



Performance of Modified MAP Scheme for Cooperative Spectrum Sensing over Fading Channels

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Abstract: Cooperative spectrum sensing in cognitive radio is a robust strategy to combat fading and shadowing effects. However, large number of secondary users reporting to the fusion center for a final decision can cause significant delay. The local detectors transmit their log-likelihood ratio values in the descending order of their magnitudes. To obtain a quick and reliable decision, sequential detection is employed at the fusion center. Modified MAP scheme is proposed in which detection thresholds are calculated using maximum a posteriori (MAP) procedure with an imposed constraint on the number of sensors. Fading and shadowing effects on the reporting channels between the local detectors and the fusion center are explored. The performance of the proposed scheme is compared for the cases of perfect and fading reporting channels using simulation.

Keywords: Cognitive radio, cooperative spectrum sensing, sequential detection, cognitive control channels, fading and shadowing effects.

I. INTRODUCTION

Cognitive radio is aimed at solving the problem of spectrum underutilization. Federal Communications Commission (FCC) defines Cognitive Radio as “Cognitive radio: A radio or system that senses its operational electromagnetic environment and can dynamically and autonomously adjust its radio operating parameters to modify system operation, such as maximize throughput, mitigate interference, facilitate interoperability, access secondary markets” [1]. The cognitive radio technology allows the Secondary User (SU), also called cognitive or unlicensed user, to access the spectrum, without disrupting the activity of the Primary (or licensed) User (PU). Among other functions, Spectrum Sensing is the most crucial task to establish Cognitive Radio Networks. It provides the key ability for secondary users to detect the unused spectrum and share it without causing harmful interference to primary users. Thus, spectrum sensing involves detection of primary activity by the secondary terminals so that the secondary operation does not interfere with the primary network. Primary users can claim their frequency bands anytime while cognitive radio is operating on their bands. In order to prevent interference to and from primary license owners, cognitive radio should be able to identify the presence of primary users as quickly as possible and should vacate the band immediately. This detection task is extremely difficult in view of the propagation conditions between the primary terminal and the secondary sensor attempting to detect primary activity. Therefore, relying on one sensor to detect the presence of primary activity is highly unreliable. Enhancement of sensing reliability requires a system of spatially distributed multiple sensors

cooperating with each other for making a combined decision at the Fusion Centre (FC).

Sequential detection is used at the FC as it is known to reduce the time of detection for specified detection reliability [2]. For the case of binary hypothesis testing, sequential detection requires two thresholds for operation. Each sensor computes the log-likelihood ratio (LLR) for its observations and reports it to the FC. Specifically, the sensors report their LLR values in the decreasing order of their magnitudes. This is accomplished through ordered transmissions from the local sensors to the FC. The FC receives the LLR measurements from the local sensors and performs sequential detection to obtain a decision on the channel occupancy. Also, a constraint is imposed on the maximum number of sensors that can send their transmissions to the FC and the detection thresholds are modified accordingly. Therefore, a decision is forced after a specified number of observations are accumulated at the FC. Assuming a slotted primary network where the primary activity switches every fixed amount of time, only few observations can be used within the primary time slot to make a decision regarding the availability of the transmission opportunity.

For the proposed Modified MAP scheme, lesser the number of sensors reporting their values to the FC, greater is the duration available for secondary transmission if the channel is declared to be free. Average number of probed sensors that are involved in the detection process is therefore, a performance parameter for the Modified MAP scheme. The local detectors transmit their LLR values to the FC over control channels, also called reporting



channels or cognitive control channels. If the control channel is prone to fading and shadowing, then the measurements acquired by the FC may not be producing an effective detection decision. This can cause the secondary users to lose a transmission opportunity. Hence, a spectrum sensing scheme should be devised in such a way that the detection decision is obtained correctly even in the presence of fading control channels. The proposed scheme shows a superior performance in preserving the transmission opportunity of the cognitive users even under worst fading conditions of the reporting channels. The analysis for the modified MAP scheme is performed using perfect reporting channels. The impact of fading and shadowing on the reporting channels is also studied. The scheme performance is simulated for various fading channels such as Rayleigh, Weibull, exponential, Gaussian and log-normal models. Simulation results also show the comparison for both fading and no-fading cases.

