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Medical Image Decomposition and Compression by using Lifting Scheme

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Abstract: In this paper an attempt has been made to decompose an image using lifting scheme so as to suite compression. Lifting based wavelets are constructed using Haar, Daubechies, Bi-orthogonal, CDF, Symlet wavelets. A large number of medical images are considered. A new design metric which is a combination of PSNR and CR is proposed. Compression ratio and PSNR are calculated and compared the results with so called traditional wavelets. It has been observed that the lifting based wavelets have produced better compression results.

Keywords: Lifting, Compression, Decomposition, Design Metric

I. INTRODUCTION

The wavelets are a family of functions generated from a single function by translation and dilation. The general form of these wavelets is described by

$$\Psi_{a,b}(t) - |a|^{-\frac{1}{2}}\, \Psi\left(\frac{t-b}{a}\right) \qquad (1)$$

Ψ is called the mother wavelet and it is used to generate all other members of the family. A common choice for a and b is

$a - 2^m$, $b - 2^n$ where n, m ϵ Z.

This reduces (1) to

$$\Psi_{m,n}(t) - 2^{-\frac{m}{2}}\, \Psi\left(2^{-m} t - n\right) \qquad (2)$$

These wavelets are used in the wavelet transform. The purpose of the wavelet transform is to represent a signal, x(t), as a superposition of wavelets. For special choices of Ψ the signal can be represented as

$$x(t) - \sum_{m,n} c_{m,n} \Psi_{m,n}(t) \qquad (3)$$

$$c_{m,n} - 2^{-\frac{m}{2}} \int x(t) \Psi_{m,n}(t) dt \qquad (4)$$

The purpose of obtaining this description is that it provides a representation of the signal x(t) in terms of both space and frequency localization. In comparison, the Fourier transform is excellent at providing a description of the frequency content of a signal. But if the signal is non-stationary the frequency characteristics vary in space, that is in different regions the signal x(t) may exhibit very different frequency characteristics, the Fourier transform does not take this into account.

The wavelet transform on the other hand produces a representation that provides information on both the frequency characteristics and where these characteristics are localized in space. The coefficients $c_{m,n}$ characterizes the projection of x onto the base formed by $\Psi_{m,n}$. For different m $\Psi_{m,n}$ represents different frequency characteristics, n is the translation of the dilated mother wavelet, therefore $c_{m,n}$ represent the combined space-frequency characteristics of the signal. The $c_{m,n}$ are called wavelet coefficients. The rest of the paper is organized as follows. The next section gives a review of classical wavelets. The section III presents decomposition of image by using lifting. Section IV presents the simulation results of traditional and lifting wavelets. Section V concludes the paper.

II. TRADITIONAL WAVELETS – A REVIEW

From an historical point of view, wavelet analysis is a new method, though its mathematical roots date back to the work of Joseph Fourier in the nineteenth century. Fourier laid the foundations with his theories of frequency analysis, which proved to be enormously important and influential. The attention of researchers gradually turned from frequency-based analysis to scale-based analysis when it started to become clear that an approach measuring average fluctuations at different scales might prove less sensitive to noise. The first recorded mention of what we now call a "wavelet" seems to be in 1909, in a thesis by Alfred Haar [1]. The concept of wavelets in its present theoretical form was first proposed by Jean Morlet and the team at the Marseille Theoretical Physics Center working under Alex Grossmann in France. The methods of wavelet analysis have been developed mainly by Y. Meyer and his colleagues, who have ensured the methods' dissemination.

The main algorithm dates back to the work of Stephane Mallat in 1988. Since then, research on wavelets has become international. Such research is particularly active in the United States, where it is spearheaded by the work of

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scientists such as Ingrid Daubechies, Ronald Coifman, and Victor Wickerhauser.

A. Haar

Any discussion of wavelets begins with Haar wavelet, the first and simplest. Haar wavelet is discontinuous, and resembles a step function. It represents the same wavelet as Daubechies db1. The basis function of Haar wavelet is shown in figure 1.

