

Robust Invisible Watermarking using Least Support Orthogonal Matching Pursuit (LS-OMP) Recovery Method

Israa Sh. Tawfic¹, Sema Koc Kayhan²

Department of Electrical & Electronics, University of Gaziantep, Gaziantep, Turkey^{1,2}

Abstract: In this paper a watermark embedding and recovery technique is proposed based on the compressed sensing theorem. Both host image and watermark are sparse, each in frequency domain using DWT. In recovery, new method called Least Support Matching Pursuit (LS-OMP) is used to recover the watermark and the host image in clean conditions. LS-OMP algorithm adaptively chooses optimum L (Least Part of support), at each iteration. This new algorithm has some important characteristics: it has a low computational complexity comparing with ordinary OMP method, the reconstruction accuracy is show better results than the other method. Second, we give the procedure for the invisible image watermarking in the presence of compressive sampling. The image reconstruction based on a set of watermarked measurements is performed using LS-OMP. While the LS-OMP offers a comparably theoretical guarantee as best optimization-based approach, simulation results show that it outperforms many algorithms especially for compressible signals.

Keywords: Compressed sensing, lest Support Orthogonal Matching Pursuit, Orthogonal matching pursuit, restricted isometry property, Watermark

I. INTRODUCTION

Digital watermarking is a process in which digital contents such as video, image, audio, and text are protected by hiding any logo or message into the content. These watermarks should be detected only by the copyright holder who has the private key [1].

In the case of image watermark, for security and robustness, digital watermark signals are commonly embedded in the spatial or frequency domain. Most watermarking algorithms, called lossy watermarking, as a result of loss of cover image quality in a watermarking process in some range not affected on the quality of cover image, especially when recovering of watermark in the process of fingerprint, the security and quality is so important in the process of transmission [1, 2].

The most challenge for the reversible watermarking lies in the difficulty to obtain the trade off between the watermark quality and the watermark robustness for resisting attacks. Higher image watermark quality means more data need for watermarking to be embedded, which yields a better watermark robustness. However, with the increasing of the watermarks quality, the quality of host image would be decreased. That means, the watermarking technology would influence the security of the watermarked image. In recent years, compressed sensing (CS) theory provides a feasible method to solve this problem [2].

II. BACKGROUND

A. Compressive Sensing

The major goal of Compressed sensing (CS) is to recover a high dimensional sparse signal from its low dimensional linear measurements. The standard CS theorem is based on a sparse signal model and uses an underdetermined system

of linear equations. Obviously, knowing that if the measurement matrix satisfies the condition so called restricted isometry property (RIP), the sparse signal can be exactly (or approximately) recovered through truly designed recovery algorithms [3].

A variety of CS reconstruction algorithms have been developed based on convex relaxation, non-convex and iterative greedy search strategies.

In practice, convex based method has heavy computation, while, the iterative greedy method has lower complexity and hence their usage may be practically applicable in solving large dimensional CS problems [3,4].

The main principle of the iterative greedy search methods is an estimation of the underlying support set of a sparse vector.

The support set contains indices that are non-zero elements of a sparse vector. To evaluate the support site, the iterative greedy search method uses linear algebra tools such as the matched filter and least square solution iteratively [5].

Orthogonal matching pursuit (OMP) greedy algorithm constructs an approximation by using an iterative process. At each iteration, the locally optimal solution is found. This is done by finding the column vector in A which most closely resembles a residual vector r [5,6].

In this study, we propose Least Support Orthogonal Matching Pursuit (LS-OMP) algorithm and CS based digital watermarking algorithm using LS-OMP in image reconstruction. The watermark embedding and detection are usually done in DFT, DCT, DWT domain [7, 8]. Here, we deal with Watermarking of compressive sampled images based on sparse DWT image representation. Further, we analyse the possibility to reconstruct images

from such a small set of data, in order to provide successful watermark detection after image reconstruction. The robustness and security of watermarking are enhanced by the usage of Arnold scrambling.

