

An Adaptive Lossy Image Compression using Singular Value Decomposition and Wavelet Difference Reduction

D. J. Ashpin Pabi¹, N. Puviarasan², P. Aruna³

Research Scholar, Department of Computer Science and Engineering, Annamalai University, Chidambaram, India¹

Associate Professor, Department of Computer Science and Engineering, Annamalai University, Chidambaram, India²

Professor, Department of Computer Science and Engineering, Annamalai University, Chidambaram, India³

Abstract: Image generation, sharing and analysis play an important role for day-to-day life. An adaptive lossy image compression using singular value decomposition (SVD) and wavelet difference reduction (WDR) is proposed for the application of colour and gray scale images. The goal is to improve the quality and minimize the storage requirement. The proposed adaptive lossy image compression is composed of three main components. The first is the application of SVD to the original input image. Next the wavelet difference reduction method is applied to the original image. Both the compressed images are fused in the final step. Such fusion of two methods increases the quality of the reconstructed images. The performance of the proposed compression method is evaluated using the parameters namely compression Ratio (CR) and Peak-Signal-to-Noise Ratio. Experimental result shows that the proposed adaptive lossy image compression using fusion of SVD and WDR outperforms the cascading of SVD and WDR.

Keywords: Image Compression, Colour Images, Gray scale Images, Singular Value Decomposition (SVD), Wavelet Difference Reduction (WDR), Fusion.

I. INTRODUCTION

It is necessary to reduce the size of images before it is being transmitted due to the rapid increase in the demand of transmission of images via public network. Social networking sites concern a great deal in the field of compression of images. There are several algorithms for performing the compression of images. Compression algorithms can be lossy, lossless and near-lossless. In Lossy image compression, there is a loss of image information, which can be visually acceptable; whereas lossless preserves image information. The lossy compression techniques achieve a high compression ratio. The output of the lossless image compressor corresponds to the original image. Lossless compression techniques cannot be used over large datasets. In picture archiving and communication systems (PACS), teleradiology and the remote dataset browsing systems, such lossless compression doesn't suit due to the limited transmission bandwidth and the storage constraints.

The methods of lossy compression schemes are chroma subsampling, Transform coding and Fractal compression. The techniques involved in lossless image compression techniques including Run-length encoding, Area image compression, Differential Pulse Code Modulation (DPCM), Entropy encoding, Adaptive dictionary algorithms, Deflation and Chain codes gives higher quality reconstructed images. Most of the compression methods involve the transformation methods include Discrete Cosine Transform (DCT), Discrete Wavelet Transform

(DWT), etc. leads to higher compression ratio and less quality reconstructed images. Compression algorithm based on the combination of Singular Value decomposition and Wavelet reduction, thus improves the quality of the reconstructed images. As the method is straight forward, it also reduces the complexity of the decompression of the method. Several compression methods have been evolved using singular value decomposition and wavelet difference reduction. They are used separately and together or combined with some other methods.[1-6].

The paper has been arranged as: section II describes the related works; section III describes the proposed adaptive compression method; the next section shows its result and their discussions; finally the work has been concluded in section V.

II. RELATED WORKS

In literature various compression algorithms have been generated for color and gray scale images using SVD and WDR methods. The singular value decomposition packs signal energy into fewer coefficients [2], [3]. It is a numerical technique which diagonalizes matrixes in numerical analysis. It is also a reliable orthogonal matrix decomposition method [5], [6]. Awwal mohammed rufai et. al developed an image compression algorithm that boosts the performance of the WDR compression by cascading singular value decomposition and wavelet difference reduction [7]. Manoj Kumar et al. developed a

compression algorithm using SVD and Huffman encoding [1].

In this paper we present an adaptive lossy image compression algorithm using SVD and WDR, capable to achieve better trade-off between the aforementioned requirements. The contribution of this work is the extension of Awwal mohammed rufai et al. method. The cascading of both the SVD and WDR method was carried out in their work. Cascading means that the output of one method is given as the input to the other. The proposed compression method replaces cascading with fusion to improve the quality of the reconstructed image. The comparisons are made by analyzing the performance measures carried out. The experimental results showed that the proposed adaptive lossy compression method in terms of fusion rather than cascading improved the aforementioned compression method.