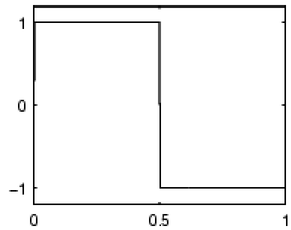

Fig. 1 Basis function of Haar Wavelet

B. Daubechies

Ingrid Daubechies, one of the brightest stars in the world of wavelet research, invented what are called compactly supported orthonormal wavelets -- thus making discrete wavelet analysis practicable [2]. The names of the Daubechies family wavelets are written dbN, where N is the order, and db the "surname" of the wavelet. The db1 wavelet, as mentioned above, is the same as Haar wavelet. The figure 2 shows the wavelet functions of the next nine members of the family.

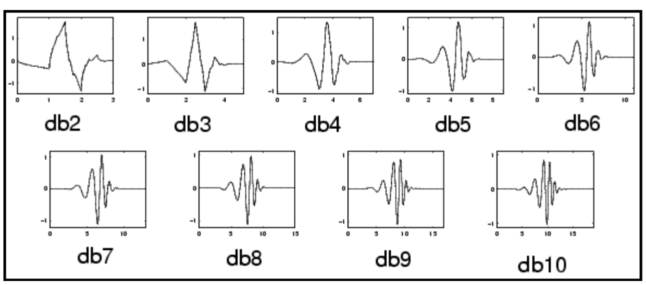

Fig. 2 Basis functions of Daubechies Wavelet Family

C. Biorthogonal

This family of wavelets exhibits the property of linear phase, which is needed for signal and image reconstruction. By using two wavelets, one for decomposition (on the left side) and the other for reconstruction (on the right side) instead of the same single one, interesting properties are derived. The basis functions of Biorthogonal wavelets are shown in figure 3.

D. Coiflet

Coiflet wavelets are built by I. Daubechies on the request of R. Coifman. The wavelet function has 2N moments equal to 0 and the scaling function has 2N-1 moments equal to 0. The two functions have a support of length 6N-1 [3]. The figure 4 shows the basis functions of the family of Coiflet wavelets.

E. Symlet

The symlets are nearly symmetrical wavelets proposed by Daubechies as modifications to the db family. The properties of the two wavelet families are similar. The figure 5 shows the wavelet functions.