B. OMP Algorithm

Notations: let the signal vector $x = \{x_1, x_2, \dots, x_N\}^t$, let the support set $T \subset \{1, 2, \dots, N\}$ denote the set of indices of the non-zero components of x (i.e. $\text{upp}(x) = \{i | x_i \neq 0\}$), $A_I \in \mathbb{R}^{M \times |I|}$ consists of the columns of A with indices $i \in I$, A^* denote the transpose of A , and A^\dagger denote the pseudo-inverse $\{(A^*A)^{-1}A^*\}$. Let us state the standard CS problem, which acquires a signal $x \in \mathbb{R}^N$ have a K sparse input, via the linear measurements

$$y = \Phi x \quad (1)$$

Where $\Phi \in \mathbb{R}^{M \times N}$ represents a random measurement (sensing) matrix, and $y \in \mathbb{R}^M$ represents the compressed measurement signal. A K sparse signal vector consists of most K nonzero indices ($K < M < N$). The aim of the algorithm is to reconstruct a sparse signal \hat{x} from y using a small number of measurements and to achieve good reconstruction quality [9].

We note that the compressed measurement signal y is the linear combination of most K atoms (atom means a column of Φ). One condition for sparse signal recovery is to use the Mutual Incoherence Property (MIP) [10,11]. The MIP requires the correlations among the column vectors Φ to be small.

The coherence parameter μ of sensing matrix is defined as,

$$\mu = \max_{i \neq j} \langle \varphi_i, \varphi_j \rangle, \quad (2)$$

Where φ_i, φ_j are two columns of Φ with a unit norm and Φ is the concatenation of two square orthogonal matrices. It was first shown by Donoho and Huo [9] in the noiseless case, for the setting where Φ is a concatenation of two square orthogonal matrices.

$$K < \frac{1}{2} \left(\frac{1}{\mu+1} \right) \quad (3)$$

It is based on the algorithmic structure of OMP [11]. Proposed algorithm, LS-OMP selects one atom in each iteration, according to its future effect on minimizing the residual norm.

Lemma 1 [12] (Consequences of RIP) $I \subset \Omega$, if $\delta_{|I|} < 1$ then for any $u \in \mathbb{R}^{|I|}$,

$$(1 - \delta_{|I|}) \|u\|_2 \leq \|\Phi_I^* \Phi_I u\|_2 \leq (1 + \delta_{|I|}) \|u\|_2 \quad (4)$$

$$\frac{1}{(1 + \delta_{|I|})} \|u\|_2 \leq \|(\Phi_I^* \Phi_I)^{-1} u\|_2 \leq \frac{1}{(1 - \delta_{|I|})} \|u\|_2 \quad (5)$$

C. Related Work

Theorem 1[13] If we have K -sparse vector x , x can be perfectly recovered from $y = \Phi x$, using the OMP algorithm, if the isometry constant δ_{K+1} satisfies

$$\delta_{K+1} < \frac{1}{\sqrt{K+1}} \quad (6)$$

Clearly, when K is very large, the proposed upper bound $\frac{1}{\sqrt{K+1}} \approx \frac{1}{\sqrt{K}}$. We depend on proofing of Theorem 1 on the mathematical induction. If the OMP algorithm chooses a correct index in the first iteration, then the iteration condition can be easily extended to the general iteration and the theorem will be confirmed.

During the first iteration, since t^1 be the index of the column maximally correlated with the residual r^0 and $r^0 = y$, we have

$$t^1 = \text{argmax}_i |\langle \varphi_i, y \rangle| \quad (7)$$

Lemma 2 [14] If the K -sparse signal x has a support T , in the first iteration of the OMP algorithm, the index chosen belongs to the support (i.e. $t^1 \in T$) if the isometry constant δ_{K+1} of a matrix Φ satisfies

$$\delta_{K+1} < \frac{1}{\sqrt{K+1}} \quad (8)$$

Lemma 3 [15, 9]: if initial k iterations $1 \leq k \leq K - 1$ of the OMP algorithm are successful (i.e. $T^k = \{t^1, t^2, \dots, t^k\} \in T$) then the $(k + 1)$ -th iteration is also successful (i.e. $t^{k+1} \in T^k$) under the initial step condition.