III. THE PROPOSED COMPRESSION METHOD

A. Singular Value Decomposition

A color image takes three components, a red, a green and a blue (RGB). Hence three times the space requires to store such images. There are several ways to compress these images. One of which is singular value decomposition. The images are compressed by computing singular values. The mathematical expression of singular value decomposition can be formulated as: any nonzero real matrix A with rank $r > 0$ can be decomposed into

$$A = P \Sigma Q^T \quad (1)$$

where P is an $m \times r$ orthonormal column matrix,

$$\Sigma = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_r) \quad (2)$$

Where Q^T is an orthonormal row matrix

Such factorization is called the singular value decomposition. This is related to the spectral theorem, such that if B is a symmetric matrix $B^T = B$, then

$$B = U \Lambda U^T \quad (3)$$

Where Λ is a diagonal matrix of eigen values and U is an orthonormal matrix of eigen vectors.

To notice the relationship, consider:

$$A^T A = Q \Sigma^T P^T P \Sigma Q^T = Q \Sigma^2 Q^T \quad (4)$$

$$A A^T = P \Sigma Q^T Q \Sigma^T P^T = P \Sigma^2 P^T \quad (5)$$

These two are separately decomposed and hence σ_i are the positive square roots of the eigen values. The matrices are arranged, so that

$$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_n \quad (6)$$

Thus, an invertible matrix A with $n \times n$ can be written as

$$A = P \Sigma Q^T = (p_1, p_2, \dots, p_n) \begin{pmatrix} \sigma_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_n \end{pmatrix} \begin{pmatrix} q_1^T \\ \vdots \\ q_n^T \end{pmatrix} \\ = p_1 \sigma_1 q_1^T + p_2 \sigma_2 q_2^T + \dots + p_n \sigma_n q_n^T \quad (7)$$

Since, $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_n$, the terms $p_i \sigma_i q_i^T$ with small i contribute the sum, which contains the most important information about the image. These terms results in a lower image quality, but lower storage size. As color images are in RGB, the reduced SVD can be computed on all the three channels separately or together.

B. Wavelet Difference Reduction

The WDR encoding combines run length encoding to produce an embedded image coder. The encoding and the decoding process of WDR compression technique is shown in Fig. 1.

The WDR algorithm is a simple procedure which comprises a bit plane encoding technique. The steps of the WDR algorithm include:

1. A wavelet transform is applied to the input image.
2. The bit plane based WDR encoding algorithm has been carried out for the wavelet coefficients.

WDR mainly consist of the following five levels

1. Initialization: An initial threshold T_0 is chosen, such that, $|x_m| < T_0$ and at least one transform value satisfies $|x_m| \geq T_0/2$.
2. Threshold has to be updated, let $T_k = T_{k-1}/2$.
3. Significance pass: index values are encoded using the difference reduction method. The difference reduction methods consist of a binary encoding of the number of steps between the index of the last significant value and the index of the current significant value. The signs of significant values along with the sequence of bits will be its output.
4. Refinement pass: generates refined bits through the standard bit-plane quantization procedure.
5. Repeat steps (2) and (4).

C. The Proposed Fusion based Compression Technique

In this paper, we propose a lossy image compression based on the fusion of SVD and WDR methods explained in the previous section. Fig. 2 shows the block diagram of the proposed fusion based image compression technique.

In order to retain the quality of the reconstructed image the fusion of the two methods are introduced. First we decompose the large image matrix into three image matrices and singular values are applied for further compression. Such singular value decomposition method compress images with lower bits to the pixels. To retain the quality of the image wavelet difference reduction is applied next to SVD. Fusion is applied to the compressed image to boost the quality of the reconstructed compressed image. Fusion between the two pixel values of the two different compressed images increases the quality of the image.

The objective metrics of the proposed image compression shows that the proposed compression technique outperforms the existing cascading method of SVD and WDR. Hence the proposed fusion based compression method is considered as the adaptive lossy image compression based on SVD and WDR methods.