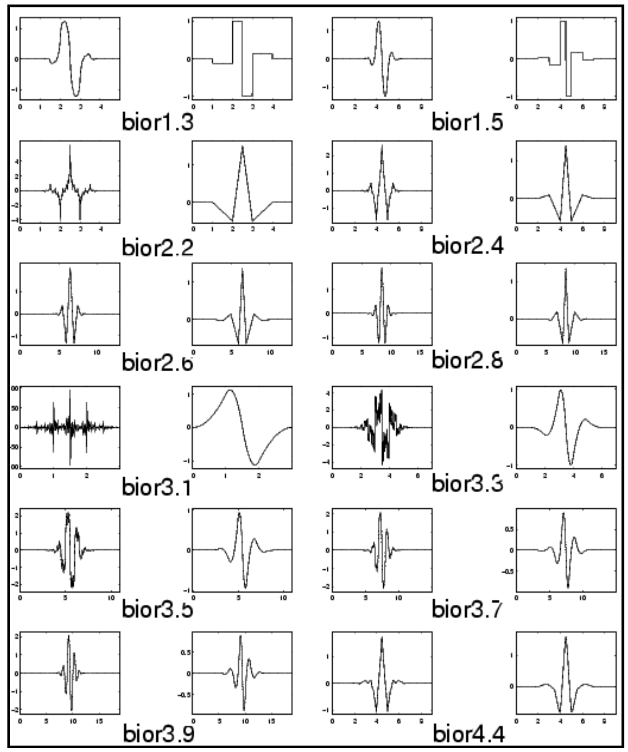

Fig. 3 Basis functions of Bi-orthogonal Wavelet Family

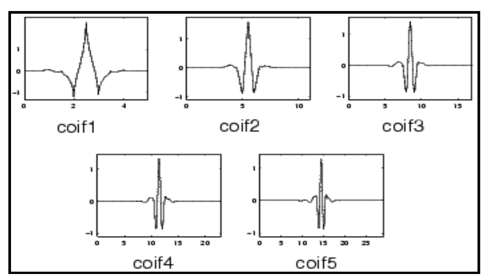

Fig. 4 Basis functions of Coiflet Family

F. Di-Meyer

The Meyer wavelet and scaling function are defined in the frequency domain [4]. The discrete version of Meyer wavelet is usually written as Di-meyer of which the basis function is plotted in the figure 6.

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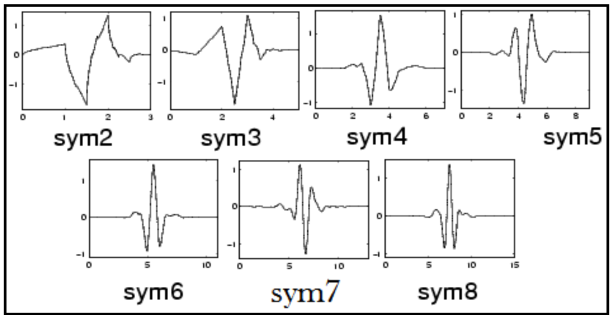

Fig. 5 Basis functions of Symlet Family

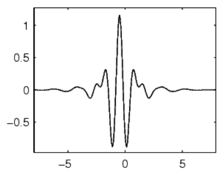

Fig. 6 Basis function of Discrete Meyer Wavelet

III. HAAR DECOMPOSITION BY LIFTING

The wavelets generated by translations and dilatations of a single or several basic functions are called first generation wavelets (classic wavelets). Since these operations represent algebraic operations in the frequency domain, the basic tool for their construction is the Fourier transform. There are a number of problems, such as problems defined on intervals, curves, surfaces or manifolds, where the Fourier transform cannot be applied, and thus neither can classic wavelets.

Classic wavelets also need to be modified when used for solving problems defined by irregular grids or where the inner product with a weight function needs to be used. Wavelets, attached to problems not allowing for translation and dilatation, are called *second generation 'wavelets.* Coefficients that correspond to these wavelets can depend on the resolution level. It is clear that working with non-constant coefficients is more complex. The basic idea of wavelet transform is to use a correlation existing in most signals in order to construct a good approximation with few addends.

A correlation is a typical local property in space (time) and frequency, meaning that neighbouring data and frequencies are far more correlated than those further moved from each other. In transformation with classic wavelets the basic tool for the space (time)-frequency localization is the Fourier transform, which cannot be applied to more complex geometries. However, localization can be performed in the physical domain space, time etc.), which is the essence of the so-called "lifting" algorithm. This algorithm was primarily developed for constructing second generation wavelets, but it is used successfully also for the construction of biorthogonal wavelets [5][6].

Lifting generalizes the idea of multiresolution to spaces that are not invariant relative to translation and dilatation, thus enabling users to create wavelets according to their needs and to speed up wavelet transform. The basic idea is to use the correlation between neighbouring data in the signal. The way in which this is performed shall be illustrated on a simple example of constructing biorthogonal wavelets. Wavelet algorithms are recursive. The output of one step of the algorithm becomes the input for the next step. The initial input data set consists of 2^p elements. Each successive step operates on 2^{p-i} elements, where i = 1 ... p-1. For example, if the initial data set contains 256 elements, the wavelet transform will consist of eight steps on 256, 128, 64, 32, 16, 8, 4, and 2 elements.

If element i in step j is being updated, the notation $step_{j,i}$ is used. The forward lifting scheme wavelet transform divides the data set being processed into an even half and an odd half. In the notation below $even_i$ is the index of the i^{th} element in the even half and odd_i is the i^{th} element in the odd half. Viewed as a continuous array the even element would be a[i] and the odd element would be a[i+(p/2)]. Another way to refer to the recursive steps is by their power of two. Here $step_{j-1}$ follows $step_j$, since each wavelet step operates on a decreasing power of two. This is a nice notation, since the references to the recursive step in a summation also correspond to the power of two being calculated.