Theorem 2 [7,8]: assumes that $\delta_{3K} = 0.165$ if sampling matrix Φ satisfies RIP, then:

$$\|x_{T-T^\ell}\|_2 \leq c_K \|x_{T-T^{\ell-1}}\|_2 \quad (9)$$

$$\|y_r^\ell\|_2 \leq \frac{c_K}{1-2\delta_{3K}} \|y_r^{\ell-1}\|_2 < \|y_r^{\ell-1}\|_2 \quad (10)$$

Where $c_K = \frac{2\delta_{3K}(1+\delta_{3K})}{1-\delta_{3K}}$

III. LEAST SUPPORT-OMP

Theorem 3: For any K -sparse vector x , where $x \in \mathbb{R}^N$ and measurement matrix $\Phi \in \mathbb{R}^{m \times N}$, and $y \in \mathbb{R}^m$ represents the measurement vector matrix, the LS-OMP algorithm perfectly recovers x from $y = \Phi x$, (depending on Fig.1) if the

$$\|y - y_r^\ell\|_2 \leq \frac{\delta_{2L}}{1-2\delta_{2L}} \|y_r^{\ell-1}\|_2$$

Assume $\delta_{2L} = 0.48$.

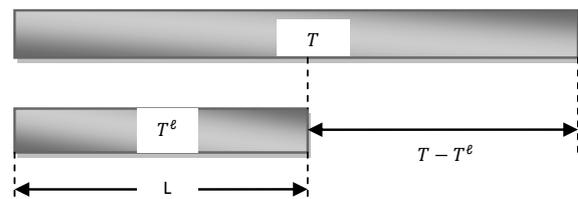


Fig.1. Illustration of support sets for our theorem

A. Least Support (LS-OMP) Algorithm

Least Support Algorithm
<p>Inputs: measurement matrix $\Phi_{m \times n}$, measurement signal $y_{m \times 1}$, sparsity K, Least Support Parameter L, stop condition</p> <p>Initialization: index set $I_0 = \phi$, initial Least Support set $J_0 = \phi$, iteration $\ell = 0$, residual signal $r_0 = y$</p> <p>Repeat Steps 1-3 for L iterations or until stop condition</p> <p>1. (Update <i>Least Support set</i>) $J_\ell = \arg \max_{\ell=1 \dots L} r_{\ell-1} , I_\ell = [I_{\ell-1} \cup J_\ell]$,</p> <p>2. (Reconstruction) $\hat{x} = \Phi_{I_\ell}^\dagger y$,</p> <p>3. (Residual update) $r_\ell = y - \Phi_{I_\ell} \hat{x}$</p> <p>Output: recovered sparse signal \hat{x} with $\hat{x}_L = \hat{x}$ and $\hat{x}_{L,c} = 0$</p>

B. Proof of Theorem 3

In Fig.1, T represents the support of whole signal that contains the index of a coefficient that has maximum correlation between y and Φ . L represents a part of support that will be used as a least support to reconstruct the original signal. The residual of current iteration can be defined as

$$\|y_r^\ell\|_2 = \|\text{resid}(y, \Phi_I)\|_2 \geq \|\text{resid}(\Phi_{T-T^\ell} x_{T-T^\ell}, \Phi_{T^\ell})\|_2 + \|\text{resid}(\Phi_{T^\ell} x_{T^\ell}, \Phi_{T^\ell})\|_2$$