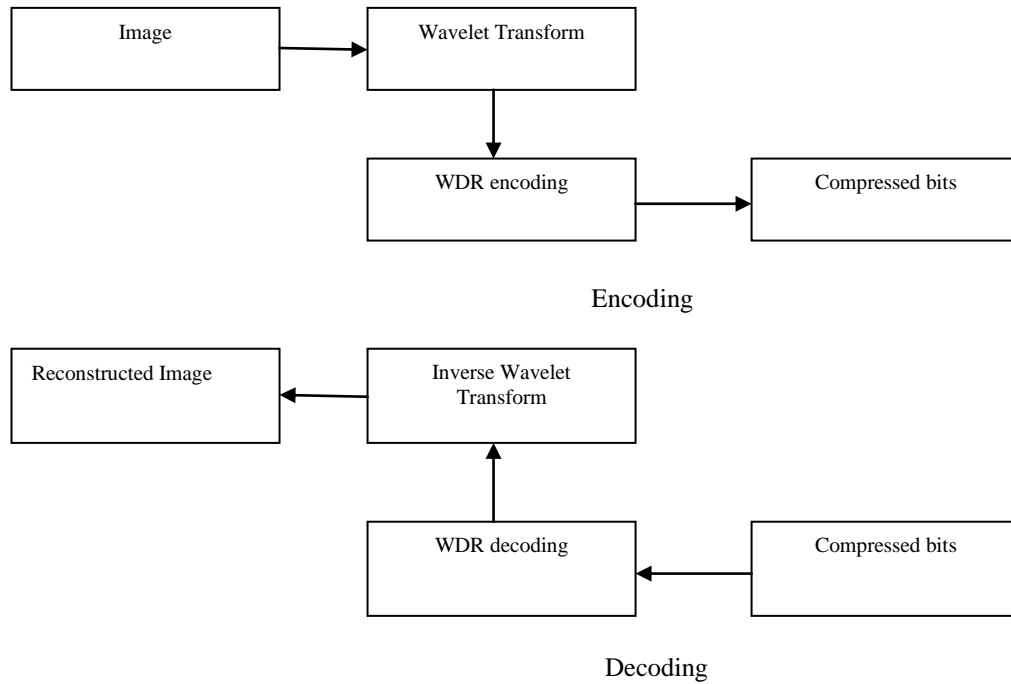


Fig. 1 Flow diagram of the WDR compression

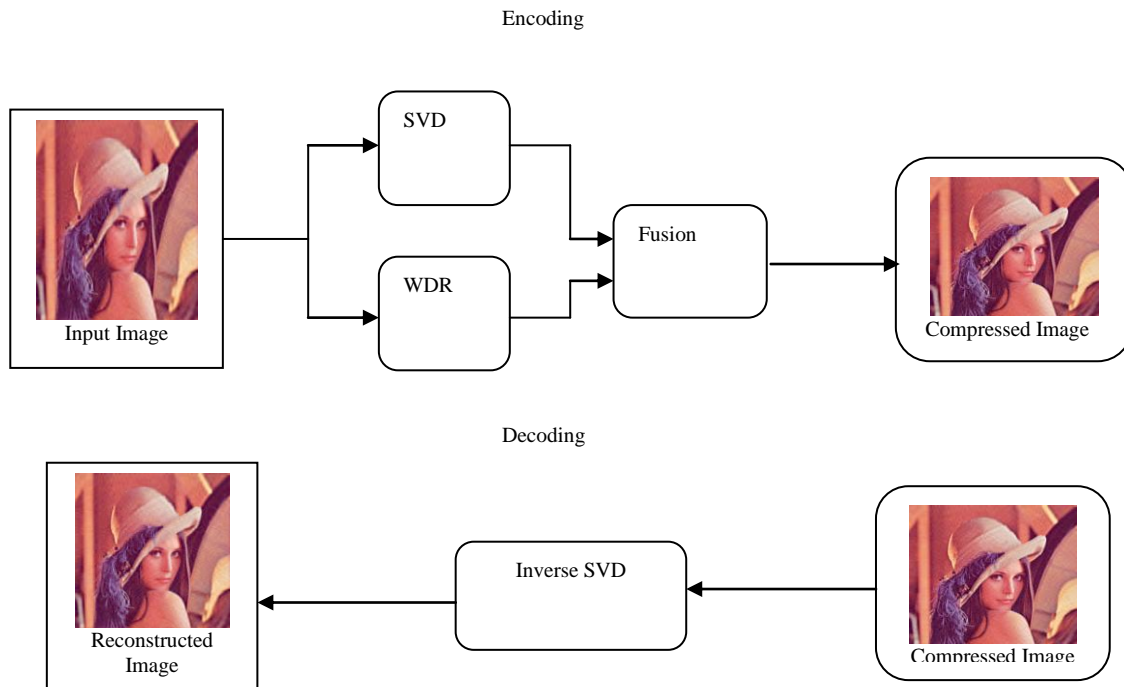


Fig. 2 Block diagram of the proposed fusion based compression technique

The algorithm of the proposed compression technique can be summarized as follows:

- 1) The image to be compressed is given as an input.
- 2) Apply SVD to the input image.
- 3) Apply WDR to the input image separately.
- 4) Both the compressed images are applied to obtain fused image.
- 5) Decompression over the above steps is performed to yield the reconstructed compressed image.

Algorithm 1: Encoding Algorithm: Algorithm of encoding using the proposed compression technique

Input : Color Image
Output : Compressed Image
Method :
 Step 1 : Get the input image
 Step 2 : Divide it into three matrices
 Step 3 : Apply SVD method.

Step 4 : Apply WDR method to the input image
 Step 5 : for i=0 to 255 && j=0 to 255
 for k=0 to 255 && l= 0 to 255
 Apply the fusion of SVD and WDR to
 both the compressed images
 Step 6 : Get the compressed image

Algorithm 2: Decoding Algorithm: Algorithm of Decoding using the proposed compression technique

Input : Compressed Image
Output : Reconstructed compressed image

Method :
 Step 1 : Input compressed image to the decoder
 Step 2 : Take Inverse WDR or Inverse SVD
 Step 4 : Get the Reconstructed images

$$PSNR = 10 * \log \left(\frac{255^2}{MSE} \right) \quad (8)$$

Mean Square Error:
 The mean square error is the cumulative squared error between the original and the compressed image.

