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A Review to the Stability of Discrete Time State-Space Filters using Saturation Non-Linearity

Shiva Jaiswal[1], Preeti Verma[2]

Assistant Professor, Department of Electronics and Communication Engineering, SRMS CET&R, Bareilly, India[1,2]

Abstract: The problem concerning the elimination of overflow oscillation in fixed-point state-space digital filter employing saturation arithmetic is considered by various researchers. In this paper a review is done to the finite procedure proposed by T. Oba [1] to test the stability of digital filters under saturation arithmetic.

Keywords: Digital filters, finite wordlength, asymptotic stability, nonlinear system.

I. INTRODUCTION

When a digital filter is implemented on a digital computer or on special-purpose digital hardware, the filter coefficients are stored in binary registers. These registers can accommodate only a finite number of bits and hence the filter coefficients have to be truncated or rounded-off in order to fit into these register. The finite-word length in recursive digital filter produces non-linearities, namely quantization and overflow. The presence of such non-linearities may result in the instability of the designed system. When dealing with the design and implementation of fixed-point state-space digital filters, it is, therefore, essential to know the conditions under which the filter will be globally asymptotically stable.

II. SYSTEM DESCRIPTION

The system under consideration is described by

$$x(r+1) = f(y(r)) = \left[f_1(y_1(r)) \quad f_2(y_2(r)) \ldots\ldots\ \ldots\ f_n(y_n(r))\right]^T \qquad (1a)$$

$$y(r) = \left[y_1(r) \quad y_2(r) \ldots\ldots\ \ldots\ y_n(r)\right]^T = Ax(r) \qquad (1b)$$

Where $x(r)$ is an n-vector space, $A = [a_{ij}]$ is the n x n coefficient matrix , and T denotes transpose. The saturation nonlinearity is given by

$$f_i(y_i(r)) = \begin{cases} 1 & y_i(r) > 1 \\ y_i(r) & |y_i(r)| \le 1 \\ -1 & y_i(r) < -1 \end{cases} \qquad (1c)$$

i=1, 2,3,………….n are under consideration.

Eq(1) is used to describe digital filters with symmetric saturation implemented with finite register length under zero external inputs.

III. THEOREM 1

The system described in (1) is asymptotically stable if there exists a positive definite matrix P satisfying

$$(P)_{i,i} - \sum_{j \ne i} (w_{|A|})_j (|P|)_{i,j} > 0 \quad for \quad all \quad i \in J^c_{|A|} \qquad (2a)$$

such that $P - A^T PA$ is positive definite.

There are some prerequisite which are to be known before stating the algorithm to calculate $w_{|A|}$ for (2a), they are

a) Stability test is to be done on matrix A, where $A \in R^{nxn}$,
b) The order of the matrix A is n.

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c) The matrix $B = |a_{ij}|, i, j = 1,2....n$

d) $J_0 = \phi$; J_k contains coordinates indices

e) $n_0 = 0$; n_k contains the number of indices of J_k

f) J_k^c contains the complement indices of J_k

g)

$$w_0 = 1 \in R^n; \qquad w_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ n \end{bmatrix}$$

IV. ALGORITHM 1

The following procedure is proposed by ref [1] with k=1, to obtain J_B, n_B and $w_B = R^n$

i) Let J_k denotes the list of coordinate indices i's satisfying $(Bw_{k-1})_i < 1$,and let n_k denotes the number of the indices in J_k

ii) If $J_k = J_{k-1}$, or if $n_k = n$, then define $J_B = J_k, n_B = n_k$,and

$$w_B = \begin{cases} w_{k-1} & if \quad n_B < n \\ 0 & if \quad n_B = n \end{cases} \qquad (2b)$$

and then exit the loop.

iii) Define $w_k \in R^n$ such that

$$\begin{cases} (w_k)_{J_k} = (I_{n_k} - B_{J_k,J_k})^{-1} \sum_{b=1}^{n-n_k} (B_{J_k,J_k^c} I_{n-n_k})_{a,b} & a = 1,2..n_k \\ (w_k)_{J_k^c} = I_{n-n_k} \end{cases} \qquad (2c)$$

and return to step (i) with k=k+1.