Since $\text{resid}(\Phi_{T^\ell} x_{T^\ell}, \Phi_{T^\ell}) = 0$, then $\|y_r^\ell\|_2 \geq \|\text{resid}(\Phi_{T-T^\ell} x_{T-T^\ell}, \Phi_{T^\ell})\|_2$ (11)

For $\ell - 1$, (11) become $\|y_r^{\ell-1}\|_2 \geq \|\text{resid}(\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}, \Phi_{T^{\ell-1}})\|_2$ (12)

$$\|\text{resid}\|_2 = \|y\| - \|y_p\| \quad (13)$$

From the definition of the residue,

$$\begin{aligned} \text{resid}(y, \Phi_I) &= y - y_p \\ \|\text{resid}(\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}, \Phi_{T^{\ell-1}})\|_2 &= \|\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}\|_2 - \|y_p\|_2 \end{aligned} \quad (14)$$

We have $\|y_p\| \leq \frac{\delta_{|I|+|J|}}{1-\delta_{\max(|I|,|J|)}} \|y\|$, since $|I|, |J|$ is the cardinality set then

$$\begin{aligned} \|y_p\|_2 &= \frac{\delta_{2L}}{1-\delta_L} \|y\|_2 \\ \therefore \|\text{resid}(\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}, \Phi_{T^{\ell-1}})\|_2 &= \|\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}\|_2 - \frac{\delta_{2L}}{1-\delta_L} \|\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}\|_2 \\ &= (1 - \frac{\delta_{2L}}{1-\delta_L}) \|\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}\|_2 \\ &= \frac{1-\delta_L-\delta_{2L}}{1-\delta_L} \|\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}\|_2 \end{aligned} \quad (15)$$

After the substitution of (15) into (12) we get,

$$\|y_r^{\ell-1}\|_2 \geq \frac{1-\delta_L-\delta_{2L}}{1-\delta_L} \|\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}\|_2 \quad (16)$$

Since we work in support set L and previous iteration is reduced in set L as $T - T^{\ell-1} = T^\ell$, then (16) become;

$$\|y_r^{\ell-1}\|_2 \geq \frac{1-\delta_L-\delta_{2L}}{1-\delta_L} \|\Phi_{T^\ell} x_{T^\ell}\|_2$$

$$\|y\|_2 \geq \|\Phi_{T^\ell} x_{T^\ell}\|_2 \quad (17)$$

After the substitution the value of $\|y\|_2$ into (17) we get

$$\|y_r^{\ell-1}\|_2 \geq \frac{1-\delta_L-\delta_{2L}}{1-\delta_L} \|y\|_2 \quad (18)$$

Or

$$\|y\|_2 \leq \frac{1-\delta_L}{1-\delta_L-\delta_{2L}} \|y_r^{\ell-1}\|_2$$

$$y_p = y - y_r^\ell \quad (19)$$

Since

$$\|y_p\|_2 = \frac{\delta_{2L}}{1-\delta_L} \|y\|_2 \quad (20)$$

After the substitution of (19) into (20) yields

$$\|y - y_r^\ell\|_2 = \frac{\delta_{2L}}{1-\delta_L} \|y\|_2 \quad (21)$$

And finally substitution of (36) into (39) yields

$$\|y - y_r^\ell\|_2 \leq \frac{\delta_{2L}}{1-\delta_{2L}} - \frac{1-\delta_{2L}}{1-2\delta_{2L}} \|y_r^{\ell-1}\|_2$$

Using monotonicity of the isometry constant $\delta_L \leq \delta_{L+1}$

$$\|y - y_r^\ell\|_2 \leq \frac{\delta_{2L}}{1-2\delta_{2L}} \|y_r^{\ell-1}\|_2$$

Where $\delta_{2L} = 0.48$

C. Proposed Schemes

Four steps were used for watermarking based CS:

First, watermark embedding and transfer: For embedding a watermark image, Singular value decomposition (SVD) was used, then a watermark scaling factor used to a adjust depth of embedding. Arnold transform was used on a watermark image to add some security, and then DWT was used.

