A Methodical Approach for the Construction of 64-QAM Golay Complementary Sequences Having Low Peak-To-Average Power Ratio

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Abstract – This paper interprets the reduction of peak-to-average power ratio (PAPR) of 64-quadrature amplitude modulation (QAM) signals in the most promising technique of high speed parallel data transmission, well-known as orthogonal frequency division multiplexing (OFDM). As one of the major impediments to deploying OFDM is the high PAPR of the uncoded OFDM signals, so the Golay complementary sequences are used to construct 64-QAM OFDM sequences to provide low PAPR. It has been explicated that a 64-QAM constellation can be written as the vector sum of three QPSK constellations respectively. Further it has been investigated that by using these sequences, PAPR is bounded above by6.69dB.

Keyword: Orthogonal Frequency Division Multiplexing (OFDM), Peak-to-Average Power Ratio (PAPR),Golay Complementary Sequences (GCS)

I.   INTRODUCTION

OFDM (Orthogonal frequency division multiplexing) is a special subset of Multicarrier communications, which is based on the principle of transmitting simultaneously many narrow- band orthogonal frequencies, often called OFDM subcarriers or subcarriers. OFDM has emerged as the standard of choice in a number of important high data applications   due to its inherent robustness against multipath fading channels, easy implementation and high spectral efficiency [1]. It is widely being used in the Digital Television Broadcasting (such as the digital ATV Terrestrial Broadcasting), European Digital Audio Broadcasting (DAB) and Digital Video Broadcasting Terrestrial (DVB-T),
PAPR reduction methods can be categorized as distortion type e.g. clipping [3], peak windowing, peak cancellation etc [4] and non-distortion type methods e.g. peak reduction carrier, selective level mapping (SLM) [5], partial transmit sequence (PTS) [6] and block coding. Coding method not only reduces PAPR to greater extent but also has the advantage of error correction associated with it. Golay codes are an excellent example of coding method whose PAPR is at most 2 [7]. Davis and Jedwab disclosed a relationship

The rest of the paper is organized as follows. Section II describes some background material on OFDM and PAPR. In section III Golay complementary sequences and their properties are reviewed. In section IV construction of 64-QAM OFDM sequences is discussed and its PAPR is calculated. Finally the paper is then concluded in V.

Wireless Asynchronous Transfer Mode (WATM). Subcarriers are made orthogonal to each other which eliminate the interference between channels and use the spectrum much more efficiently by spacing the channels more closely together [2].

High Peak to Average Power Ratio is one of the major practical complications of the uncoded OFDM signals. A higher PAPR means the system requires large linearity range for the analog circuitry used and increase in the dynamic range of the analog-to-digital converter. This causes expensive system elements, higher power consumption and lower efficiency of the power amplifier (PA).

between Golay Complementary Sequences (GCS) and second order Reed-Muller (RM) codes that the large sets of binary length Golay complementary pairs can be obtained from certain second-order cosets of the classical first order RM code. More precisely the 2^h-PSK Golay sequences can be obtained from certain cosets of the first order Reed-Muller codes within the second order Reed-Muller codes over the integer ring $\square_{2^h}$.

II.   PRELIMINARIES

A.   OFDM

In OFDM a high rate data stream is divided into many low data streams. These binary data streams are then mapped to form digital symbols using modulation techniques such as BPSK, QPSK, and QAM etc. These mapped symbols are

then superimposed onto the orthogonal carriers in IFFT block. A composite signal so formed by multiplexing these modulated signals is called the OFDM signal. Such a scheme has several advantages like easier frequency-domain equalization, low sensitivity to impulse noise, good modularity etc. Moreover, it is possible to choose the constellation size and energy for each subcarrier, thus approaching the theoretical capacity of the channel.

