

An Overview and Analysis of the higher order QAM Modulation Scheme over Rayleigh Fading Channel with RSBC, Alamouti STBC and Interleaving for Wireless Communication Applications

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Abstract: The aim of the paper is to analyze the performance under various schemes and coding techniques for one of the most widely used modulation technique for wireless & mobile communication applications i.e. QAM (Quadrature Amplitude Modulation) in Rayleigh fading channel. First of all, we observe and analyze the effect of Rayleigh fading on the communication channel using QAM (Quadrature Amplitude Modulation) technique that can provide the higher rate of transmission without any incremental changes or variation in the bandwidth of the utilizing wireless communication systems. Whereas the performance of any modulation technique is measured on the basis of or in terms of its bit error probability therefore we find the performance of QAM in fading channel in terms of its bit error probability then we will try to improve & observe the effect on the value of bit error rate for the QAM technique by employing various schemes and coding techniques like Reed-Soloman block coding, interleaving and alamouti STBC. In order to overcome the fading which is one of the major problems in the wireless communications, a diversity modulation technique is used for the efficient transfer of message signals whereas for error detection and correction the Reed-Solomon codes are used and the error rate is improved by the interleaver.

Keywords: Bit error probability, QAM, STBC, and Interleaver.

I. INTRODUCTION

Radio signals generally propagate according to three mechanisms; reflection, diffraction, and scattering. Reflection is occurs when the plane waves are incident upon any surface with dimensions that are very large when compared to the wavelength. Whereas diffraction arises as per the Huygen's principle whenever there is any object between the transmitting and receiving antennas, and as a result, secondary waves are get generated that arises behind the obstructing body. The scattering is occurred due to the reflection of the plane waves when incident on the object, whose dimensions are on the order of a wavelength or less. As a result energy of the waves gets radiated in the various directions. In a terrestrial environment, the multiple signals are present as a result Multipath signals are received i.e. different variations of the signals are propagated in the environment that reaches to the receiver from the transmitter through a various path. In addition, the movement of Tx or Rx or the surrounding clutter in it results in the amplitude or phase changes in the overall received signal for the small amount of time that causes the fading. The main effects of the multipath fading are rapid variations in the strength of a signal when travelled over a small distance or interval of time. On different multipath signals due to variations in the Doppler shifts results into a random frequency modulation. Multipath propagation delays result into the echoes or time dispersion. The problem can be solved by following two different ways firstly by adding margin to the fading on the transmitter but it is not considered to be an effective solution for this, therefore another method that can be used is to receive the two or more variations of the signal at the receiver ensuring the correlation between them. So, the system performance in fading channels is improved by the diversity technique. Rather than receiving and transmitting the required signal using one channel, we use the multiple copies of the required signal that are transmitted over the different channels. In order to provide a diversity gain, the space-time block coding utilizes the high algebraic structure. The multiple transmit & receiver antennas increase the diversity, which results in high SNR. The QAM is one of the digital modulations type or scheme that provides the greater rate of transmission for the wireless communication systems along with the high spectral efficiency and for that it is not necessary to increase the bandwidth of the channel. QAM, combined with other schemes, has gained great attention in overcoming detrimental channel impairments. QAM is widely used because it comprises of amplitude as