Arnold transform can be found as follows [16]:

$$\begin{pmatrix} i' \\ j' \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} i \\ j \end{pmatrix} \text{ mod } N \quad (22)$$

Where $i, j \in \{0, 1 \dots N - 1\}$

Second, compression sensing step: sparsify image by using DWT, add it to the results of the first step, built sensing matrix, then find linear measurement vector.

Third, compressed sensing recovery step: use proposed LS-OMP to recover the signal, inverse DWT.

Fourth, Extracting watermark image: extract watermark image, inverse Arnold transforms, show results.

Fig.2 illustrate the entire steps above

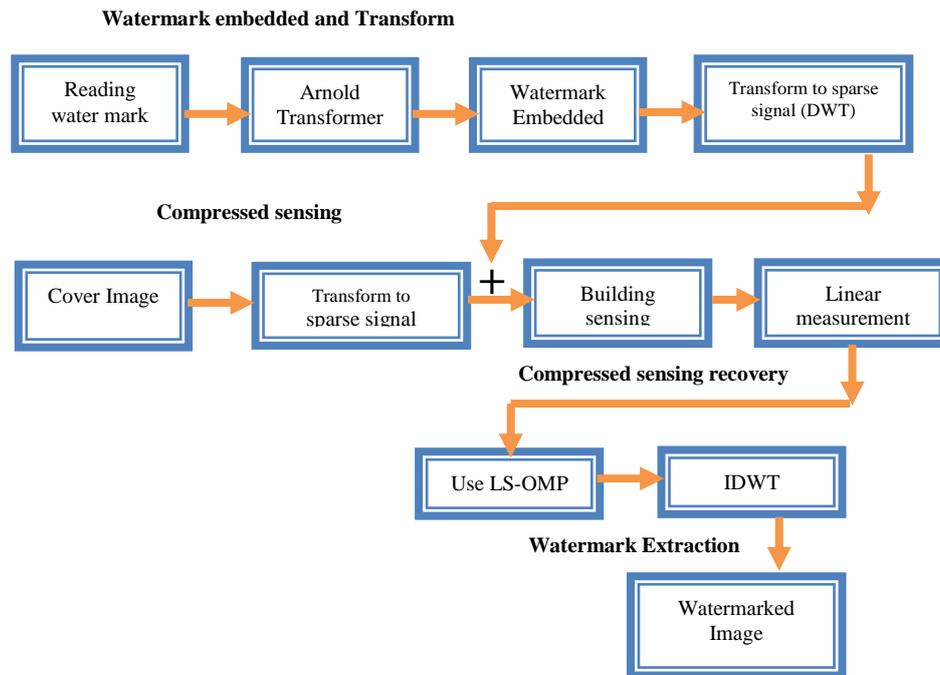


Fig.2 Proposed Schemes for watermark embedded and extraction

IV. EXPERIMENTS RESULTS

Fig.3 shows the results of using the three theorems given above for the sparse signal having length=1000, number of spikes in the signal = 50, measurement size(M) = 200, and Least Support parameter (L) =20. As it can be seen from Fig 3, even when the value of L is less than observation, the new stopping condition can calculate the exact original signal after a small number of iterations. This property is very important, especially for application used consumed time as one of important requirement such as MRI in medical application.

While Fig.4 illustrated the comparison between the ordinary OMP method and our LS-OMP method for the time spent to get back the original signal. Number of spikes in signal =40, measurement size(M) = 200, and Least Support parameter (L) =20. Reconstructed -Signal-to-Noise ratio (R-SNR) is used to measure performance of the reconstructed signal for different measurements value as a comparison between LS-OMP and OMP as shown in Fig.5.

