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A Graphic User Interface for 3D Segmentation of Nonlinear Functions

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Abstract: In this paper, we introduce a Graphic User interface (GUI) which performs segmentation of nonlinear 3D functions which approximate the 3D functions within a desired degree of accuracy. Due to the approximation and prediction difficulty of 3D functions, it is advantageous to segment these nonlinear shapes into sub regions which can easily be approximated. The GUI systematically segments nonlinear functions into smaller regions and applies the Least Square Estimate (LSE) method to approximate the parameters of each subsequent linear plane. The GUI has the ability to regulate the approximation error to be within a desired range chosen by the user. We demonstrate, through simulation, the functionality and capabilities of the GUI and provide results of user-defined parameters variations and how they affect the linear region parameter approximation.

Keywords: Nonlinear functions, 3-dimensional segmentation, MATLAB Graphic User Interface (GUI), Linear Approximation, Least Square Estimate Method.

I. INTRODUCTION

There have been great efforts in engineering to linearize nonlinear functions to approximate system states, model dynamic behaviour, simplify computational complexity, and provide control methods and strategies [1-10]. In all the aforementioned references, different methods of linearization and approximation are presented. This same approach is also beneficial in fields outside of engineering where there is a need to approximate nonlinear curves. Approximation by 'segmenting' or essentially breaking up a nonlinear function into smaller regions has shown to be a very efficient method [11-12]. The approximation task becomes especially difficult for 3D profiles compared to 2D profiles which can be approximated by linear segments [13]. This segmentation method, referred to in control engineering as Piecewise Linearization, has shown in recent literature to be very useful because it allows for linear control and estimation methods on nonlinear systems. Linear control and estimation theory is found in abundance, and is less computationally complex. In practice, the control costs are less due to the ability to utilize less powerful processors compared to those needed for the more complex nonlinear problems.

A Graphic User Interface (GUI) is presented in this work and its functionality and capabilities are outlined in detail. The user-defined inputs of the GUI are as follows:

- Defines a 2-variable (3D) nonlinear function
- Defines the range of the variables of the nonlinear function
- Sets the number of randomly generated points used for approximation for each linear plane
- Sets an acceptable error range (%)
- Set the maximum amount of trials for the GUI to achieve the acceptable error percentage requirements

The GUI will use the randomly chosen points within the defined ranges and error allowance, and it uses the Least Square Estimate (LSE) method to approximate the parameters for the linear approximation of each region. A flowchart of the GUI is presented in Figure 1. In section II, we discuss segmentation and how it is performed. In section III, we outline the Least Squared Estimate (LSE) method used to find the linear segmentation parameters. In Section IV, we provide the analysis of the results of how the user-defined GUI parameters affect the approximation error. This is done by varying the number of randomly generated points used for parameter approximation and the number of segments chosen for region approximation. Lastly, we provide our final thoughts on the work that was presented in this paper. The GUI, along with a tutorial document, can be found at the following link:

tiny.cc/MATLAB_3D_LinearSeg_GUI

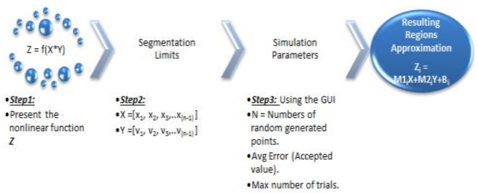

Fig. 1 Segmentation GUI Procedure Flowchart

II. DIMENSIONAL SEGMENTATION

Segmentation is a method by which a 3-dimensional (3D) profile is broken up into sub regions. The main advantage is in the ability to approximate the (3D) profile of a system to a high degree of accuracy without using complex nonlinear computational methods. The simplest form of segmentation is for 2-dimensional profiles (2D) in the form shown in Eq. 1 where $f(x)$ is a nonlinear function dependent on one variable x. Fig. 2 and Fig. 3 demonstrate segmentation of a 2D nonlinear profile. For this example we picked the cosine function. From Figures 2 and 3, it is easy to see that the more segments that are taken then the better the approximation. This is evident by the decrease in the Root Mean Squared Error (RMSE) recorded for each segmented function and shown in Table I. q denotes the number of segments chosen to approximate the nonlinear profile. Each of these line

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segments are represented as a linear equation of the form presented in Eq. 2. The slope of each line segment is given by m_i, where $i = 1.....q$. The boundary point for each segment is b_i and $\hat{y}$ is the approximation of nonlinear profile.