well as phase, that helps in correcting the QAM signal when get corrupted. The Quadrature Amplitude Modulation (QAM) is simply a combination of AM and PSK, in which two carriers out of phase by 90° are amplitude modulated. The 16QAM is used in various applications such as DVB-C (digital video broadcasting—cable), modems and microwave digital radio. The 16-QAM or other higher-order QAMs (64-QAM, 256-QAM) are more bandwidth efficient than BPSK, QPSK, or 8PSK and are used to gain high-speed transmission. The performance of Quadrature Amplitude Modulation (QAM) scheme is measured by estimating its probability of error produced by noise and interference induced in the channel. Because of the fading in a wireless communication system during transmission leads to an error at the receiver end. The number of errors that are likely to occur in the system is expressed as the bit error rate (BER). The bit error ratio or bit error rate (BER) is the number of bit errors corresponding to the total number of transferred bits in a studied time interval. The BER is given as the ratio between the Errors and the total Number of Bits, whereas the bit error probability indicates the value of the expectation for the bit error rate. The bit error rate, which is the approximation of the bit error probability is said to be accurate in the long time interval and for a high number of bit errors. In this paper, we analyze the performance of QAM which is degraded due to multipath fading by employing various schemes and coding techniques like reed-soloman block coding, interleaving, and almouti STBC. For that, we calculate the probability of bit error rate of Quadrature Amplitude Modulation (QAM) over Rayleigh fading and additive white Gaussian channels with Reed-Soloman block coding, interleaving and almouti STBC. For the above simulation and analysis, a 16-QAM modulation technique is used along with (15, 7) reed soloman block coding, 30 X 30 interleaver and almouti 2 X 2 STBC while the MATLAB software with relevant Toolboxes is used for developing Simulink model and analysis.

The rest of the paper is organized as follows. The major elements of the systems are described in Section II. The BER expression for 16-QAM along with the BER expression for 16-QAM with Reed Soloman block coding are presented in Section III. Whereas the Simulation & Analysis is presented in Section IV & the Conclusions are given in Section V.

II. SYSTEM ELEMENTS

A. Quadrature Amplitude Modulation

QAM schemes with rectangular signal constellations are generally favored in wireless communications protocols because they are less complex to implement in the modulation/demodulation process than non-rectangular constellations. Rectangular QAM signal constellations can be easily generated by means of a using the two PAM signals such that the PAM signals are impressed on the in-phase and quadrature carriers. In addition, they are easily demodulated. Even though it is not one of the best M-ary QAM signal constellations for the $M = 16$, the average amount of transmitted power required to achieve a given minimum distance is just slightly greater than the average power, which is required for the best M-ary QAM signal constellation. This makes the rectangular M-ary QAM signals to get used more widely in the applications. M-ary QAM and are widely used in digital terrestrial microwave links, Internet communication via twisted-pair telephone wires, and in wireless local area networks (WLAN) such as IEEE 802.11a,g and IEEE 802.16.

B. Alamouti 2 X 2 STBC

Space-time block code (STBC) is one of the effective diversity method and Space-time coding (STC) techniques to combat the fading effect using multiple transmit/receive antennas in wireless multiple-input-multiple-output (MIMO) channels without bandwidth expansion, having one of the important advantage and feature of linear decoding/detection algorithms as compared to the other Space-time coding (STC) techniques. The STBC is a widely used technique in wireless communications for transmitting different copies of information from multiple antennas and the use of different versions of the same information to the receiver in such a way as to improve the system performance. The space-time coding basically combines all the copies of the original signal, obtained in the most appropriate manner in order to recover the best possible information from them. Space-time block coding is a channel coding techniques simple yet ingenious transmit diversity technique in MIMO technology. An STBC can be assumed as one of the modulation technique for multiple transmitting antennas that provides the full diversity as well as very low complexity encoding and decoding. The goal of space-time coding is to achieve the maximum diversity of $M_t \times M_r$, the maximum coding gain, and the highest possible throughput. Where M_t and M_r are a number of transmitting & receiving antennas respectively. STBCs with two transmit and two receive antennas are among different MIMO types that have been widely utilized by wireless communications standards. For a 2×2 system, Alamouti Basically proposed a simple scheme that provides a full diversity gain along with the maximum likelihood decoding algorithm. In which Combination of both the receive and transmit antenna diversity results into to the full diversity benefit as a formation of multiple-input multiple-output (MIMO) system.