The paper is organised as follows: Section II defines the system model and Section III gives the details of the proposed Modified MAP Scheme. Finally, section IV presents the simulation results and conclusion.

II. SYSTEM MODEL

A slotted primary system is considered, where the primary activity does not change during the time slot, τ_s and primary activity switches independently from one slot to the next. A total of N samples are measured and processed by all sensors in the network over a time duration τ_N . There are M cognitive sensors which take those measurements at the beginning of each time slot and compute a function of them. A maximum of K sensors among the total sensors in the network forward the results sequentially to a fusion center at which the final decision regarding primary activity is taken. It is assumed that the procedure of seizing the control channel and sending the observation of one sensor requires an amount of time τ . The slotted primary system is shown in Fig 1.

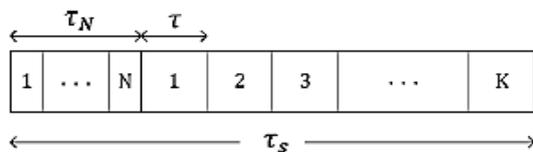


Fig 1: The primary time slot

At the fusion centre, binary hypothesis testing is considered as following:

H_0 : Sensed channel is free; H_1 : Sensed channel is busy.

The priori probabilities of each, are denoted by π_0 and $(1-\pi_0)$ respectively and are assumed to be known. Let $X_i(n)$ be the received signal at the i^{th} sensor at instant n , where $i=1,2,\dots,M$. At each sensor i , $X_i(n)$ are independent given each hypothesis and are identically distributed. Under the two hypotheses, $X_i(n)$ is given by

$$\begin{aligned} H_0 : X_i(n) &= W_i(n), \quad n = 1, 2, 3, \dots, N \\ H_1 : X_i(n) &= S_i(n) + W_i(n), \quad n = 1, 2, 3, \dots, N \end{aligned} \quad (1)$$

Where $W_i(n)$ is Additive White Gaussian Noise (AWGN) having the same noise power, σ^2 , at all sensors. The received primary signal $S_i(n)$ is assumed to be real zero-mean Gaussian random variable. The conditional probability distributions of $X_i(n)$ given H_0 and H_1 are described by $f_{X_i(n)}(x_n | H_0)$ and $f_{X_i(n)}(x_n | H_1)$, respectively, such that

$$\begin{aligned} f_{X_i(n)}(x_n | H_0) &\sim N(0, \sigma^2) \\ f_{X_i(n)}(x_n | H_1) &\sim N(0, \sigma_{s_i}^2 + \sigma^2) \end{aligned} \quad (2)$$

where $\sigma_{s_i}^2$ is defined as the average received primary signal power at the i^{th} local sensor. The LLR at the i^{th} sensor [2] is defined as

$$Y_i = \sum_{n=1}^N \log \left[\frac{f_{X_i(n)}(x_n | H_1)}{f_{X_i(n)}(x_n | H_0)} \right] \quad (3)$$

On substituting the likelihood functions of (2) in (3), the LLR value can be simplified as

$$Y_i = \frac{1}{2\sigma^2} \cdot \left(\frac{\gamma_i}{\gamma_i + 1} \right) \sum_{n=1}^N |X_i(n)|^2 - \frac{N}{2} \log(\gamma_i + 1) \quad (4)$$

where $\gamma_i = \frac{\sigma_{s_i}^2}{\sigma^2}$ is defined as the local signal-to-noise ratio

(SNR). Thus, the LLR value computed at each local sensor is a shifted and scaled chi-square distribution with N degrees of freedom, both under H_1 and H_0 .

The LLR values from the sensors are transmitted to the fusion center sequentially through the cognitive control channels, also called the reporting channels. The reporting channels are assumed to be perfect and are different from the channel being sensed. Sequential detection is carried out at the FC using two thresholds. The sensors having most reliable observations send their LLR values using ordered transmission [3]. The LLR with maximum magnitude is transmitted first to the FC. A decision can be made, but if the value is between the two thresholds, the second highest LLR is transmitted and is combined with the first and a decision is attempted again. This continues until a decision is made or K LLRs are accumulated at the FC. This imposes a constraint on the maximum number of sensors that can participate in the detection activity. If ' k ' sensors are probed before a decision is made, where $1 \leq k \leq K$, the time taken to make a decision is $\tau_N + k\tau$. Since the primary slot duration is τ_s , this leaves $\tau_s - \tau_N - k\tau$ for transmission. Thus K satisfies the inequality $\tau_s - \tau_N - K\tau \geq 0$.