Table 1 shows the results for comparison between ordinary OMP method and LS-OMP for the different value of watermark scaling factor, when applying both methods on a gray cover image of size 256x256 using DWT (wavelet filter Coiflets5) and gray watermark image having size 64x64 with uses Arnold transform before adding it to cover image as some kind of security in addition to hiding the watermark.

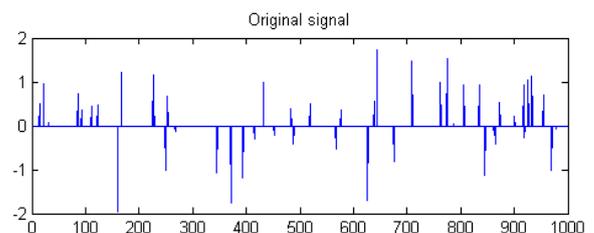
The stability of embedded watermark is reduced if the value of watermark factor is too small, however, the too big value will decrease the quality of the watermark and cover image as well.

For watermarking verification PSNR is used to evaluate the accuracy of the reconstructed image. The performance of the blind or non-blind watermark extraction result is evaluated in terms of Normalized Correlation Coefficient (NCC), for the extracted watermark W' and the original watermark W [16]:

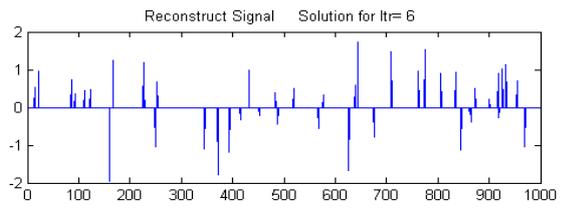
$$MSE = \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N (X(i, j) - \hat{X}(i, j))^2 \quad (23)$$

$$PSNR = 10 \cdot \log_{10} \left(\frac{MAX^2}{MSE} \right) \text{ (db)} \quad (24)$$

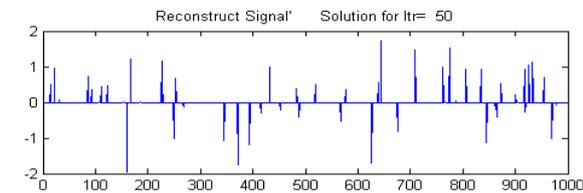
$$NCC(W, W') = \frac{\sum_{i=1}^n W(i)W'(i)}{\sqrt{\sum_{i=1}^n W(i)^2} \sqrt{\sum_{i=1}^n W'(i)^2}} \quad (25)$$



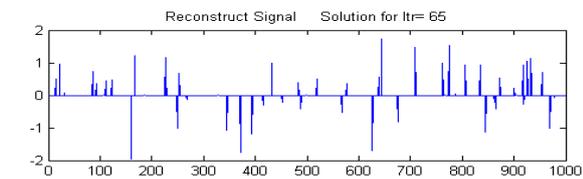
Original signal, signal length=1000, No. of sparse=50



Reconstructed Original signal after 6 iterations using condition $\|y - y_r^l\|_2 \leq \frac{\delta_{2K}}{1 - 2\delta_{2K}} \|y_r^{l-1}\|_2$



Reconstructed Original signal after 50 iteration using condition $\|y_r^l\|_2 \leq \|y_r^{l-1}\|_2$



Reconstructed Original signal after 65 iteration using condition $\delta_{K+1} < \frac{1}{\sqrt{K+1}}$

Fig.3. Illustrated the effect of stopping condition on number of iteration needed to reconstruct original signal

Fig.6 shows the visual reconstruction of test image having 256x256 size, with watermark image 64x64, for different sampling rate (M/N) using OMP and LS-OMP; when Watermark factor=0.08, Number of Iteration =50, wavelet filter type is Coiflets5on. Fig6 show the quality of reconstructed cover image and watermark logo image using both aforementioned two methods.