Let us assume that transmit diversity via space-time coding is implemented by the M_t transmitting antennas while receive diversity is implemented by the M_r receiving antennas with the proper maximal ratio combining it gives the full diversity of order $M = M_t M_r$. for each l^{th} receive antenna, there corresponds an orthogonal channel tap matrix H_l



obtained from the mapping $Gh_l \rightarrow H_l s$ where $h_l = [h_{l,1} h_{l,2} \dots h_{l,M_t}]^t$ is the l^{th} channel tap vector representing M_t paths from M_t transmit antennas to the l^{th} receive antenna. The receivers compute the following pre-combining vectors

$$X_l = H_l^*(H_l s + N_l) = \|h_l\|^2 s + H_l^* N_l, \quad l = 1, 2, \dots, M_r, \quad (1)$$

Define the channel tap vector as h follows $h = [h_1 h_2 \dots h_{M_r}]^t$. The normalized sum of the pre-combining vectors, X , exhibits the maximal ratio combining for all $M = M_t M_r$ paths and is given by,

$$X = \frac{1}{\|h\|} \sum_1^{M_r} X_l = \frac{1}{\|h\|} \sum_1^{M_r} \|h_l\|^2 s + \frac{1}{\|h\|} \sum_1^{M_r} H_l^* N_l, \quad (2)$$

The decision vector X can be detected by m parallel minimum Euclidean distance detectors. The noise vector has the covariance $2\sigma^2 I_m$. For each l^{th} receive antenna there corresponds a m channel tap vectors $h_{i,l}^*$, $i = 1, 2, \dots, m$. the receiver computes $X_{i,l} = h_{i,l}^* Y_{i,l} = \|h_l\|^2 s_i + \|h_l\| N_{i,l}$, $i = 1, 2, \dots, m; l = 1, 2, \dots, M_r$, where $h_l = [h_{l,1} h_{l,2} \dots h_{l,M_t}]^t$ is the l^{th} channel tap vector representing M_t paths from M_t transmit antennas to the l^{th} receive antenna. The receiver computes the following combining variables based on the channel tap vector $h = [h_1 h_2 \dots h_{M_r}]^t$.

$$X_i = \frac{1}{\|h\|} \sum_1^{M_r} X_{i,l} = \frac{1}{\|h\|} \sum_1^{M_r} \|h_l\|^2 s_i + \frac{1}{\|h\|} \sum_1^{M_r} \|h_l\| N_{i,l}, \quad (3)$$

The m decision variables X_i can be detected by m parallel minimum Euclidean detectors. The noise vector has the covariance $2\sigma^2$.

2 x 2 MIMO System

Consider a MIMO system with two transmit antennas utilizing the Alamouti code and two receive antennas. Let h_{ij} be the channel tap between the j^{th} ($j = 1, 2$) transmit antenna and the l^{th} ($l = 1, 2$) receive antenna. The sum of the channel tap weighted code word at the first receive antenna ($l = 1$) is,

$$Z_1 = h_{11} \begin{bmatrix} s_1 \\ -s_2^* \end{bmatrix} + h_{12} \begin{bmatrix} s_2 \\ -s_1^* \end{bmatrix} + \begin{bmatrix} n_{11} \\ n_{12} \end{bmatrix}, \quad (4)$$

$$Z_1 = Gh_1 + n_1.$$

Where, $h_1 = [h_{11} \ h_{12}]^t$.

Similarly, for the second receive antenna ($l = 2$) we have,

$$Z_2 = h_{21} \begin{bmatrix} s_1 \\ -s_2^* \end{bmatrix} + h_{22} \begin{bmatrix} s_2 \\ -s_1^* \end{bmatrix} + \begin{bmatrix} n_{21} \\ n_{22} \end{bmatrix}, \quad (5)$$

$$Z_2 = Gh_2 + n_2.$$

Where, $h_2 = [h_{21} \ h_{22}]^t$.