A simple block diagram of an OFDM system is shown in Figure1. The encoded data is converted to parallel form, and is mapped in the constellation used for modulation (e.g., QAM or PSK). The parallel data is inserted into an inverse discrete Fourier transform (IDFT) block, which outputs the time equi-distributed samples of the baseband signal.
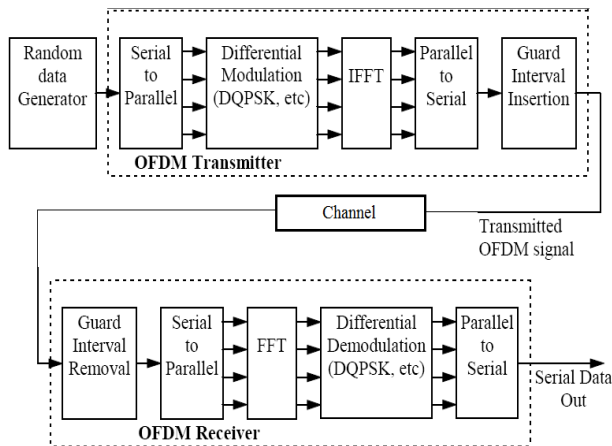

Fig.1 Block diagram of OFDM system

The next block introduces a guard interval (GI) used to abate the effect of the multipath propagation delay. The cyclic prefix (CP) makes the transmit signal periodic, and the subcarriers remain orthogonal at the receiver. The samples are then converted back to the serial form and transformed by a low-pass filter (LPF) to give a continuous signal. This continuous signal is amplified by a high-power amplifier (HPA) and transmitted. . The reverse steps are performed by the receiver.

Mathematically the transmitted OFDM signal can be represented as the real part of the complex signal and is given as

$$S(t) = \sum_{i=0}^{n-1} c_i(t) e^{2j\pi f_i t} \qquad (1)$$

Where f_i is the frequency of the ith carrier, $c_i(t)$ is constant over a symbol period of duration T. To maintain orthogonality, the carrier frequencies are related by

$$f_i = f_0 + i\Delta f \qquad (2)$$

f_0 is the smallest carrier frequency and Δf is the integer multiple of the OFDM symbol rate.

B.   PAPR

When the N sinusoidal signals after modulation by their carriers, adds mostly constructively the peak envelop power is as much as N times the mean power. As a result the amplitude of such a signal can have very large values. When high-peak power signals pass through power amplifiers and Analog to Digital (A/D) and Digital to Analog (D/A) converters, peaks are distorted non-linearly because of amplifier and converters imperfection. Thus, the output signal will suffer from intermodulation distortion resulting in energy being generated at frequencies outside the allocated bandwidth. So, PAPR is defined as the ratio between the instantaneous power of the peaks and the average power of the signal.

Let $c_i(t)$ in eq.(1) takes the value c_i over a given symbol period, then the corresponding OFDM signal is denoted by $S_c(t)$ and can be expressed as

$$S_c(t) = \sum_{i=0}^{n-1} c_i e^{2j\pi f_i t} \qquad (3)$$

Instantaneous envelope power associated with signal is givens

$$P_c(t) = \left| S_c(t) \right|^2 = S_c(t).S_c^*(t) \qquad (4)$$

Also the average power of c can be expressed as

$$\frac{1}{T}\int_0^T P_c(t) dt = \|c\|^2 = \sum_{k=0}^{n-1} |c_k|^2$$

$$\|c\|^2 = \sum_{k=0}^{n-1} |c_k|^2 \qquad (5)$$

Where,

$$\text{PAPR}(c) = \frac{\max_{0 \le t \le T} P_c(t)}{P_{av}} \qquad (6)$$

We would like to design codes C such that $PAPR(C) = \max_{C \in c} PAPR(c)$ are small. So ideally it is usually required that the percentage of codewords with high PAPR be an order of magnitude less than the probability of decoding error.