For fading/shadowing channels between the local sensors and the FC, the statistical distribution of the channel gain between the i^{th} sensor and the FC is given by $f_{g_i}(x)$, possibly different for different sensors. The instantaneous values of the channel gains are known and fixed over the duration T_c , which is multiples of primary slot duration τ_c . For the transmission from the i^{th} sensor to be decodable, the rate of transmission r_i should be less than the link capacity C_i . For a fixed and known channel gain g_i at the transmitter and additive white Gaussian noise at the receiver, the link capacity is given as [4],



$$C_i = W \log \left(1 + \frac{P_i g_i}{\sigma_{f^2}} \right) \quad (5)$$

where W is the reporting channel bandwidth, σ_{f^2} is the noise variance at the receiver of the FC and P_i is the transmitted power at the i^{th} sensor. The rate of transmission is given by

$$r_i = W \log \left(1 + \frac{P_i g_i}{\Gamma_i \sigma_{f^2}} \right) \quad (6)$$

where $(\Gamma_i > 1)$ is the SNR gap to capacity. If 'b' information bits are to be transferred to the FC, the time duration needed for this transfer conditioned that the information is decodable is given by b/r_i . Assume that this transmission takes place in the duration τ_b which is smaller than τ . Thus,

$$\frac{b}{W \log \left(1 + \frac{P_i g_i}{\Gamma_i \sigma_{f^2}} \right)} \leq \tau_b \quad (7)$$

This can be written as

$$g_i \geq \frac{\Gamma_i \sigma_{f^2}}{P_i} \left(2^{\frac{b}{W \tau_b}} - 1 \right) \quad (8)$$

The probability that a sensor would report its LLR value to the FC is given by

$$\delta_i = \int_{g_i}^{\infty} f_{g_i}(x) dx \quad (9)$$

where \bar{g}_i is the right-hand-side of the inequality (8). Since the channels are assumed to be known at the sensors as well as at the FC, each sensor can decide whether it should participate in the next sensing epoch and also the FC can know the subset of participating sensors. Thus, the average number of sensors participating in the decision making process is $\sum_{i=1}^M \delta_i$.

III. MODIFIED MAP SCHEME

In this scheme, ordered transmissions from the local sensors are accumulated sequentially at the fusion center. The thresholds for the sequential detection are calculated using Maximum a Posteriori (MAP) procedure with a constraint on the maximum number of sensors. A back-off timer is set at each of the M sensors and specifically, the timer is decreased as the magnitude (absolute value) of the locally computed LLR increases. Thus, the most informative measurements are sent to the fusion center. The FC then compares the accumulated sum of LLRs to the dynamically calculated thresholds and makes a decision accordingly. Let $Y^{[m]}$ denote the LLR of rank $m=1, 2, 3, \dots, K$ received during m^{th} mini-slot, where $m=1$ denotes the highest rank. For $1 \leq k \leq K$, the decision strategy is given by

$$\text{If } \sum_{m=1}^k Y^{[m]} \begin{cases} < t_{Lk} \\ \in (t_{Lk}, t_{Hk}) \\ > t_{Hk} \end{cases} \begin{matrix} \text{Declare } H_0 \\ \text{Continue} \\ \text{Declare } H_1 \end{matrix}$$

The data-dependent thresholds t_{Lk} and t_{Hk} for k^{th} stage are given in [3] as

$$\begin{aligned} t_{Lk} &= \log \left(\frac{\pi_0}{1 - \pi_0} \right) - (M - k) |Y^{[k]}| \\ t_{Hk} &= \log \left(\frac{\pi_0}{1 - \pi_0} \right) + (M - k) |Y^{[k]}| \end{aligned} \quad (11)$$