$$MSE = \frac{1}{MN} \sum_{i=0}^M \sum_{j=0}^N (I(i,j) - J(i,j))^2 \quad (9)$$

Compression Ratio:
 It is the ratio between uncompressed image size and the compressed image size.

$$Compression\ Ratio = \frac{Uncompressed\ size}{Compressed\ size} \quad (10)$$

Bits Per Pixel:
 Number of bits used to represent single pixel define bits per pixel.

$$bpp = \frac{Size\ of\ the\ compressed\ image\ in\ bits}{Total\ No.\ of\ pixels} \quad (11)$$

IV. EXPERIMENTAL RESULTS AND DISCUSSIONS

The study explained the efficiency of the proposed adaptive compression method using the parameters includes MSE, PSNR, CR and bpp, defined as

Peak Signal-to-noise ratio:

The Peak Signal-to-noise ratio (PSNR) measures the difference in pixel value between two images, widely used to measure the quality of compressed or reconstructed images. The definition of PSNR is [8]:

The proposed lossy compression technique is applied to some true color images, including Lenna, Mandrill, Strawberry, City1, City2, Bus, Rose etc. The metrics such as peak signal-to-noise ratio (PSNR), the compression ratio (CR) and bits-per-pixel (bpp) significantly predicts the performance of the proposed techniques with the already existing techniques. Fig. 3 shows some of the test images used in this work.



Fig. 3 Test Images



Fig. 4 (a) Original image (b) Reconstructed images using the proposed compression technique for sv: 40 (c) sv:50 (d) sv:100 (e) sv:200 (f) sv:300