III.   GOLAY COMPLEMENTARY SEQUENCES

More than sixty years ago, efforts by Marcel Golay led to the discovery of complementary sequences which were later named after him as Golay complementary sequences. .These codes are capable of correcting any combination of three or fewer bit errors. . The perfect autocorrelation property of GCS has proved to be of value in a variety of applications. Golay complementary pairs have the property that the sum of their autocorrelation functions vanishes at all delays other than zero. Let $c = (c_0, c_1, ..., c_{n-1})$ be a sequence of length n such that $c_i \in \{+1, -1\}$ Then, Aperiodic Auto-Correlation Function (AACF) of c given as

$$A_c(u) = \sum_{i=0}^{n-u-1} c_i . c_{i+u} = c_u \otimes c^*_{-u} \qquad (7)$$

Let us consider two sequences 'a' and 'b' of length 'n', i.e.

$$a = (a_0, a_1, ..., a_{n-1})$$

$$b = (b_0, b_1, ..., b_{n-1}) \quad , \quad a_i, b_i \in \square_4$$

Where, $\square_h = (0, 1, ..., h-1)$ is an integer ring of size h. These two sequences are said to be Golay complementary pair if sum of their aperiodic autocorrelation can be written as, $A_a(u) + A_b(u) = (\|a\|^2 + \|b\|^2)\delta(u)$ (8)

$$\delta(u) = \begin{cases} 1 \text{ , if } u = 0 \\ 0 \text{ , if } u \neq 0 \end{cases}$$

$\delta(.)$ represents Kronecker function. Since the complex envelope signal $S_c(t)$ and the input vector $c = [c_k]_{k=0}^{n-1}$ forms the Fourier series pair, their conjugates also forms a Fourier pair. Also, $A_c(u)$ and $P_c(t)$ forms the Fourier transform pair, therefore aperiodic autocorrelation can be studied in frequency domain to obtain the knowledge about $P_c(t)$ in time domain.

Theorem 1. Let a be one of the sequence which belongs to a complementary pair. Then

$$PAPR(a) \leq 2.$$

Proof: Since $P_a(t) + P_b(t)$ and

$A_a(u) + A_b(u) = (\|a\|^2 + \|b\|^2)\delta(u)$ form a pair of Fourier series, taking Fourier transform we have $P_a(t) + P_b(t) = \|a\|^2 + \|b\|^2$, $0 \leq t \leq T$. For PSK modulation of unit energy, the average power $\|c\|^2$ of any sequence c is equal to $\sum_{k=0}^{n-1} |c_k|^2 = n$. Since $P_a(t)$ and $P_b(t)$ are non-negative and, $P_a(t) + P_b(t) = 2n$ thus we have $PAPR(c) \leq \dfrac{2n}{n} = 2$. So it is concluded that PAPR is upper bound by 2 for any Golay sequence c in PSK modulation.

IV.   64-QAM OFDM SEQUENCES

Rößing and Tarokh [8], Chong et.al [9] demonstrated a construction of 16-QAM sequences from QPSK Golay complementary sequences, and derived bounds for the PMEPR of the 16-QAM sequences. They observed that any 16-QAM symbol can be decomposed uniquely into a pair of QPSK symbols because any point on the 16-QAM constellation can be written as

$$S_{16-QAM} = \frac{1}{\sqrt{2}} S_{QPSK} + \sqrt{2} S_{QPSK} \qquad (9)$$

Similarly, the construction 64-QAM Golay sequences requires three QPSK symbols where each symbol can have four phase values as coefficients, that is fourth roots of unity; so as to get a total of 64 constellation points. So, three QPSK Golay complementary sequences are required to have 64 constellation points. Let S_{64-QAM} denotes the 64-QAM constellation symbols and can be written as the sum of three QPSK symbols.

$$S_{64-QAM} = \frac{1}{\sqrt{2}} S_{QPSK} + \sqrt{2} S_{QPSK} + 2\sqrt{2} S_{QPSK}$$