$$z = f(x) \qquad (1)$$

$$\hat{y} = m_i x + b_i \qquad (2)$$

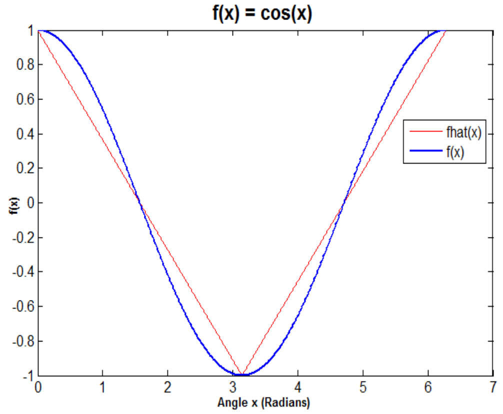

Fig. 2 2D Segmentation $f(x) = \cos(x)$ with $q = 2$

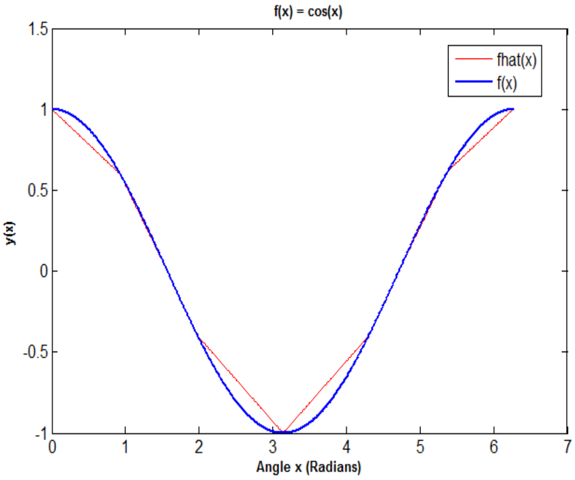

Fig. 3 2D Segmentation $f(x) = \cos(x)$ with $q = 8$

TABLE I
2D SEGMENTATION OF COSINE FUNCTION

Number of Linear Segments	Root Mean Square Error (RMSE)
2	0.0506
7	0.0214

The regions can be chosen intelligently by having a prior knowledge of the symmetry of the profile. Things such as minima, maxima, zero crossings and curvature are all good boundary points for segmenting. Even though, it is clear that more segments means a better approximation. One has to take into account the increase in complexity factor that more segments brings. The more pieces taken, then the more computations need to be performed. That trade-off between accuracy and complexity needs to be taken into account considering the particular problem at hand.

3D segmentation is a tougher task than 2D segmentation due to the increase in the dimensions and the shape of the profile because the task now must take two variables into account as shown in Eq. 3. The linear approximation of what are now planes, as opposed to linear segments, takes the form shown in Eq. 4.

$$z = f(x, y) \qquad (3)$$

$$\hat{y} = m_{1i} x + m_{2i} y + b_i \qquad (4)$$

The approximations of 3D profiles, such as the one shown in Fig. 4, are of great importance. This is because it is known that many systems in engineering, and other fields such as medicine, are 3 dimensional and dependent on more than one variable. 3D segmentation, therefore, allows us to approximate these profiles for control, analysis or diagnosis of the system.

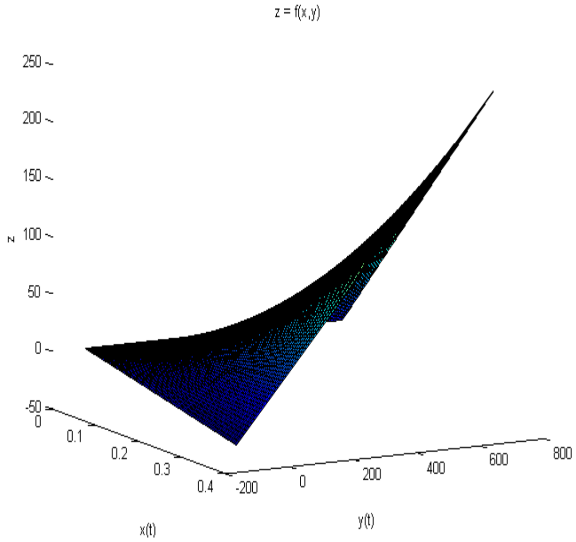

Fig. 4 3D Nonlinear profile

III. LEAST SQUARE ESTIMATE METHOD

The Least Square Estimate (LSE) method is an estimation method that finds the solution which reduced the squared error of a function based on statistical data [14-15]. It is often used for parameter determination where the function is known and much statistical data can be collected. In our GUI, we use this method to determine the parameters m_1, m_2 and b_1 for the linear planes which approximate the nonlinear function. While there exists other methods of estimation, such as Taylor Series [16], the LSE method is the most optimal because it inherently minimizes an error criterion. Taylor series has the advantage over other methods in that it doesn't require an extensive amount of data, however, an operating point must be known. This is not practical for 3D nonlinear systems because of their two variables nature the operating point often changes dynamically and/or is not well known. This can lead to high approximation error due to limited knowledge of the operating point.

