

$$\operatorname{sat}(s(t), \varphi) = \begin{cases} +1 & \text{if } s(t) \geq \varphi \\ -1 & \text{if } s(t) \leq -\varphi \\ \frac{s(t)}{\varphi}, & \text{otherwise} \end{cases} \quad (26)$$

IV. HYBRID DYNAMICAL MODEL AND CONTROL DESIGN

The ODE model for the switched hybrid ramp metering system [30] is given by:

$$\frac{d\rho(t)}{dt} = F(\rho_{lf}, \rho) - F(\rho, \rho_{rg}) + u_{rp}(t) \quad (27)$$

Here the switching takes place based on the values of ρ_{lf}, ρ and ρ_{rg} . The function $F(\rho_{lf}, \rho)$ can have three different values, $f(\rho_{lf}), f(\rho)$ or $f(\rho_s)$. Similarly, the function $F(\rho, \rho_{rg})$ can have three different values $f(\rho), f(\rho_{rg})$ or $f(\rho_s)$.

Hence dynamics can be written as:

$$\frac{d\rho(t)}{dt} = G_n(\rho_{lf}, \rho, \rho_{rg}) + u_{rp}(t) \quad (28)$$

where $n \in \{1, 2, 3, \dots, 9\}$ and different G_n function can be obtained from (11), (27), (28) applying Godunov dynamics.

To keep the freeway traffic density at ρ_s (which is $\rho_m/2$ as per Greenshield's model to maximize the flow), the feedback linearization model for ramp metering control is given

$$u_{rp}(t) = -G_n(\rho_{lf}, \rho, \rho_{rg}) - k(\rho(t) - \rho_s), k > 0 \quad (29)$$

where, k is control discontinuity.

We propose the control law using sliding model as:

$$u_{rp}(t) = -G_n(\rho_{lf}, \rho, \rho_{rg}) - k \operatorname{sgn}(s(t)), k > 0 \quad (30)$$

where $s(t) = (\rho(t) - \rho_s)$, which is the sliding surface. As the inflow and outflow given in (27) are not exactly known, their estimated values are taken.

The function G_n in (30) can be obtained by taking the estimated values of upstream traffic flow, $\hat{u}_{in}(t)$ and downstream traffic flow, $\hat{u}_{out}(t)$. The control law given by $u_{rp}(t)$ in (30) satisfies the sliding condition given in (14) is discontinuous across $s(t)$. This control law leads to chattering which is undesirable. Chattering is removed by using a non linear saturation function described in (26) which smoothens the control discontinuity in a boundary layer neighbouring the switching surface.

The modified control law is given as:

$$u_{rp}(t) = -G_n(\rho_{lf}, \rho, \rho_{rg}) - k \operatorname{sat}(s(t), \varphi), k > 0 \quad (31)$$

which reduces the chattering.



V. SIMULATIONS

The modified control design has been implemented by using MATLAB simulator. The feedback linearization based hybrid dynamical model for the ramp metering control is consistent with the conservation law as well as the Godunov conditions. Further to deal with the functional uncertainties in the dynamics of the nonlinear system and to have a robust control design, sliding mode control is applied using sign function. But this leads to chattering. The chattering has been removed by using a non linear saturation function in the control law.

The ordinary differential condition (ODE) given in (27) which uses the hybrid control scheme developed in this study is implemented in this simulation. The lower and upper limits of traffic flow are taken as zero and 75% of the maximum flow is applied at the inflow of the control. The simulation was run with different initial values of traffic density viz. $\rho_0 = 50, 20, 10$ (vehicles / km) with jam density, $\rho_m = 86$ (vehicles / km) and the simulation results are depicted in the following figures. The free low speed is taken as 70 km/hour. As shown in the Traffic Density using Hybrid Control Plot, the traffic density converges to the desired critical density that maximizes the flow. The desired result is obtained as the steady state traffic density of 43 (i.e. $\rho_m/2$) is achieved. The chattering phenomenon is clearly shown in Traffic Density with parametric uncertainties using Sliding Mode Control Plot, where high control activity is observed around the steady state value of traffic density i.e. 43. The chattering reduction using the saturation function is also shown in Traffic Density using Sliding Mode Control with Saturation Function Plot. Here the value of φ is taken as 1.0, 2.0 & 2.25 for applying boundary layer conditions. The effect of increasing the boundary layer smoothes the steady state value and eliminates chattering as is clearly depicted in the simulation results. However, increasing the boundary layer shifts the steady state value to little lower than 43. The simulation results are shown in Fig. 7 to Fig. 18.