where $(M-k)$ is the number of sensors that have not yet transmitted their LLRs at the k^{th} stage and $|Y^{[k]}|$ is the absolute value of the LLR received at k^{th} stage. The transmission is ordered in terms of the absolute LLR value, i.e. $|Y^{[m]}| < |Y^{[k]}|, \forall m > k$. The MAP scheme is extended for $K \leq M$ sensors such that only K out of M LLRs with highest magnitude are processed. Given the sequence of observations $Y^{[1]} = y_1, Y^{[2]} = y_2, \dots, Y^{[K]} = y_K$, the decision rule is given by

$$\begin{aligned} \log \left[\frac{f_{Y^{[1]} \dots Y^{[K]} | H_1}(y_1, y_2, \dots, y_K | H_1)}{f_{Y^{[1]} \dots Y^{[K]} | H_0}(y_1, y_2, \dots, y_K | H_0)} \right] &> \log \frac{\pi_0}{1 - \pi_0} \\ \log \left[\frac{f_{Y^{[1]} \dots Y^{[K]} | H_1}(y_1, y_2, \dots, y_K | H_1)}{f_{Y^{[1]} \dots Y^{[K]} | H_0}(y_1, y_2, \dots, y_K | H_0)} \right] &< \log \frac{\pi_0}{1 - \pi_0} \end{aligned} \quad (12)$$

where $f_{Y^{[1]} \dots Y^{[K]} | H}(y_1, y_2, \dots, y_K | H)$ is the joint density function of the magnitude-ordered LLRs given hypothesis H . Let the probability density function of LLR value y under hypothesis H be given by $f_Y(y|H)$. Assuming identical and statistically independent sensors, the joint density function is given by

$$\begin{aligned} f_{Y^{[1]} \dots Y^{[K]} | H}(y_1, y_2, \dots, y_K | H) \\ = f_Y(y_1 | H) \dots f_Y(y_K | H) (\Pr\{|Y| \leq |y_K|\})^{M-K} \end{aligned} \quad (13)$$

In order to perform sequential detection, the accumulated sum of LLRs at the fusion center at stage k is computed as $\sum_{m=1}^k \log \left[\frac{f_Y(y_m | H_1)}{f_Y(y_m | H_0)} \right]$ which can be replaced by $\sum_{m=1}^k y_m$ [5]. This accumulated sum is then compared to two thresholds \hat{t}_{Lk} and \hat{t}_{Hk} , if it is smaller than \hat{t}_{Lk} , then H_0 is declared and if it is greater than \hat{t}_{Hk} , then H_1 is declared. If it does not cross either of the thresholds, then the fusion center continues to accumulate the LLRs from the next ranked sensors. The thresholds are given by

$$\begin{aligned} \hat{t}_{Lk} &= \log \frac{\pi_0}{1 - \pi_0} - (K - k) |y_k| + (M - K) \max_{0 \leq y \leq |y_k|} \rho(y) \\ \hat{t}_{Hk} &= \log \frac{\pi_0}{1 - \pi_0} + (K - k) |y_k| - (M - K) \min_{0 \leq y \leq |y_k|} \rho(y) \end{aligned} \quad (14)$$

where for LLR Y ,

$$\rho(y) = \log \frac{\Pr\{|Y| \leq y | H_1\}}{\Pr\{|Y| \leq y | H_0\}} \quad (15)$$

is a correction term to (11) that appears at each stage k to account for the probability that all the other $(M-K)$ sensors would have LLRs less than $|y_k|$. Since $\rho(y)$ is not a monotonic function, it is spanned over the range $0 \leq y \leq |y_k|$ and maximum and minimum values are chosen.



The thresholds of (14) are formulated such that the upper threshold is much higher and the lower threshold is much lower than the MAP reference value $\log(\pi_0 / (1 - \pi_0))$. It is noteworthy that these thresholds are different from those mentioned in [5]. The thresholds do not converge at $k = K$. The average probability of error of the modified MAP scheme is given by,

$$\begin{aligned}
 P_e &= \Pr(H_1) \Pr(H_0 | H_1) + \Pr(H_0) \Pr(H_1 | H_0) \\
 &= (1 - \pi_0) \Pr(H_0 | H_1) + \pi_0 \Pr(H_1 | H_0) \quad (16) \\
 &= (1 - \pi_0) \int f_y(y | H_1) dy + \pi_0 \int f_y(y | H_0) dy
 \end{aligned}$$