For any function represented as in Eq. 5, with parameters x which need to be estimated, an approximation $\hat{x}$ can be found. The approximation error e, is written in Eq. 6.

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$$y \approx Ax \quad (5)$$

$$e = y - A\hat{x} \quad (6)$$

The LSE decreases the approximation error by finding the minimum solution to the error criterion denoted, $\min_{\hat{x}_{LS}}(J)$. (J) is the squared error which is shown in Eq. 7.

$$J = \frac{1}{2}e^T e = \frac{1}{2}(y - A\hat{x})^T (y - A\hat{x}) \quad (7)$$

The solution to this problem, $\hat{x}_{LS}$, is the least squared estimate given in Eq. 8 and is found by taking the derivative of Eq. 7 and equating it to Eq.8.

$$\hat{x}_{LS} = A^{-1} y \quad (8)$$

In case that there are more data point taken, y, then the unknown parameters, x, Eq. 8 will not hold due to dimensions mismatch. To solve this issue, the pseudo-inverse of A is used as follow,

$$A^{-1} = (A^T A)^{-1} A^T \quad (9)$$

IV. ANALYSIS AND RESULTS

In this section, we provide an overview of the functionality of the GUI, while conducting a test to show its capabilities. The most important user-defined command in this GUI, in terms of parameter determination and the segmentation approximation, is the total amount of segments chosen. While we vary the amount of segments taken, the number of data points taken for the LSE of the parameters, the acceptable error, and the maximum number of trials to reach the acceptable error for each segment remain unchanged. These constant parameters for this test are shown in Table II.

TABLE II
USER DEFINED TEST PARAMETERS

Number of random data points	**Acceptable error** (±%)	**Max. number of trials**
100	10	100

In Table III, we show the three different combinations of region segmentations that were applied on the same nonlinear function (Z= *X*Y*). Each row provides the approximation error for each particular segment. For the first trial, 2 segments were chosen for Var1 and Var2 as shown in Fig.5. This gives a total of 4 segments. For the second trial, 2 segments were taken for var1 and 3 segments for Var2 given us a total of 5 segments as shown in Fig. 6. In the third and final trial 3 segments were used for Var1 and 5 segments for Var2 or a total of 8 segments as shown in Fig. 7. The results are shown in the table IV. They clearly demonstrate that using more segments for the nonlinear function approximation yields less error. We calculate the root mean squared error for every segment in each trial and the percent error of each segment for the individual trials to find their average in order to get the average error for each trial. For each increasing number of segments, from trial 1 to trial 3, the average error percentage as well as the RSME decreased. As discussed in section II, the segmentation procedure improves with the increasing amount of segments taken. This is the main parameter a user needs to adjust when wanting to get better results using this GUI.

TABLE III
ERROR APPROXIMATION

TRIAL 1 Approximation Error	**TRIAL 2 Approximation Error**	**TRIAL 3 Approximation Error**
9.985110208	7.244302229	5.188132174
5.134776836	0.780429228	9.647742165
8.174976413	0.801097449	7.780393755
-0.621739724	8.439138971	4.743422568
	-0.132472445	5.960491566
	-0.488394563	-7.877453238
		-0.147799208
		-0.069140105
		-0.049439983
		-0.033067977
		3.049246746
		-0.016993497
		-0.010973732
		-0.005255483
		-0.005234502

TABLE IV
AVERAGE ERRPR RESI;TS PF THE TRIALS

	TRIAL 1	**TRIAL 2**	**TRIAL 3**
Average Error	5.668280933	2.774016811	1.876938083
Root mean Square Error	3.366981121	2.355426421	1.937492237