A. *Predict Wavelets*

Like all lifting scheme wavelets the predict wavelet transform starts with a split step, which divides the data set into odd and even elements. The predict step uses a function that approximates the data set. The difference between the approximation and the actual data replaces the odd elements of the data set. The even elements are left unchanged and become the input for the next step in the transform. The predict step, where the odd value is "predicted" from the even value is described by the equation

$$odd_{j+1,i} = odd_{j,i} - P(\ even_{j,i}\ )$$

The inverse predict transform adds the prediction value to the odd element. In the inverse transform the predict step is followed by a merge step which interleaves the odd and even elements back into a single data stream. The simple predict wavelets are not useful for most wavelet applications. The even elements that are used to "predict" the odd elements result from sampling the original data set by powers of two (e.g., 2, 4, 8...).

B. *The update step*

The update step replaces the even elements with an average. This result in a smoother input for the next step of the wavelet transform. The odd elements also represent an approximation of the original data set, which allows filters to be constructed. A simple lifting scheme forward transform is shown in Fig. 7.

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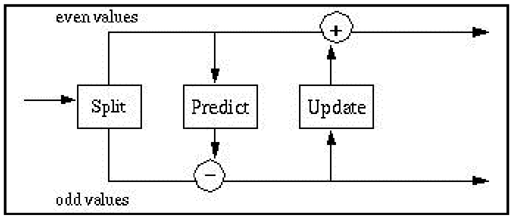

Fig. 7. Lifting Scheme – Forward wavelet transform

The update phase follows the predict phase. The original value of the odd elements has been overwritten by the difference between the odd element and its even "predictor". So in calculating an average the update phase must operate on the differences that are stored in the odd elements:

$$even_{j+1,i} = even_{j,i} + \boldsymbol{U}(\, odd_{j+1,i}\,)$$

C. Lifting Scheme Haar Transform

In the lifting scheme version of the Haar transform, the prediction step predicts that the odd element will be equal to the even element. The difference between the predicted value (the even element) and the actual value of the odd element replaces the odd element. For the forward transform iteration *j* and element *i*, the new odd element, *j+1,i* would be

$$odd_{j+1,i} = odd_{j,i} - even_{j,i}$$

In the lifting scheme version of the Haar transform the update step replaces an even element with the average of the even/odd pair (e.g., the even element s_i and its odd successor, s_{i+1}):

$$even_{j+1,i} = \frac{even_{j,i} + odd_{j,i}}{2}$$

The original value of the $odd_{j,i}$ element has been replaced by the difference between this element and its even predecessor. Simple algebra lets us recover the original value:

$$odd_{j,i} = even_{j,i} + odd_{j+1,i}$$

Substituting this into the average, we get

$$even_{j+1,i} = \frac{even_{i,i} + even_{j,i} + odd_{j+1,i}}{2}$$

$$even_{j+1,i} = even_{j,i} + \frac{odd_{j+1,i}}{2}$$

The averages (even elements) become the input for the next recursive step of the forward transform. This is shown in Fig. 8, below.

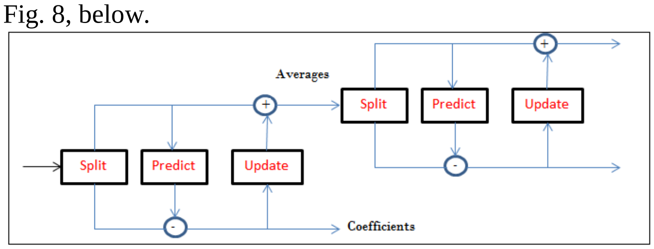

Fig. 8. Two Steps in Lifting forward transform

The number of data elements processed by the wavelet transform must be a power of two. If there are 2^p data elements, the first step of the forward transform will produce 2^{p-1} averages and 2^{p-1} differences. These differences are sometimes referred to as wavelet coefficients. Fig. 9 shows a 4-steps forward wavelet transform on a 16-element data set.