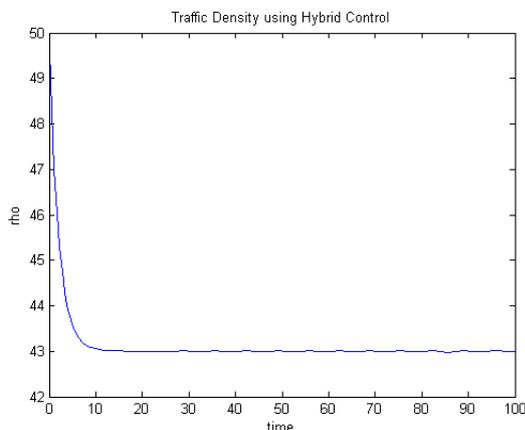


Fig. 7. Freeway segment traffic density using the hybrid based control, $\rho_0 = 50$ and $v_f = 70$

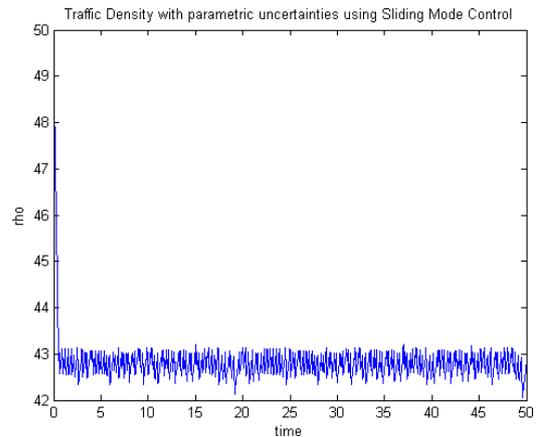


Fig. 8. Freeway segment traffic density using the sliding mode control depicting chattering phenomenon, $\rho_0 = 50, v_f = 70, \hat{v}_f = 69$

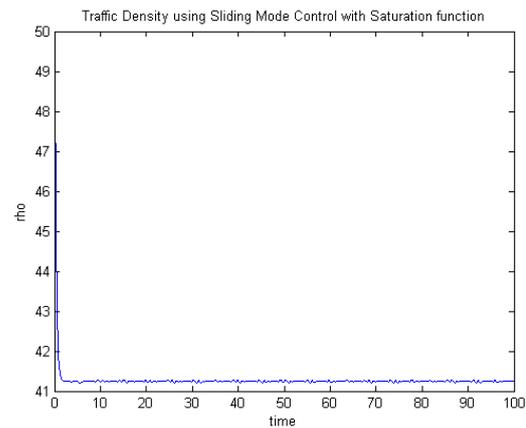


Fig. 9. Freeway segment traffic density using sliding mode control by adopting non linear saturation function, $\rho_0 = 50, v_f = 70, \hat{v}_f = 69$ and $\varphi = 2.25$