V. NUMERICAL EXAMPLE 1

To illustrate the algorithm for the stability test of fixed-point state-space digital filter with saturation arithmetic, a specific example of a third-order digital filter is considered with

$$A = \frac{1}{10}\begin{bmatrix} 1 & 5 & 4 \\ 10 & -2 & 5 \\ 0 & -3 & -1 \end{bmatrix}$$

According to the prerequisite of the algorithm Order of the matrix A is 3

$$B = \frac{1}{10}\begin{bmatrix} 1 & 5 & 4 \\ 10 & 2 & 5 \\ 0 & 3 & 1 \end{bmatrix}$$

$J_0 = \phi$

$n_0 = 0$, and

$$w_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

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Iteration1

Step (i)

$$Bw_0 = \frac{1}{10}\begin{bmatrix}1 & 5 & 4\\10 & 2 & 5\\0 & 3 & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}1\\1.7\\0.4\end{bmatrix}$$

$(Bw_0)_i < 1$, where 'i' is the indices values satisfying the given condition, in this step it is $\{3\}$ Therefore $J_1 = \{3\}$ and n_1=1(number of indexes in J_1)

Step (ii)

$J_0 \neq J_1$ and $n_1 \neq n$

Step(iii)

$$\begin{cases}(w_1)_{J_1} = \left(I_{n_1} - B_{J_1,J_1}\right)^{-1}\sum_{b=1}^{3-1}(B_{J_1,J_1^c} I_{n-n_1})_{a,b} & a = 1\\ (w_k)_{J_1^c} = I_{n-n_1}\end{cases}$$

Where J_1^c contains the complement indexes of J_1 i.e. $J_1^c = \{1,2\}$

Now

$$(w_1)_{J_1} = \left[1 - \frac{1}{10}\right]^{-1}\sum_{b=1}^{2}\left(\begin{bmatrix}0 & \frac{3}{10}\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right)_{1,b}$$

$$(w_1)_{J_1} = \left[\frac{9}{10}\right]^{-1}\sum_{b=1}^{2}(\begin{bmatrix}0 & 0.3\end{bmatrix})_{1,b}$$

$$(w_1)_{J_1} = \left[\frac{10}{9}\right]\left[0.3\right] = 0.3333$$

$$(w_1)_{J_1^c} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

Now return to step (i) of the algorithm, with k=k+1 i.e. k=2

Iteration 2

Step (i)

$$w_1 = \begin{bmatrix}1\\1\\0.3333\end{bmatrix}; \text{ since } J_1 = \{3\} \text{ therefore}$$

$$w_1(3) = (w_1)_{J_1}$$

$$Bw_1 = \frac{1}{10}\begin{bmatrix}1 & 5 & 4\\10 & 2 & 5\\0 & 3 & 1\end{bmatrix}\begin{bmatrix}1\\1\\0.3333\end{bmatrix} = \begin{bmatrix}0.7333\\1.3666\\0.3333\end{bmatrix}$$

$(Bw_1)_i < 1$, where 'i' is the indices values satisfying the given condition, in this step it is $\{1,3\}$

Therefore $J_2 = \{1,3\}$ and $n_2 = 2$ (the number of indices in J_2)

Step (ii)

$J_1 \neq J_2$ and $n_2 \neq n$

Step (iii)

$$\begin{cases}(w_2)_{J_2} = \left(I_{n_2} - B_{J_2,J_2}\right)^{-1}\sum_{b=1}^{3-2}(B_{J_2,J_2^c} I_{n-n_2})_{a,b} & a = 1,2\\ (w_2)_{J_2^c} = I_{n-n_2}\end{cases}$$

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Where J_2^c contains the complement indexes of J_2 i.e. $J_2^c = \{2\}$
Now

$$(w_2)_{J_2} = \left[\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} - \frac{1}{10}\begin{bmatrix}1 & 4\\0 & 1\end{bmatrix}\right]^{-1} \sum_{b=1}^{1}\left(\frac{1}{10}\begin{bmatrix}5\\3\end{bmatrix}[1]\right)_{a,b} \qquad a = 1,2 \qquad (w_2)_{J_2^c} = [1]$$

$$(w_2)_{J_2} = \begin{bmatrix}0.9 & -0.4\\0 & 0.9\end{bmatrix}^{-1}\begin{pmatrix}0.5\\0.3\end{pmatrix}_{a,1} \qquad a = 1,2$$

$$(w_2)_{J_2} = \begin{bmatrix}0.7037\\0.0333\end{bmatrix}$$

Return to step (i) of the algorithm with k=k+1, i. e k=3

Iteration 3

Step (i)

$$w_2 = \begin{bmatrix}0.7037\\1\\0.0333\end{bmatrix}$$; since $J_2 = \{1,3\}$ therefore

$$Bw_2 = \frac{1}{10}\begin{bmatrix}1 & 5 & 4\\10 & 2 & 5\\0 & 3 & 1\end{bmatrix}\begin{bmatrix}0.7037\\1\\0.0333\end{bmatrix} = \begin{bmatrix}0.7037\\1.0703\\0.3333\end{bmatrix}$$

$$w_2\begin{pmatrix}1\\3\end{pmatrix} = (w_2)_{J_2}$$

$(Bw_2)_i < 1$, where 'i' is the indices values satisfying the given condition, in this step, it is again {1, 3}.Therefore $J_3 = \{1,3\}$ and $n_3 = 2$ (the number of indices in J_3)