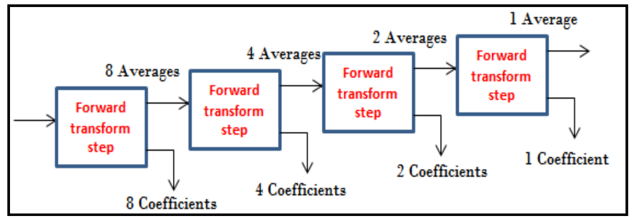

Fig. 9. 4 steps of a 16 element wavelet transform

The split phase that starts each forward transform step moves the odd elements to the second half of the array, leaving the even elements in the lower half. At the end of the transform step the odd elements are replaced by the differences and the even elements are replaced by the averages. The even elements become the input for the next step, which again starts with the split phase. One of the elegant features of the lifting scheme is that the inverse transform is a mirror of the forward transform which is shown is Fig. 10. In the case of the Haar transform, additions are substituted for subtractions and subtractions for additions. The merge step replaces the split step.

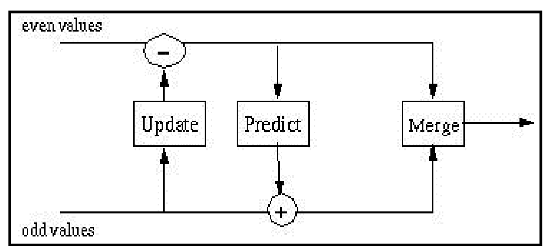

Fig. 10. Inverse Lifting scheme

IV. Simulation Results

In this section simulation results of implemented techniques are presented. A broad range of medical images are considered and results on five images are presented here. A number of lifting based wavelets are designed. The designed wavelets are “haar, db2, db4, db5, db8, bior4.4, cdf1.3, cdf5.3, sym3 and sym5”. All these wavelets are implemented using lifting scheme. The results are tabulated in table I.

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TABLE I
PERFORMANCE OF DIFFERENT LIFTING BASED WAVELETS ON MEDICAL IMAGES

Image	ct.jpg		mri.tif		mri3.tif		mri4.tif		mri5.tif	
Wavelet	CR	PSNR	CR	PSNR	CR	PSNR	CR	PSNR	CR	PSNR
Haar	3.19	39.11	3.23	39.76	2.79	34.69	2.81	34.85	2.81	34.89
db2	3.20	38.48	3.21	38.83	2.85	34.41	2.87	34.49	2.87	34.57
db4	3.54	38.90	3.25	39.68	3.16	37.40	3.17	37.52	3.20	37.46
db5	3.53	38.95	3.24	39.54	3.02	36.11	3.02	36.22	3.05	36.27
db8	3.13	33.98	2.79	31.29	3.06	32.46	3.06	32.46	3.09	32.83
bior4.4	3.51	38.71	3.24	39.59	2.91	35.11	2.92	35.18	2.93	35.37
cdf1.3	3.19	39.31	3.21	39.59	2.79	34.70	2.81	34.85	2.80	34.94
cdf5.3	3.20	34.79	2.74	29.24	2.89	35.98	2.90	35.78	2.91	36.33
sym3	3.19	38.90	3.25	39.67	2.81	34.66	2.82	34.75	2.83	34.95
sym4	3.22	38.59	3.24	39.22	2.94	35.00	2.96	35.16	2.97	35.26

Table II shows the average values of CR and PSNR obtained on the five images.