Figure 2 below shows the constellation diagram of 64-QAM as the sum of three QPSK symbols.
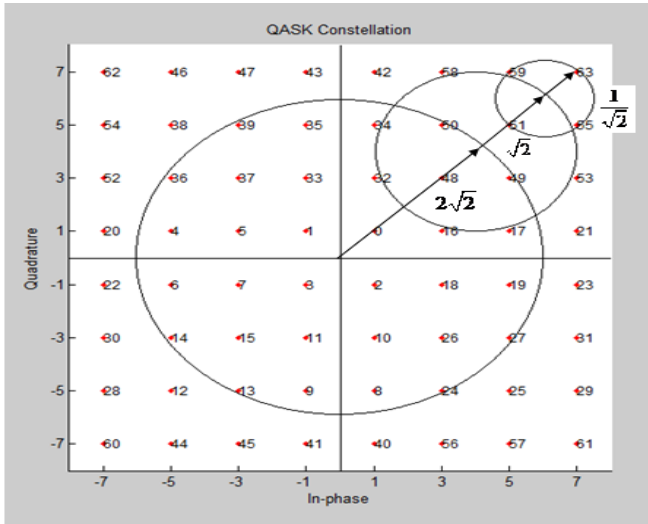
Figure2: 64-QAM symbols as sum of three QPSK symbols

Let $c = (c_0, c_1, ..., c_{n-1})$, $c_i \in S_{64-QAM}$ be the 64-QAM sequence and can be associated with three QPSK sequences $S_x(t)$, $S_y(t)$ and $S_z(t)$, where

$$S_{QPSK} = \left\{ e^{\frac{j\pi}{4}}, je^{\frac{j\pi}{4}}, -e^{\frac{j\pi}{4}}, -je^{\frac{j\pi}{4}} \right\}$$ So, c_k can be represented as

$$c_k = (\frac{1}{\sqrt{2}} j^{x_k} + \sqrt{2} j^{y_k} + 2\sqrt{2} j^{z_k}) e^{j\pi/4} \quad (10)$$

A 64-QAM OFDM signal may be written as the weighted sum of three QPSK OFDM signals as $S_c(t) = \sum_{k=0}^{n-1} c_k . e^{\frac{j2\pi kt}{T}}$

From (10),

$$S_c(t) = \sum_{k=0}^{n-1} (\frac{1}{\sqrt{2}} j^{x_k} + \sqrt{2} j^{y_k} + 2\sqrt{2} j^{z_k}) e^{j\pi/4} . e^{\frac{j2\pi kt}{T}}$$

$$\therefore S_c(t) = \frac{1}{\sqrt{2}} S_x(t) + \sqrt{2} S_y(t) + 2\sqrt{2} S_z(t)$$

The minimum Euclidean distance between signal points of 64-QAM constellation is assumed to be $E = 1$. Thus for each signal point with equal probability, the average energy of 64-QAM constellation is calculated to be 10.5. Using (4) the instantaneous envelope power is given as

$$P_c(t) = |S_c(t)|^2 = \left| \frac{1}{\sqrt{2}} S_x(t) + \sqrt{2} S_y(t) + 2\sqrt{2} S_z(t) \right|^2$$

If three QPSK Golay sequences of length 2^m are used as the component codes, the resulting 64-QAM code C of length $n = 2^m$ has PAPR bounded by

$$PAPR(c) \leq \frac{\left| \frac{1}{\sqrt{2}}.\sqrt{2n} + \sqrt{2}.\sqrt{2n} + 2\sqrt{2}.\sqrt{2n} \right|^2}{10.5n} = \frac{14}{3} = 6.69 dB \quad (11)$$

As with 16-QAM, we can trade off the size, the PAPR, and the squared Euclidean distance of a 64-QAM OFDM code by an appropriate selection of three QPSK component codes.

V.   CONCLUSION

OFDM is a very attractive technique for wireless communications due to its spectrum efficiency and channel robustness. This paper presents a method for the construction of M-QAM sequences particularly 64-QAM from QPSK sequences. By using Golay sequences the Peak to Average Power Ratio is bounded by 6.69 dB and it is possible to increase the transmission rate as compared to 4-PSK transmission. The methods developed in this paper can be applied to generate OFDM sequences with low PAPR and large squared Euclidean distance for 256 or 1024-QAM constellations because 256-QAM or 1024-QAM can also be written as vector sum of four or five QPSK respectively.

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