The vector Z_l can be mapped into the vector $Y_l = H_l s + N_l$ by changing the second element Z_{l2} of $Z_l = [Z_{l1} \ Z_{l2}]^t$ into its complex conjugate. Thus for $Z_l = [s_1 \ s_2]^t$ we have $Y_l = H_l s + N_l = \begin{bmatrix} Z_{l1} \\ Z_{l2}^* \end{bmatrix}$, $l = 1, 2$ specifically, we obtain

$$Y_1 = \begin{bmatrix} h_{11} s_1 + h_{12} s_2 \\ h_{12}^* s_2 - h_{11}^* s_1 \end{bmatrix} + \begin{bmatrix} n_{11} \\ n_{12} \end{bmatrix}, \quad (6)$$

$$Y_1 = H_1 s + N_1.$$

Where, $H_1 = \begin{bmatrix} h_{11} & h_{12} \\ h_{12}^* & -h_{11}^* \end{bmatrix}$, $H_1^* H_1 = \|h_1\|^2 I_2$.

Also,

$$Y_2 = \begin{bmatrix} h_{21} s_1 + h_{22} s_2 \\ h_{22}^* s_2 - h_{21}^* s_1 \end{bmatrix} + \begin{bmatrix} n_{21} \\ n_{22} \end{bmatrix},$$



$$Y_2 = \begin{bmatrix} h_{21} & h_{22} \\ h_{22}^* & -h_{21}^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_{21} \\ n_{22}^* \end{bmatrix},$$

$$Y_2 = H_2 s + N_2. \tag{7}$$

Where, $H_2 = \begin{bmatrix} h_{21} & h_{22} \\ h_{22}^* & -h_{21}^* \end{bmatrix}$, $H_2^* H_2 = \|h_2\|^2 I_2$.

Therefore, the decision variable

$$X_l = H_l^* (H_l s + N_l) = \|h_l\|^2 s + H_l^* N_l, \quad l = 1, 2.$$

Specifically, we have

$$X_1 = \|h_1\|^2 s + H_1^* N_1. \tag{8}$$

$$X_2 = \|h_2\|^2 s + H_2^* N_2. \tag{9}$$

The MRC decision variable is the normalized sum of X_1 and X_2 . For the channel tap vector $h = [h_{11} \ h_{12} \ h_{21} \ h_{22}]^t$ we have,

$$X = \frac{1}{\|h\|} (X_1 + X_2) = \frac{\|h_1\|^2 + \|h_2\|^2}{\|h\|} s + \frac{1}{\|h\|} (H_1^* N_1 + H_2^* N_2) \tag{10}$$

Since, $\|h_1\|^2 + \|h_2\|^2 = |h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2 = \|h\|^2$, We obtain the following MRC decision vector, $X = \|h\|s + N$. Where the noise vector is $N = (H_1^* N_1 + H_2^* N_2) / \|h\|$. The covariance of noise N is given by $E(NN^*) = H_1^* E(N_1 N_1^*) H_1 + H_2^* E(N_2 N_2^*) H_2$. Since $E(N_l N_l^*) = 2\sigma^2 I_2, l = 1, 2$, we obtain $E(NN^*) = 2\sigma^2 (\|h_1\|^2 + \|h_2\|^2) I_2 / \|h\|^2 = 2\sigma^2 I_2$.

- { G - Is the space-time block code Matrix.
- H - Is the channel tap matrix.
- Z - Is the premapped vector at the input of the combiner.
- X - Is the decision vector.
- h - Is the channel tap vector $h = [h_1 \ h_2 \ \dots \ h_{M_r}]^t$.
- n - Is the gaussian noise vector $n = [n_1 \ n_2 \ \dots \ n_{M_r}]^t$.