TABLE II
AVERAGE VALUES OF CR AND PSNR

Wavelet	CR	PSNR
Haar	2.97	36.66
db2	3.00	36.16
db4	3.26	38.19
db5	3.17	37.42
db8	3.02	32.61
bior4.4	3.10	36.79
cdf1.3	2.96	36.68
cdf5.3	2.93	34.43
sym3	2.98	36.59
sym4	3.07	36.65

The average CR and PSNR are plotted in Fig. 11 and 12. From the figures it can be understood that except with db8 and cdf5.3 the PSNR is greater than 35dB and CR is around 3bpp. The compression can be verified by the well-established design metrics PSNR and CR [7]. But by considering any one of the above, one cannot come to an opinion on how far the compression successful. Consider a case where PSNR is very high, but the CR is very less, say approximately 1. In this first case, even though the PSNR is very high, it can be well said that the image was not compressed. Think of calculating PSNR of one image with the same image. In the second case, consider CR is very high, but PSNR is very less. In this case the original image could not be reconstructed or recognized from the compressed image. Hence, one may think that this compression corrupted the image. So, in compression both PSNR and CR should be high [8][9][10].

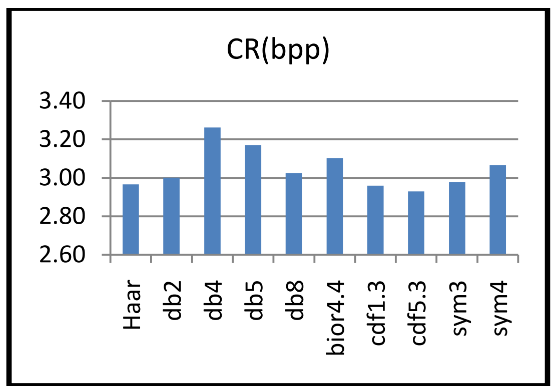

Fig. 11. Average CR

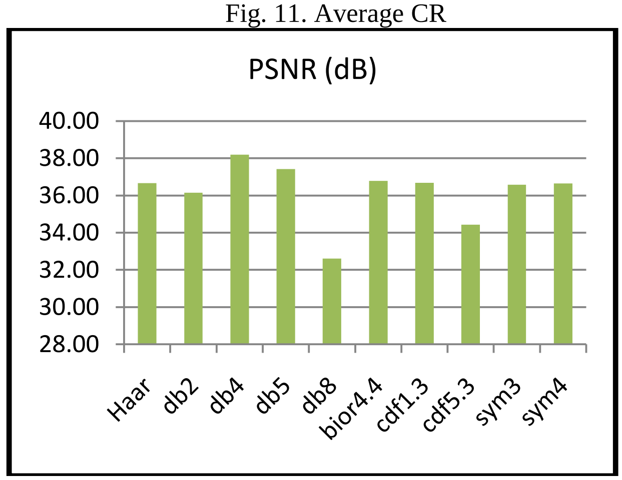

Fig. 12. Average PSNR

Because, for better compression performance, both PSNR and CR should be high, let us multiply these two. The new

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parameter will give us an idea on which wavelet is performing better. The PSNR*CR values are calculated for the lifting scheme and plotted in the Fig. 13. Of course, the discussion is not on finding better wavelet here for compression, but to verify how the lifting based wavelets are performing over the traditional wavelets.

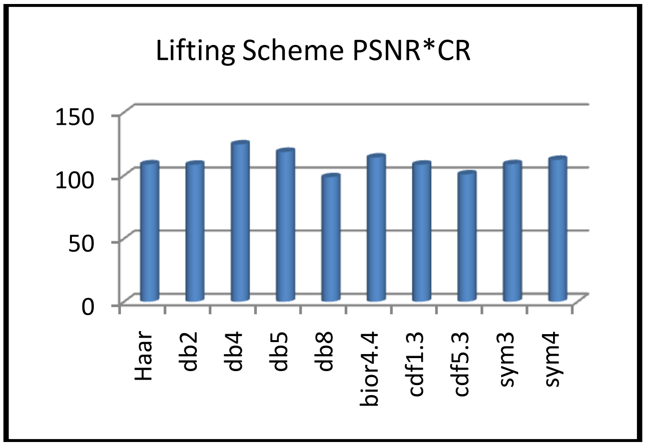

Fig. 13. PSNR*CR of Lifting Scheme

For that purpose consider the traditional wavelets results obtained and presented in [11] for reference. The PSNR*CR is calculated for the traditional wavelets of table III and plotted in figure 14. By comparing the figures 13 and 14, one can easily state that the lifting based transforms outperform the traditional wavelets.