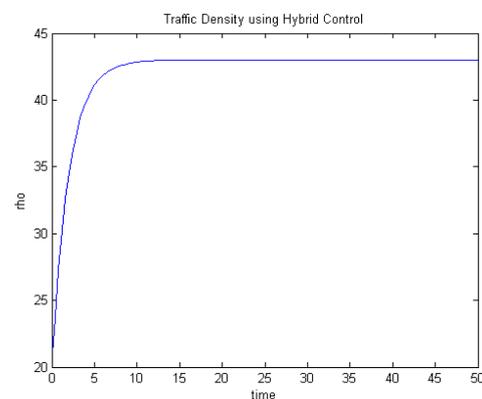


Fig. 10. Freeway segment traffic density using the hybrid based control, $\rho_0 = 20$ and $v_f = 70$

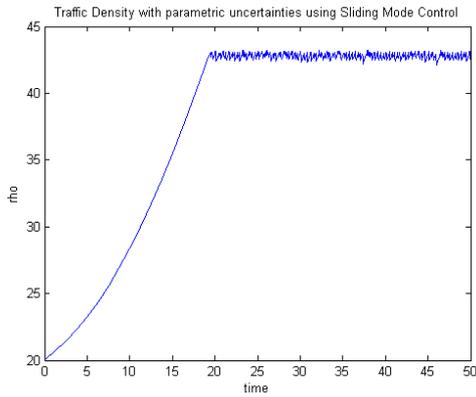


Fig. 11. Freeway segment traffic density using the sliding mode control depicting chattering phenomenon, $\rho_0 = 20$, $v_f = 70$, $\hat{v}_f = 69$

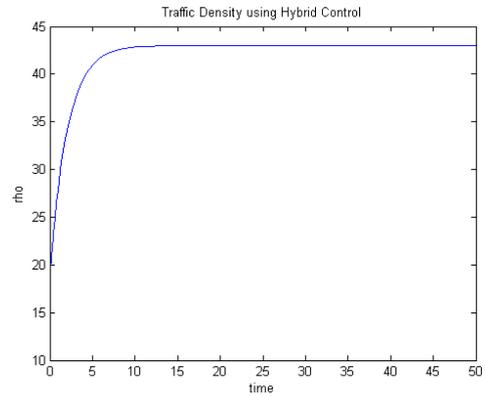


Fig. 14. Freeway segment traffic density using the hybrid based control, $\rho_0 = 10$ and $v_f = 70$

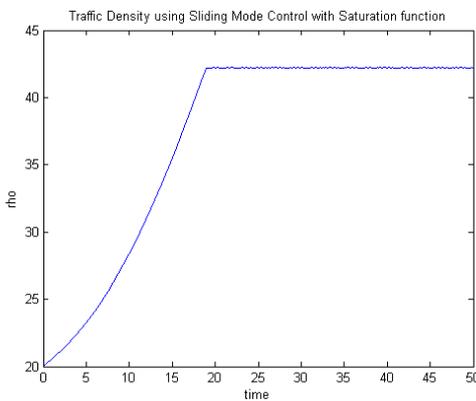


Fig. 12. Freeway segment traffic density using sliding mode control by adopting non linear saturation function, $\rho_0 = 20$, $v_f = 70$, $\hat{v}_f = 69$ and $\varphi = 1$

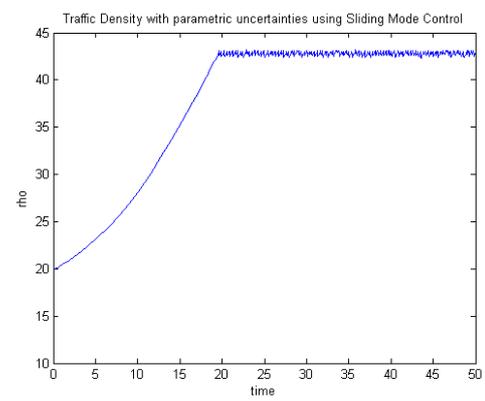


Fig. 15. Freeway segment traffic density using the sliding mode control depicting chattering phenomenon, $\rho_0 = 10$, $v_f = 70$, $\hat{v}_f = 69$