Step (ii)

$J_2 = J_3$, and $n_3 \neq n$

In step (ii) of iteration 3, one of the conditions stated in step (ii) of the algorithm is satisfied. Therefore we will define

$J_B = J_3 = \{1,3\}$

$n_B = n_3 = 2$ and

$w_B = w_2$, since $n_B < n$

$$(w_B) = (w)_2 = \begin{bmatrix}0.7037\\1\\0.3333\end{bmatrix}$$

Exit the loop.

To calculate the value of P for the given A in Numerical Example 1, we will use MATLAB LMI tool box. The matrix P for given A in Numerical Example 1 comes out to be

$$P = \begin{bmatrix}0.8951 & -0.0789 & 0.1553\\-0.0789 & 0.5423 & -0.0186\\0.1553 & -0.0186 & 1.1414\end{bmatrix}$$

Following the algorithm stated in IV, for the A given in numerical example 1 we have $J_{|A|} = \{1,3\}$ and $J_{|A|}^c = \{2\}$.

Considering $J_{|A|}$, $J_{|A|}^c$ and P, for the given A in Numerical example 1, we will check whether Theorem 1 is satisfied, i.e.

$$(P)_{i,i} - \sum_{j\neq i}(w_{|A|})_j(|P|)_{i,j} > 0 \qquad for \quad all \qquad i \in J_{|A|}^c \qquad (3a)$$

In our case

$$(P)_{i,i} - \sum_{j\neq i}(w_{|A|})_j(|P|)_{i,j} > 0 \qquad for \quad all \qquad i \in J_{|A|}^c = \{2\} \qquad (3b)$$

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$$(P)_{2,2} - (w_{|A|})_1 (|P|)_{2,1} - (w_{|A|})_3 (|P|)_{2,3} \qquad (3c)$$

(0.5423)-(0.7037)*(|-0.0789|)-(0.3333)*(|-0.0186|) = 0.4806

Thus the value of (3c) comes out to be greater than zero. Hence the system considered in the numerical example 1 is judged to be asymptotically stable according to Theorem 1.The same can also be verified by plotting the state trajectories of the numerical example 1. The figure1 shows that the system under consideration is stable, as the next state of the system reaches zero with increasing iterations i.e. the output reaches zero with zero input

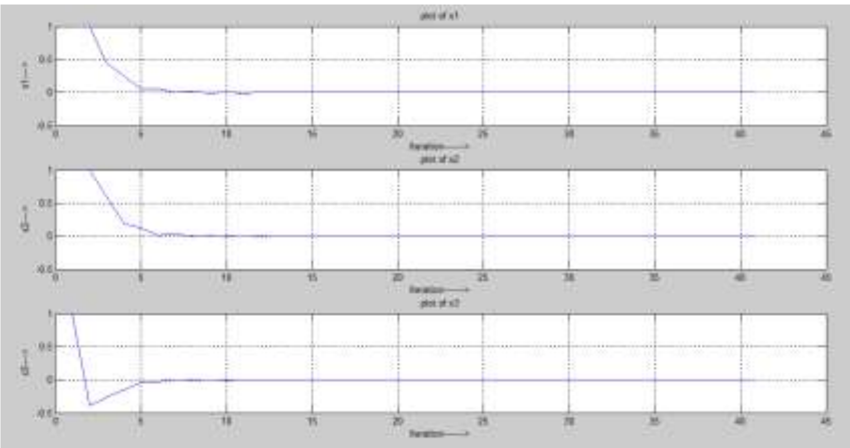

Figure1 Dynamical behavior of the system considered in numerical example

VI. CONCLUSION

The criteria for the global asymptotic stability of fixed-point state-space digital filters with saturation nonlinearity have been given by several researchers. A finite procedure proposed by Ooba.T [1] ascertains the global asymptotic stability of the system considered in the numerical example. Modification is done to the algorithm proposed by [1] ,which is reasonably required and it broaden the scope of stability test from those of earlier results.

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