C. Reed-Solomon Codes

Reed-Solomon (RS) codes are probably the most widely used codes in practice. These codes are used in data storage and mainly in communication systems. They are one the kind of non binary BCH codes defined by the Reed and Solomon in 1960 that provides the solution for the problem of burst errors i.e. they are helpful in correcting the burst errors along with the advantage of high coding efficiency that increases as the length of the coding increases and can be configured with the large block lengths in bits that requires lesser decoding time when compared with the other codes that has the same lengths this is because the decoding logic works with the symbol-based approach rather than the bit-based arithmetic approach. With the increase in the redundancy of the R-S code that has the lower code rate the complexity of implementation increases in case of devices that are operating at a higher speed that result in the improvement of the bit-error performance same as when obtained due to the increased in the size, of the symbol. The R-S codes have the advantage of great power and utility and thus found application in electronics & communication such as disc players to deep-space communication applications. The Reed-Solomon codes are non binary cyclic codes, in which symbols are consist of the $m - bit$ sequences, where m is defined as any positive integer that has the value greater than 2. The (n, k) R-S codes on $m - bit$ symbols exist for the value of n and k for which $0 < k < n < 2^m + 2$ where n is called as the total no. of code symbols in an encoded block while the k is called as the total no. of data symbols getting encoded. For the most generalize R-S (n, k) code $(n, k) = (2^m - 1, 2^m - 1 - 2t)$ where t defined as the symbol-error correcting capability of the code and $n - k = 2t$ is called as the number of parity symbols. The Reed-Solomon codes achieve the better largest possible code minimum distance as compared to any linear code having the same encoder input and output block lengths. For non binary codes, the distance between two codewords is defined (analogous to Hamming distance) as the number of symbols in which the sequences differ. For Reed-Solomon codes, the minimum code distance is given by the,

$$d_{min} = n - k + 1.$$

That is capable of correcting the fewer errors or composition of t errors, where t is given by the expression below.

$$t = \left\lfloor \frac{d_{min} - 1}{2} \right\rfloor = \left\lfloor \frac{n - k}{2} \right\rfloor \tag{11}$$

Where $\lfloor x \rfloor$ means the largest integer not to exceed x . The above equation illustrates that in order to correct the t symbol errors, the Reed-Soloman code does not require the parity symbols more than that of $2t$. Which means that the decoder



can spend $n - k$ redundant symbols which are exactly twice than that of a number of correctable errors. For the each error the two redundant symbols are used of which one of the redundant symbol is used to locate the error whereas another redundant symbol is used to find its correct value.

The erasure-correcting capability of the code is given by

$$\rho = d_{min} - 1 = n - k \quad (12)$$

Whereas simultaneously error and erasure correcting capability can be given as

$$2\alpha + \gamma < d_{min} < n - k \quad (13)$$

Where α and γ are called as the number of symbol error and erasure patterns which can be corrected. An advantage of non binary codes such as a Reed-Solomon code can be seen by the following considerations.

Let us consider a binary code that has the values of the n and k as 15 and 7 respectively therefore for the said case there will be total $2^n = 2^{15} = 32768$ n-tuples out of which there are $2^k = 2^7 = 128$ (Or 1/256 of the n-tuples) are codewords. While when the same values of n and k have been considered for the non binary code where the symbol consists of the m -bit that has the value of 4 there will be total $2^{nm} = 2^{60} = 1152921504606846976$ n-tuples out of which there are $2^{km} = 2^{28} = 268435456$ (or 1/4294967296 of the n-tuples) are codewords. When dealing with non binary symbols, each made up of m bits, only a small fraction (i.e. 2^{km} of the large number 2^{nm}) of possible n-tuples are codewords. This fraction decreases with increasing values of m . The important point here is that when a small fraction of the n-tuple space is used for codewords, a large d_{min} can be created. Any linear code is capable of correcting $n - k$ symbol erasure patterns if the $n - k$ erased symbols all happen to lie on the parity symbols. However, the R-S codes have one of the remarkable property that they are able to correct $n - k$ symbol erasures in a set over the block. The R-S codes can be designed to have any redundancy. However, the complexity of a high-speed implementation increases with redundancy. As a result, most useable R-S codes have the high code rates (low redundancy).