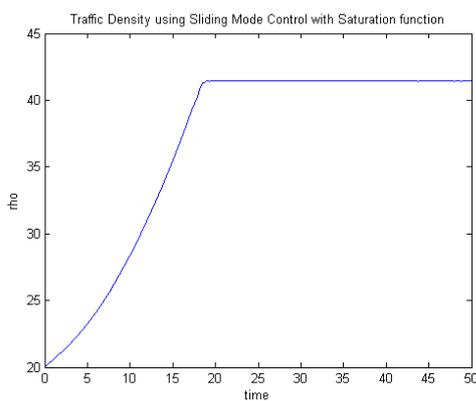


Fig. 13. Freeway segment traffic density using sliding mode control by adopting non linear saturation function, $\rho_0 = 20$, $v_f = 70$, $\hat{v}_f = 69$ and $\varphi = 2$

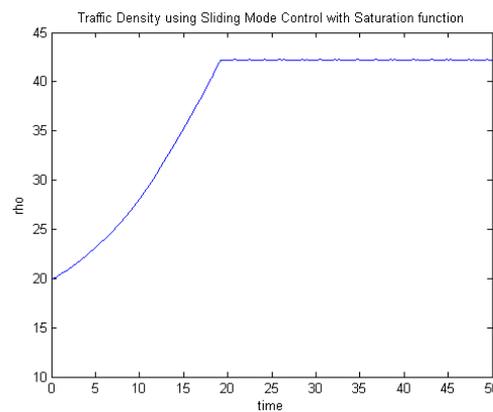


Fig. 16. Freeway segment traffic density using sliding mode control by adopting non linear saturation function, $\rho_0 = 10$, $v_f = 70$, $\hat{v}_f = 69$ and $\varphi = 1$

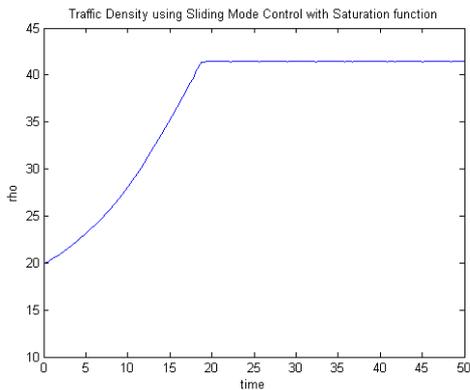


Fig. 17. Freeway segment traffic density using sliding mode control by adopting non linear saturation function, $\rho_0 = 10$, $v_f = 70$, $\hat{v}_f = 69$ and $\varphi = 2$

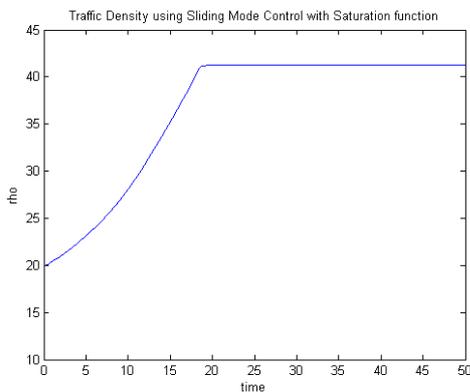


Fig. 18. Freeway segment traffic density using sliding mode control by adopting non linear saturation function, $\rho_0 = 10$, $v_f = 70$, $\hat{v}_f = 69$ and $\varphi = 2.25$

VI. CONCLUSIONS

The paper presents a robust hybrid feedback control design of an isolated ramp based on Godunov dynamics and sliding mode control methodology. Functional uncertainties are taken into account and sliding mode control is used to present a design which is stable and shows consistent performance. The control design presented maximizes the flow of traffic on the freeways and traffic congestion is avoided. Since the control function in real life applications is imperfect, it leads to chattering. Therefore, the control law was further modified by using a boundary layer approach neighbouring the switching surface by introducing a non linear saturation function which smoothes the control discontinuity. Simulations for the model have been performed that show the effectiveness of the proposed novel approach.

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