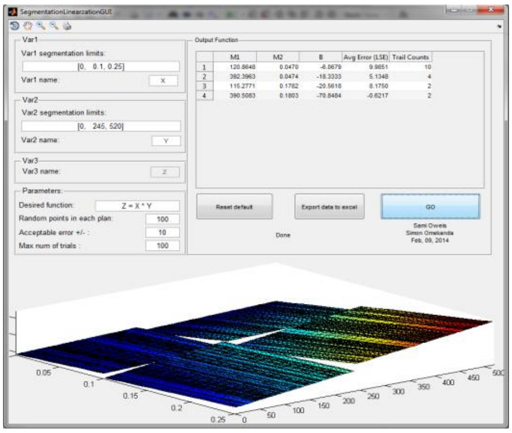

Fig. 5 Trial 1 (4 segments)

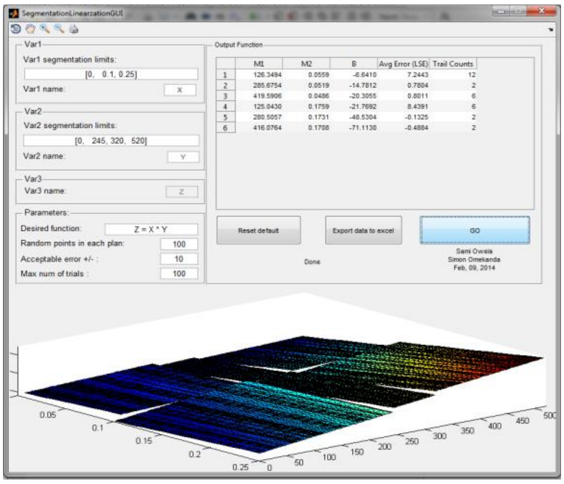

Fig. 6 Trial 2 (5 segments)

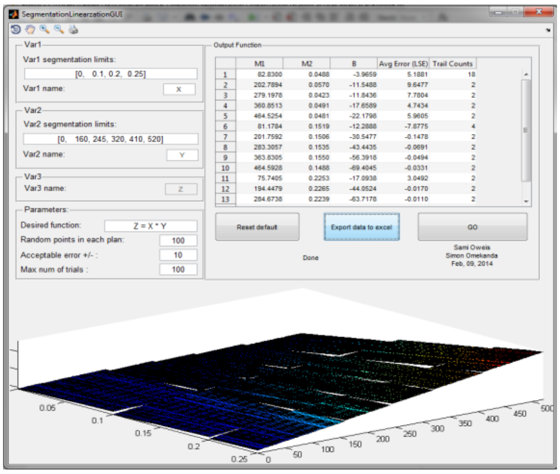

Fig. 7 Trial 3 (8 segments)

V. CONCLUSION

In this work, we provided a Graphic User Interface which performs segmentation and estimation of 3D nonlinear functions. Indeed, segmentation is a good way to approximate any nonlinear function. This GUI can span various fields and applications where the approximation of a 3D profile is beneficial. The tasks involved in finding the equations for the segments of a 3D nonlinear function can be tedious, time consuming and mathematically extensive. Therefore, the GUI we provide is of great service to the technical community. It has already been shown in literature that this segmentation method of splitting a nonlinear function into sub regions has favourable advantages. In the engineering field it decreases the mathematical, control and estimation complexity of a problem.

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BIOGRAPHIES

Simon O. Omekanda was born in Rabat, Morocco, on September 15, 1984. He received his Bachelor in Science (B.S.) degree in electrical engineering from Pennsylvania State University, State College, in 2009 and the Masters in Science (M.S.) degree in electrical and computer engineering from Oakland University, Rochester, MI, in 2011. He is currently pursuing his Ph.D in controls engineering from Oakland University. He has been working as a teaching and research assistant for the electrical and computer engineering department at Oakland University during this time. He has taught and helped students in the areas of controls and estimation theory as well as using MATLAB/Simulink for simulation. His research areas primarily include design, implementation and simulation of nonlinear observers for state estimation and control, multivariable control, subsystem interconnection with application to internal combustion engines and electric machine in automotive applications.

Sami Oweis (seweis@oakland.edu) received his Masters in Science (M.S.) degree in Electrical and Computer Engineering, from Oakland University. He is currently pursuing his Ph.D in artificial intelligence and controls Engineering from Oakland University. He has been working as a teaching and research assistant for the Electrical and Computer Engineering department at Oakland University during this time. He has taught and mentored students in the areas of digital signal and imaging processing, microcontrollers, micro-based computer systems, embedded application, artificial intelligent and control theory as well as using MATLAB/Simulink. More information is at: https://www.linkedin.com/in/samioweis & https://www.researchgate.net/profile/Sami_Oweis