D. Interleaving.

An Interleaving is a form of data scrambling that spreads bursts of bit errors evenly over the received data allowing efficient forward error correction. An effective method for correction of error burst is to interleave the coded data such that the locations of errors look random & is distributed over many code words rather than few codeword. In this way, the number of errors that occur in each block is low & can be corrected by using a random error correcting code. On the receiver end, the deinterleaver block is used in order to alter the effect of the interleaving operation. The error-correcting codes are generally capable of correcting individual data-bit errors, but not a burst error involving a group of adjacent data bits. However, in the wireless and mobile channel environment, the burst error occurs quite frequently. In order to correct the burst error, interleaving exploits time diversity without adding any overhead in wireless digital cellular communication systems such as GSM, IS-95 CDMA, 3G cellular systems. Interleavers disperse the burst error into multiple individual errors which can then be handled by an error-correcting code. Furthermore, interleaving does not have the error-correcting capability. Therefore, interleaving is always used in conjunction with an error-correcting code. In other words, interleaving does not introduce any redundancy into the information sequence, so it does not add to extra bandwidth requirement. The disadvantage of interleaving is an additional delay as the information data sequence needs to be processed block by block. Therefore, small memory size interleaving is preferred in delay-sensitive applications. Correlative channel fading usually creates bursty errors during fading periods, which reduces the forward error-correction capability of channel coding. Interleaving is usually followed by channel coding and is quite effective in combating bursty errors. The coded symbols are first interleaved before being mapped to modulated waveforms in the transmitter. At the receiver, the demodulator output symbols are then de-interleaved before being applied to the decoder. The interleaver length is kept sufficient enough so as to eliminate the negative effect of the channel-fading correlation on the coding gain. When the normalized channel fading rate is low, interleaving becomes unnecessary and adds extra processing delay and hardware memory complexity which may not be acceptable in real-time communications applications. Interleaving the coded message before transmission and deinterleaving after reception causes bursts of channel errors to be spread out in time. An interleaver permutes the symbols, according to the mapping. In order to restore the original sequence of the symbols a corresponding deinterleaver uses the inverse mapping. An Interleaving is a technique that requires only the knowledge of the duration of the channel memory and not its exact statistical characterization. An interleaver main functionality is to protect the transmitted data from the error bursts. Due to the use of speech coders, many important bits are produced together. The interleaver spreads these bits out in time so that all these bits are not corrupted at the same time by deep fade or noise burst. There are many types of interleavers such as block interleaver, semi random and random interleaver, circular interleaver, odd-even interleaver, and near-optimal interleaver. Each one has its advantages and drawbacks in the context of noise. Block interleaver is the most commonly used interleaver in wireless communication systems. A set of symbols is accepted by



the block interleaver. The block interleaver then rearranges the symbols in such a way that none of the symbols is omitted or repeated. In the case of given interleaver in each set, the number of symbols is fixed. Table I summarizes various types of block interleavers along with their functions. Each one of the special-case interleaver function uses the same kind of computational code that is used by the general block interleaver function.

Sr. No	Block interleave types	Description
1	General block interleaver	Uses the permutation table given explicitly as an input argument.
2	Algebraic interleaver	In this the permutation table is derived algebraically.
3	Helical scan interleaver	In which an interleaver fills the data in the elements of the matrix row by row after which the completed matrix contents is being send to the output in a helical fashion.
4	Matrix interleaver	In which an interleaver fills the data in the elements of the matrix row by row after which the completed matrix contents is being send to the output in a column by column fashion.
5	Random interleaver	In which the initial state of the input provides the permutation table which is chosen randomly.

TABLE I.

The basic idea is to write data row-wise from top to bottom and left to right and read out column-wise from left to right and top to bottom. The concept of the interleaver is shown in Figure 1.