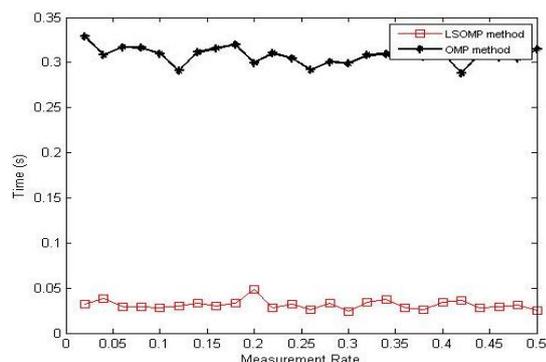


Fig.4 Time consume for OMP and LS-OMP

Also effect for some attack was tested for both OMP and LS-OMP methods to focus on the ability of the new method to get back the invisible watermark even when the image force some attack. The effect of these attacks is shown in Table.2, while Fig.7 shows the results of some attacked mansion in Table.2

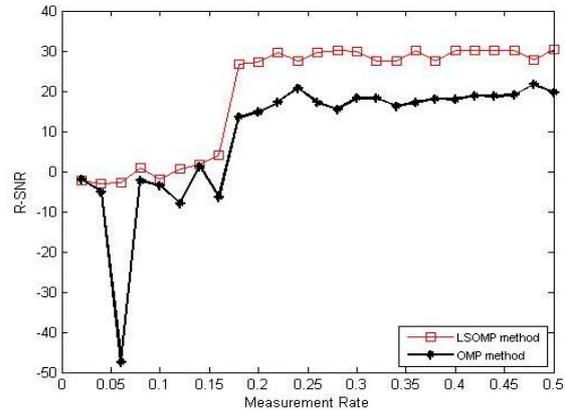


Fig.5 R-SNR for both method OMP and LS-OMP

Table 1. Effect of changing measurement ratio on reconstructed Lena image and watermark logo for both OMP and LS-OMP methods if Watermark factor=0.05 , and wavelet filter= Coiflets5

M/N	OMP		LS-OMP	
	PSNR	NCC	PSNR	NCC
0.7212	33.0943	1	35.6433	1
0.6031	32.4041	1	34.2223	1
0.5850	30.9211	0.9673	32.3115	1
0.4687	28.3020	0.8502	29.2227	0.9445
0.3906	13.1214	0.5231	21.0438	0.7035

Table 2. Effect of different types of attack on reconstructed Lena image and watermark logo for both OMP and LS-OMP methods if Watermark factor=0.05, M/N=0.4, and wavelet filter= Coiflets5

Attack type	OMP		LS-OMP	
	PSNR	NC C	PSNR	NCC
Gaussian noise (mean=0.02; variance=0)	20.4150	0.7021	21.6741	0.8521
Salt & pepper (Noise density=0.02)	24.0199	0.7640	24.0831	0.9933
Poisson noise	26.0039	0.8072	26.3446	0.9873
Rotate image	29.2093	0.8953	30.8547	0.9431
Crop image	30.3438	0.9987	32.5655	1
Resize image	28.9145	0.9867	30.4336	1
Cut some part	29.9782	1	31.5361	1



Figure.6. illustrate comparison between OMP and LS-OMP when both methods are used for watermarking process

Some attacks from Table.2

		LS-OMP NCC=0.8521	OMP NCC=0.7021
	Gaussian noise (mean=0.02; variance=0)		
	salt & pepper (noise density=0.02)	NCC=0.9933 	NCC=0.7640 
	Rotate image	NCC=0.9431 	NCC=0.8953 
	Crop image	NCC=1 	NCC=1 
	Cut some part	NCC=1 	NCC=1 

Fig.7 Effect of some attack on watermark reconstructed

V. CONCLUSION

In this paper, we modify iterative algorithm orthogonal matching pursuit (OMP) with sensing terms LS-OMP. The new algorithm reduces the computational complexity significantly and performs better than OMP. Results show that the new proof algorithm improves the performance, thereby needs fewer samples to approximate reconstruction.