The advantage of ordered transmissions is that the average number of transmissions needed to reach a decision is about half the total number of sensors communicating with the fusion center [3]. This applies for the modified MAP scheme also. In this case, the number is $K/2$, which is shown through simulation. When the simulation is run over Q slots, the normalized secondary throughput is given by

$$\frac{1}{Q} \sum_{q=1}^Q I_q^{(S)} R_s \left(1 - \frac{\tau_N + k_q \tau}{\tau_s} \right) \quad (17)$$

where $I_q^{(S)}$ is equal to unity if one of the secondary users transmits successfully over the q^{th} time slot and zero otherwise, and k_q is the number of probed sensors in the q^{th} slot. The normalized primary throughput is given by,

$$\frac{1}{Q} \sum_{q=1}^Q I_q^{(P)} R_p \quad (18)$$

where $I_q^{(P)}$ is equal to unity when the primary user transmits successfully over the q^{th} time slot and zero otherwise. R_p and R_s are the primary and secondary rates of transmission respectively and are set to unity. Normalized weighted sum throughput is given by

$$\omega \frac{1}{Q} \sum_{q=1}^Q I_q^{(P)} R_p + (1 - \omega) \frac{1}{Q} \sum_{q=1}^Q I_q^{(S)} R_s \left(1 - \frac{\tau_N + k_q \tau}{\tau_s} \right) \quad (19)$$

Increasing the weight puts more emphasis on the primary throughput. The performance of the Modified MAP Scheme can be evaluated in terms of throughput and average number of probed sensors per detection cycle amongst other factors.

IV. RESULTS

In this section, the Modified MAP scheme performance is evaluated via simulation. Comparison is presented for perfect reporting channels as well as fading/shadowing channels. The simulation parameters are as follows.

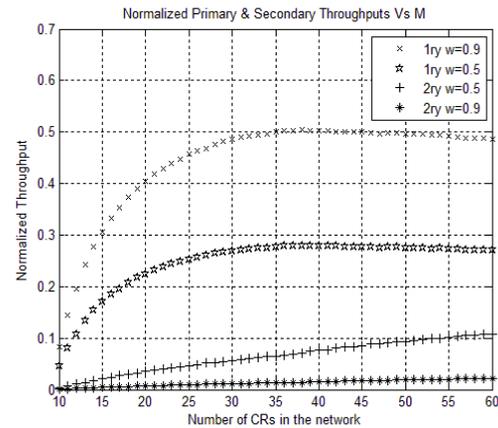


Fig 2: Normalized primary and secondary throughputs versus M.

The primary slot duration $\tau_s = 1$ and the transmission of each observation takes $\tau = 0.1$ units of time. The time taken to collect $N=3$ observations is $\tau_N = 2\tau$. These parameters define the constraint on the number of sensors, i.e. the maximum number of sensors, K that can participate in the detection activity. Therefore there are $K=8$ stages in the primary slot in which FC takes the ordered LLR observations.

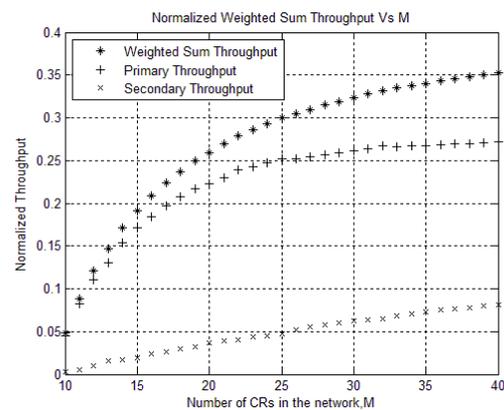


Fig 3: Normalized primary, secondary and weighted sum throughputs versus no. of CRs in the network, M for $\omega=0.5$.

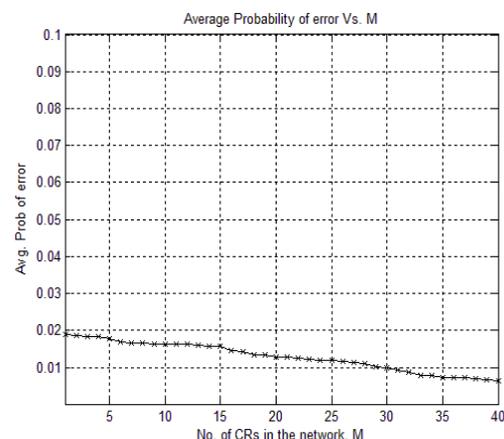


Fig 4: Average probability of error versus number of CRs in the network, M.