TABLE III
COMPRESSION PERFORMANCE OF TRADITIONAL WAVELETS

Image	Parameter	HAAR	DAUBECHIE	BIORTHOGONAL	DEMEYER	COIFLET	SYMLET
CT	**CR**	3.4	1.77	3.05	1.9	2.89	1.88
	PSNR	24.7	24.3	26.7	24.6	45.5	24.6
MRI-1	**CR**	3	2.6	2.7	1.62	2.4	2.7
	PSNR	24.8	30.9	32.18	30.8	49.8	30.9
MRI-2	**CR**	2.9	2.6	2.7	1.61	2.4	2.7
	PSNR	25.2	31	32.8	31.3	50	31.5
MRI-3	**CR**	3.62	3.478	3.53	2.12	3.2	3.57
	PSNR	23.1	23.74	24.07	23.77	40	23.7
MRI-4	**CR**	3.03	2.6	2.73	1.63	2.4	2.72
	PSNR	24.2	30.54	31.39	30.4	49.2	30.4
MRI-5	**CR**	3.05	2.6	2.7	1.63	2.46	2.73
	PSNR	23.9	30.1	31	30.05	48.8	30.42
MRI-6	**CR**	3.05	2.69	2.75	1.64	2.47	2.75
	PSNR	24.7	30.87	32.44	30.62	50.1	30.91

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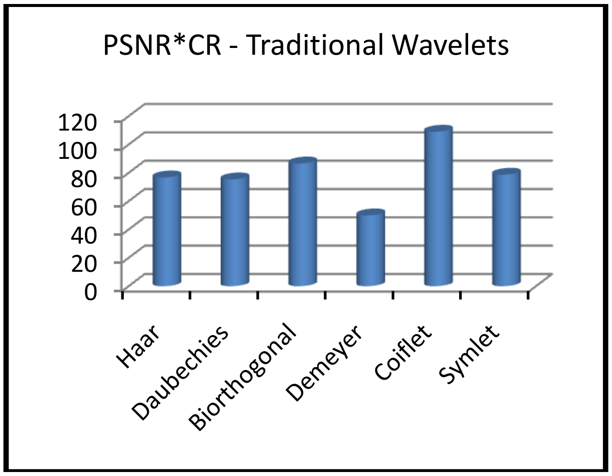

Fig. 14. PSNR*CR in case of Traditional Wavelets

V. CONCLUSIONS

This paper presents the lifting version of various classical wavelets. It has been observed that the compression performance was improved compared to that of classical wavelets. A new design metric in terms of PSNR and CR was proposed and used to analyse and compare the traditional and lifting based wavelets. With the traditional wavelets the PSNR*CR ranges between 65 and 100. Except with Coiflet wavelet, the PSNR*CR is less than 80. In the case of lifting based wavelets the PSNR*CR is greater than 100. With db4, the PSNR*CR is 124.5, which is very high compared to that of traditional db4's just 80+. With this analysis one can state that the lifting scheme outperforms classical wavelets.

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BIOGRAPHY

K.Gopi is working as an Assistant Professor in Department of ECE, Sreenivasa Institute of Technology and Management Studies, chittoor. He has 9 years of teaching experience. He has done his M.E in applied electronics in the year 2007. He is currently doing his research in digital image processing in the field of Medical Image Compression in S.V. University, Tirupati.

Dr.T.Rama Shri is working as Professor in the Department of ECE, S.V University College of Engineering, Tirupati. She has more than 18 years of Teaching Experience. She has published more than ten research papers in national and international conferences. Her research interests include Digital Image Processing. She is a life member of ISTE and IETE.