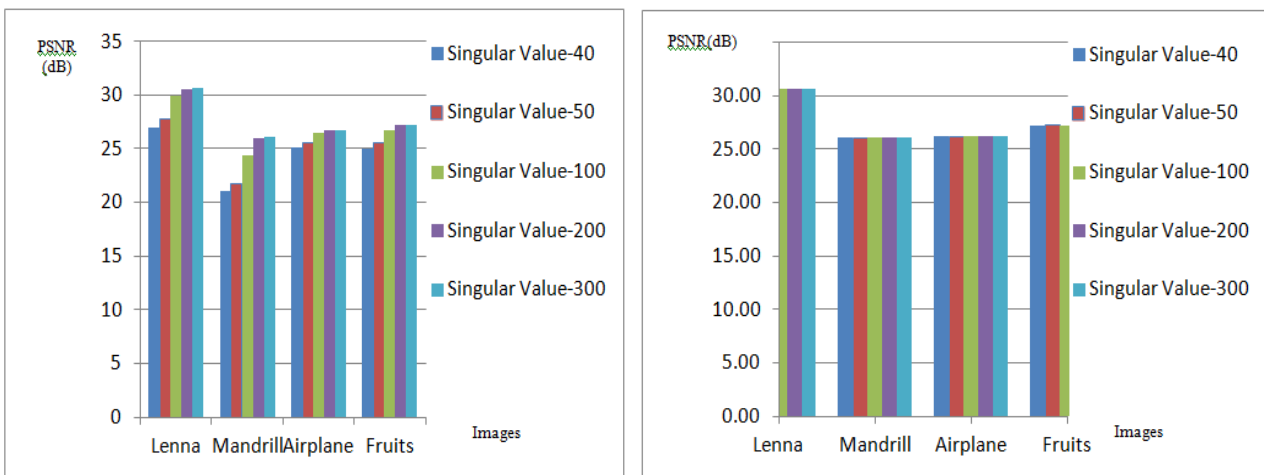


Fig. 5 Average performance of the proposed compression technique for various images (a) using cascading of SVD and WDR (b) using the proposed fusion based compression technique

Fig. 4 shows the subjective value of the quality of the compressed Lena image with different singular values (sv). The quality of the image does not detect by the human visual evaluation.

Figure 5 shows the average PSNR value for Lenna, Mandrill, Airplane and Fruits images at various singular values. The average PSNR value is 30 dB for those images. In Fig. 5(a) we notice that the quality of the images increases with increase in singular values. Hence it takes more time to compress the image to predict the optimum singular value. Fig. 5 (b) shows the performance

of the proposed fusion based compression technique. From the figure it could be seen that the PSNR value remains same for all the singular values sv. Hence the proposed method outperforms the existing cascading of SVD and WDR in terms of fast in compression.

Fig. 6 shows the Comparison of the compression ratio and the PSNR of the proposed compression method over the cascading of SVD and WDR. It is found that the PSNR value is high for the proposed fusion of SVD and WDR compression technique, also the PSNR value remains same for different singular values.

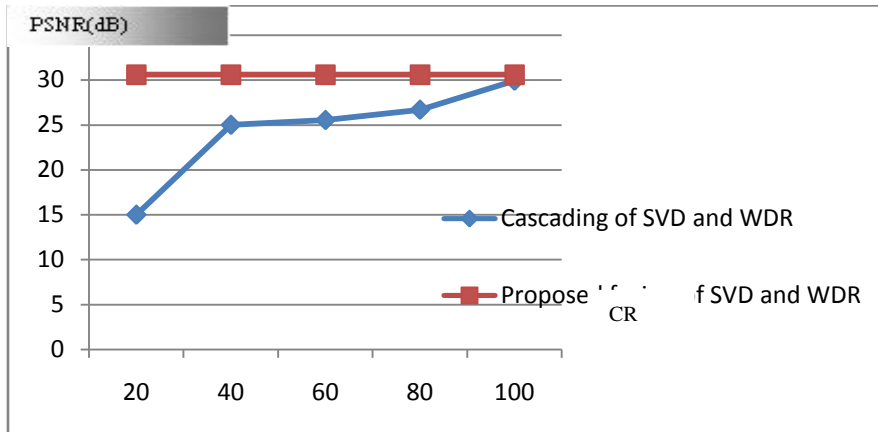






Fig. 6 Comparison of the CR and the PSNR of the proposed compression method

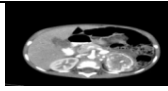

TABLE I BPP AND PSNR OF VARIOUS TEST IMAGE WITH VARYING SINGULAR VALUES ON CASCADING OF SVD AND WDR AND FUSION OF SVD AND WDR