The proposed algorithm is applied to the CS based image watermarking algorithm for image reconstruction. Compared with the previous reversible watermarking algorithm, the new method gives better results and improves the robustness to achieve reversible watermarking. Meanwhile, algorithm complexity prevents breakthrough with better practical performance.

Future work includes analysing to proofing theoretical some other iterative algorithms for Compressed Sensing like Partially Known Support method.

REFERENCES

- [1] Huimin ZHAO Dong ZHANG. A Novel Image Watermarking Algorithm Based on Block Compressed Sensing with Variable Sampling Rates. *Journal of Pattern Recognition & Image Processing* 4:4(2013)467-476.
- [2] Hsiang-Cheh Huang .Robust Image Watermarking Based on Compressed Sensing Techniques. Feng-Cheng Chang Department of Electrical Engineering National University of Kaohsiung. *Journal of Information Hiding and Multimedia Signal Processing* Volume 5, Number 2, April 2014
- [3] Richard Baraniuk,. Compressive sensing. *IEEE Signal Processing Magazine*, 24(4), pp. 118-121, July 2007
- [4] S. Chatterjee, D. Sundman and M. Skoglund,. Robust matching pursuit for recovery of Gaussian sparse signal. *DSP/SPE Workshop 2011*, Sedona, USA
- [5] Mark A. Davenport, and Michael B. Wakin. Analysis of Orthogonal Matching Pursuit Using the Restricted Isometry Property. *IEEE Transactions On Information Theory*, Vol. 56, No. 9, Sept. (2010).
- [6] Entao Liu and V.N. Orthogonal Super Greedy Algorithm and Applications in Compressed Sensing. *IEEE Trans. Inform. Theory* 58 (4), 2040-2047, 2011.
- [7] Mehmet Yamaç, Çağatay Dikici, Bülent Sankur. Robust Watermarking Of Compressive Sensed Measurements Under Impulsive And Gaussian Attacks. supported by Tubitak Bideb-2232, (2013).
- [8] Irena Orović and Srdjan Stanković . Combined Compressive Sampling and Image Watermarking. *55th International Symposium ELMAR 09/2013*
- [9] Wei Dai, Member. Subspace Pursuit for Compressive Sensing Signal Reconstruction. *IEEE*, and Olgica Milenkovic, Member, *IEEE*, 5, May (2009).
- [10] Tony Tony Cai, Lie Wang, and Guangwu Xu. Stable Recovery of Sparse Signals and an Oracle Inequality. *IEEE Transactions On Information Theory*, Vol. 56, No. 7, July (2010).
- [11] T. Tony Cai and Lie Wang. Orthogonal Matching Pursuit for Sparse Signal Recovery. *IEEE Transaction on Information Theory*, VOL. 57, NO. 7, JULY 2011
- [12] Jian Wang and Byonghyo Shim. A Simple Proof of the Mutual Incoherence Condition for Orthogonal Matching Pursuit. *arXiv 1105.4408v1 [cs.IT]*, 23 May (2011).
- [13] Yi Shen and Song Li. Sparse Signals Recovery from Noisy Measurements by Orthogonal Matching Pursuit. *arXiv 1105.6177v1 [math.FA]*, 31 May (2011).
- [14] J. Wang, S. Kwon, and B. Shim. Generalized orthogonal matching pursuit. *IEEE Trans. Signal Process.*, vol. 60, no. 12, pp. 6202–6216, Dec. 2012.
- [15] Wei Dai and Olgica Milenkovic. Subspace Pursuit for Compressive Sensing: Closing the Gap Between Performance and Complexity. *International Journal of Electronics and Computer Science Engineering* 2010.
- [16] Divya saxena. Digital Watermarking Algorithm based on Singular Value Decomposition and Arnold Transform. *International Journal of Electronics and Computer Science Engineering* 2012