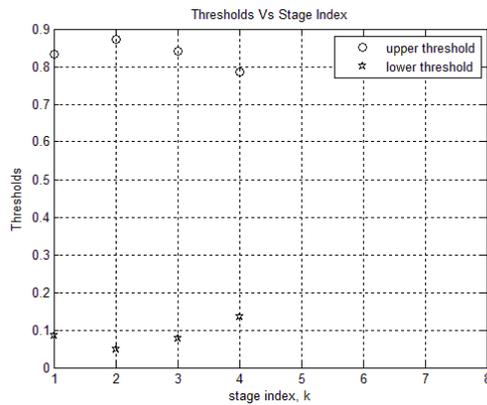


Fig 5: Lower and upper thresholds of sequential detection versus the stage index, k. Channel decision is obtained in four stages.

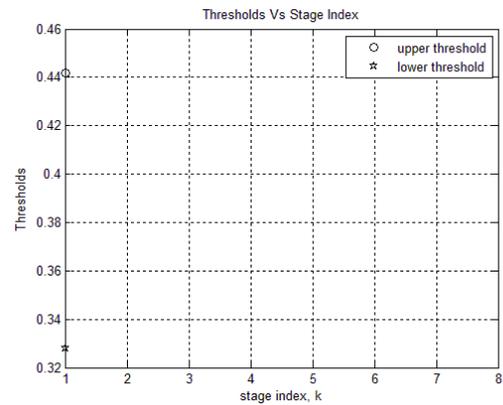


Fig 7: Lower and upper thresholds of sequential detection versus the stage index, k. Channel decision is obtained in single stage.

Equal priori probabilities are considered for both the hypotheses i.e. $\pi_0 = 0.5$ and equal local SNR is considered for the channels between the local sensors and the primary user.

For fading/shadowing channels, the number of information bits transferred by the local sensor to the FC is considered as $b=4096$ and the time required to transmit 'b' bits is taken as $\tau_b = 0.09$ units of time, which is less than τ . Fig. 2 shows the normalized primary and secondary throughputs for weights $\omega = 0.5$ and 0.9 . In Fig. 3, normalized weighted sum throughput of (19) is plotted along with normalized primary and secondary throughputs for a weight of $\omega = 0.5$.

In Fig. 4, the average probability of error is plotted against the number of CRs in the network. The error probability decreases with the increased number of CRs in the cognitive network Fig. 5 gives the plot of lower and upper thresholds of sequential detection against the stage index, k.

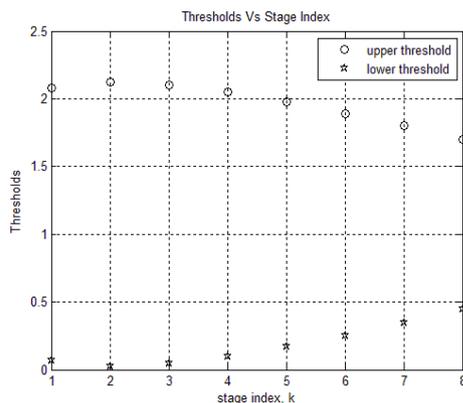


Fig 6: Lower and upper thresholds of sequential detection versus the stage index, k. Channel decision is obtained in eight stages.

In this case, the channel decision is obtained in four stages which implies that $(1-(0.2+4*0.1)) = 0.4$ units of time is available for secondary transmission, as mentioned in section II. The similar plot in Fig. 6 reveals that all the 8 stages in the primary slot are utilized for detection process and hence, the channel decision is forced in the last stage and the channel is declared to be busy.

In contrast, in Fig. 7, the channel decision is obtained in a single stage with the channel declared free, leaving 0.7 units of time for secondary transmission which is the maximum transmission opportunity for the secondary network in the available primary slot duration. Fig. 8 shows the plot of the average number of probed sensors against the maximum number of sensors that can participate in the detection decision, K. $\sigma_{s_1}^2 = 2$ for low SNR and $\sigma_{s_1}^2 = 20$ for high SNR. The maximum number of probed sensors is observed to be $K/2$.