| Test Images | Metrics | Existing method [7] | | | | Proposed Method | | | |
|---|---------|---------------------|----------|----------|----------|-----------------|----------|----------|----------|
| | | SV=50 | SV=100 | SV=200 | SV=300 | SV=50 | SV=100 | SV=200 | SV=300 |
|  | PSNR | 27.7245 | 29.9139 | 29.9717 | 29.9717 | 30.586 | 30.586 | 30.586 | 30.586 |
| | bpp | 0.479889 | 0.493546 | 0.502879 | 0.502879 | 0.503448 | 0.503448 | 0.503448 | 0.503448 |
|  | PSNR | 21.7284 | 24.3332 | 25.9704 | 25.9859 | 26.0359 | 26.0359 | 26.0359 | 26.0359 |
| | bpp | 0.678019 | 0.750722 | 0.785273 | 0.786577 | 0.787577 | 0.787577 | 0.787577 | 0.787577 |
|  | PSNR | 25.5276 | 26.4539 | 26.6848 | 26.6876 | 26.6916 | 26.6916 | 26.6916 | 26.6916 |
| | bpp | 0.301687 | 0.450536 | 0.455638 | 0.455768 | 0.455968 | 0.455968 | 0.455968 | 0.455968 |
|  | PSNR | 25.5376 | 26.7484 | 27.2034 | 27.2097 | 27.2217 | 27.2217 | 27.2217 | 27.2217 |
| | bpp | 0.438787 | 0.46787 | 0.481099 | 0.481323 | 0.481623 | 0.481623 | 0.481623 | 0.481623 |

The quantitative results of the proposed compression technique and the existing compression technique for color and medical images are shown in Table I and Table II. As the scheme proposed by Awwal in [7] is the state of the art technique for compression of images, to give the comparison we have compared the proposed fusion based compression technique with the cascading of SVD and WDR scheme of Awwal [7]. The parameters PSNR and bpp are used for Awwal's scheme. With these parameters for the test image Lena, Awwal's scheme gives bpp=0.502948 and PSNR=29.9717 dB. However in the proposed fusion based compression method sv=50,100,200,300 are used in order to achieve good

compression performance, as a result we achieved PSNR=30.586 and bpp=0.503448. Hence from the quantitative analysis of the results, it is evident that the proposed work has maintained the good quality of the reconstructed image. Table III shows the objective results of the proposed compression method. The quality of the Lena image is 30.586 dB with the compression ratio CR=75.1823. The compression ratio is lower for higher quality image. The comparison of proposed work with the state of art SVD and cascading of SVD and WDR is shown in Fig. 7. The bits per pixel increase with increase in the PSNR values. The proposed method shows better improvement over the other existing techniques.

TABLE II BPP AND PSNR OF VARIOUS MEDICAL MRI BRAIN IMAGE WITH VARYING SINGULAR VALUES ON CASCADING OF SVD AND WDR AND FUSION OF SVD AND WDR

| Test Images | Metric s | Existing method [7] | | | | Proposed Method | | | |
|---|----------|---------------------|----------|----------|----------|-----------------|----------|----------|----------|
| | | SV=50 | SV=100 | SV=200 | SV=300 | SV=50 | SV=100 | SV=200 | SV=300 |
|  | PSNR | 34.8654 | 36.5174 | 38.181 | 38.181 | 40.181 | 40.181 | 40.181 | 40.181 |
| | bpp | 0.34081 | 0.340403 | 0.178156 | 0.178156 | 0.198156 | 0.198156 | 0.198156 | 0.198156 |
|  | PSNR | 30.1056 | 39.0476 | 40.2067 | 40.2067 | 42.2038 | 42.2038 | 42.2038 | 42.2038 |
| | bpp | 0.232844 | 0.310369 | 0.2157 | 0.2157 | 0.2857 | 0.2857 | 0.2857 | 0.2857 |

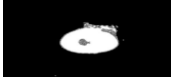
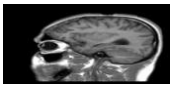
| | | | | | | | | | |
|---|------|----------|----------|----------|----------|----------|----------|----------|----------|
|  | PSNR | 43.6576 | 45.7253 | 45.7253 | 45.7253 | 48.8453 | 48.8453 | 48.8453 | 48.8453 |
| | bpp | 0.108419 | 0.103511 | 0.103511 | 0.103511 | 0.097351 | 0.097351 | 0.097351 | 0.097351 |
|  | PSNR | 33.7753 | 36.757 | 36.9723 | 36.9723 | 37.2253 | 37.2253 | 37.2253 | 37.2253 |
| | bpp | 0.423889 | 0.425522 | 0.42345 | 0.42345 | 0.4123 | 0.4123 | 0.4123 | 0.4123 |

TABLE III QUALITY METRICS OF THE PROPOSED COMPRESSION METHOD SVD AND WDR

| Images | CR(%) | bpp | PSNR[dB] |
|----------|---------|----------|----------|
| Lenna | 75.1823 | 0.503448 | 30.586 |
| Mandrill | 91.6817 | 0.7875 | 26.0359 |
| Airplane | 72.4479 | 0.455968 | 26.6916 |
| Fruits | 72.5329 | 0.481623 | 27.2217 |

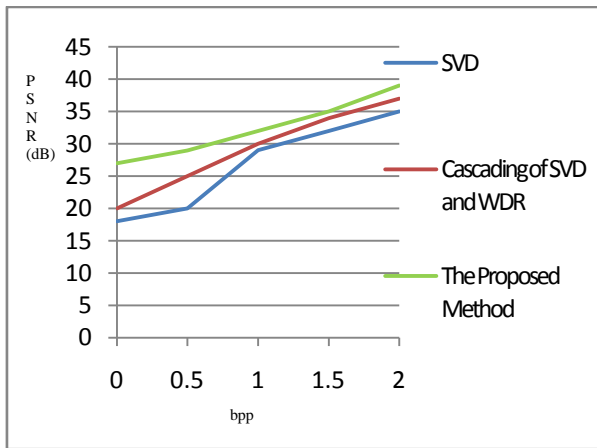


Fig. 7 Performance of proposed method over other techniques

Fig. 8 shows the Graphical User Interface of the proposed fusion based compression technique.



Fig. 8 GUI of the proposed compression technique

V. CONCLUSIONS

A new image compression technique using the fusion of SVD and WDR is proposed. First, the image was compressed using SVD, which offers high compression ratios and then WDR is applied to the same image to retain the quality of the image. Finally fusion of SVD and WDR is applied to increase the PSNR of the reconstructed

compressed image. The results were compared with SVD and WDR exclusive methods in terms of some compression metrics. The proposed technique is tested over the test images in order to measure the quality. The average PSNR value of the proposed method gets higher when compared with other techniques. Thus, it is found that the proposed system shows the superiority over other techniques. Regarding future work directions, aiming at further enhancing the quality of reconstructed image and to extend this research to all types of medical images.

REFERENCES

- [1] Manoj Kumar, Ankita Vaish. An efficient encryption-then-compression technique for encrypted images using SVD 2016; -.
- [2] Moonen M, Van Dooren P, Vandewalle J. Singular value decomposition updating algorithm for subspace tracking. SIAM Journal on Matrix Analysis and Applications 1992;13(4):1015-38.
- [3] Konda T, Nakamura Y. A new algorithm for singular value decomposition and its parallelization. Parallel Comput 2009;35(6):331-344.
- [4] Andrews H C, Patterson C L. Singular value decompositions and digital image processing. IEEE Trans. On Acoustics, Speech, and Signal Processing 1976;24(1):26-53.
- [5] Julie Kamm L. SVD-Based Methods For Signal And Image Restoration. PhD Thesis,1998.
- [6] Yang J F, Lu C L. Combined Techniques of Singular Value Decomposition and Vector Quantization for Image Coding. IEEE Trans. Image Processing,1995, 4(8) :1141 – 11.
- [7] Awwal Mohammed Rufai, Gholamreza Anbarjafari, Hasan Demirel, Lossy Image Compression using singular value decomposition and wavelet difference reduction, Digital signal Processing (2014) 117- 123.
- [8] Nanpreet Saroya, Prabhpreet Kaur. Anaysis of image compression algorithm using DCT and DWT transforms. International journal of advanced Research in Computer Science and Software Engineering, 2012, 4(2):897-900.