Fig. 9 gives a comparison of scheme performance for various fading and shadowing channels along with the perfect reporting channels. As observed from the plot, the scheme performance is the best for the "no fading" case, where the number of probed sensors is less than two.

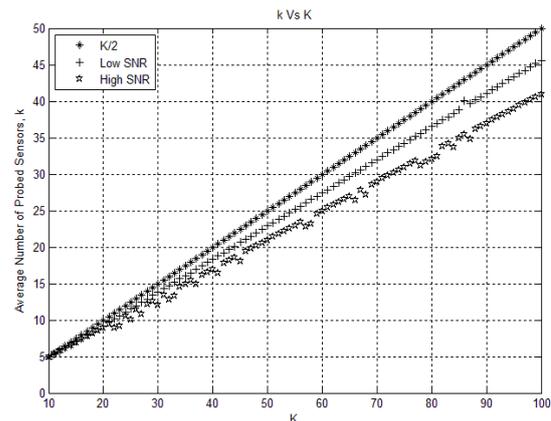


Fig 8: Average number of probed sensors versus maximum number of sensors involved in detection decision, K.

The performance of AWGN channel stands next to that of the perfect reporting channel. Maximum number of sensors is required for detection decision when the cognitive control channel suffers from log-normal shadowing. This is because the receiver loses the signal due to shadowing [6].

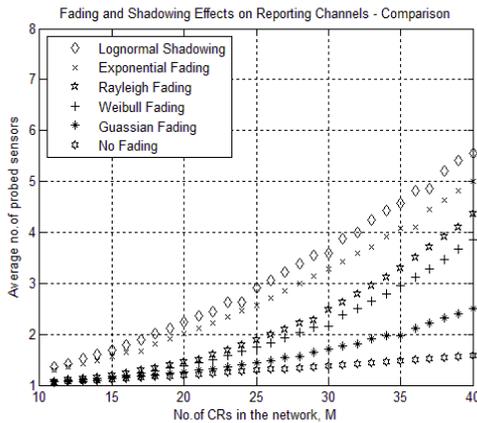


Fig 9: Comparison of Average number of probed sensors versus M for perfect reporting channel and various fading/shadowing channels.

Exponential fading channel also requires a maximum of up to five sensors, the performance of which is inferior to that of other fading channels such as Weibull and Rayleigh. Among Weibull and Rayleigh fading channels, the Weibull channel shows better performance than the Rayleigh channel.

V. CONCLUSION

In this paper, a cooperative spectrum sensing scheme called the Modified MAP Scheme is proposed. The analysis for the scheme is performed considering perfect reporting channels and the scheme is further extended for enhancing the performance of the cognitive system over fading control channels. As observed from the simulation results, the proposed scheme efficiently combats the effects of fading control channels. The scheme provides more than 25% of primary slot duration for secondary transmission when the signal is shadowed in the worst case and much better transmission opportunities over other fading channels. In conclusion, the Modified MAP scheme shows optimum performance even in worst fading conditions.

REFERENCES

- [1] Federal Communications Commission, "Notice of proposed rulemaking and order: Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies," ET Docket No. 03-108, Feb. 2005.
- [2] Mischa Schwartz and Leonard Shaw, Signal Processing—Discrete spectral analysis, detection and estimation, Mc Graw Hill book company, 1975.
- [3] R.S.Blum and B.M.Sadler, "Energy efficient signal detection in sensor networks using ordered transmissions", IEEE Trans. Signal Process., vol. 56, no. 7, pp. 3229-3235, July 2008.
- [4] Simon Haykin, Communication systems, 3/e, John Wiley and sons, 1995.
- [5] Laila Hesham, Ahmed Sultan, Mohammed Nafie and Fadel Digham, "Distributed spectrum sensing with sequential ordered transmissions to a cognitive fusion center", IEEE Trans. Signal Process., vol. 60, no. 5, May 2012.
- [6] Marvin K Simon and M S Alouini, *Digital communications over fading channels – A unified approach to performance analysis*, John Wiley and sons, 1976.

BIOGRAPHIES



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