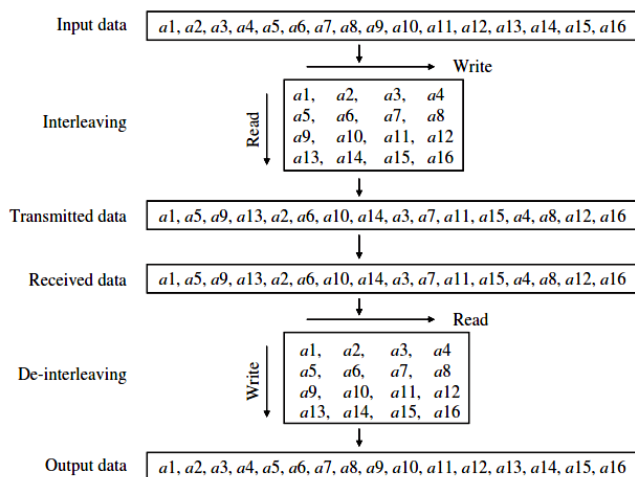


Fig. 1. A basic concept of an Interleaver.

Figure 2 shows an example in which there are four burst error bits {0001111000000000} in the received data sequence. After interweaving, the error is dispersed and the output data sequence becomes {0100010001001000}. We can see that the burst error of length 4 is transformed into multiple individual errors. The error-correcting codes generally are capable of correcting individual errors, but not a burst error. However, in the wireless and mobile channel environment, the burst error occurs frequently. In order to correct the burst error, interleaving is needed to disperse the burst error into multiple individual errors, which can be handled by the error-correcting code.

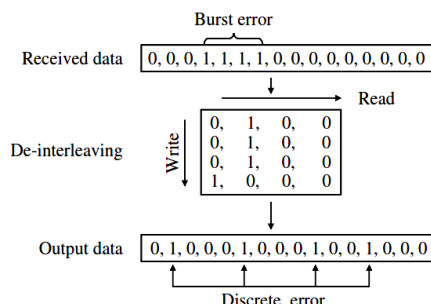


Fig. 2. An example of an Interleaver.

III. THEROTICAL ANALYSIS

A. Expression for the probability of error of QAM is given as,

To determine the probability of error for QAM for the case of rectangular constellation the signal space diagram is shown in Figure 3, in which the minimum Euclidean distance between the adjacent points is given by $d = \sqrt{2E_g}$.

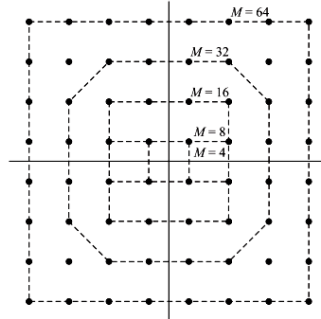


Fig. 3. Several signal space diagrams for rectangular QAM.

The average signal energy for the rectangular constellation is,

$$E_{av} = \frac{M-1}{3} E_g. \tag{14}$$

Where E_g is the signal Energy.

Therefore, the average signal energy per bit,

$$E_b = \frac{M-1}{3 \log_2 M} E_g. \tag{15}$$

Thus, the minimum Euclidean distance between adjacent points is given by,

$$d = \sqrt{\frac{6 \log_2 M}{M-1} E_b}. \tag{16}$$

For the 16-QAM constellation, the distance between any two adjacent points on the constellation is,

$$d = \sqrt{\frac{6 \log_2 M}{M-1} E_b} = \sqrt{\frac{8}{5} E_b}. \tag{17}$$

Note that this constellation can be considered as two \sqrt{M} -ary PAM constellations in the in-phase and quadrature directions. The probability of a correct detection for this QAM constellation is, therefore, the product of correct decision probabilities for constituent PAM systems, i.e.

$$P_{c,M-QAM} = P_{c,\sqrt{M}-PAM}^2 = (1 - P_{bs,\sqrt{M}-PAM})^2, \\ P_{bs,M-QAM} = 1 - (1 - P_{bs,\sqrt{M}-PAM})^2, \\ P_{bs,M-QAM} = 2P_{bs,\sqrt{M}-PAM} \left(1 - \frac{1}{2}P_{bs,\sqrt{M}-PAM}\right). \tag{18}$$

Whereas the probability of error for PAM is given by,

$$P_{bs,\sqrt{M}-PAM} = 2 \left(1 - \frac{1}{\sqrt{M}}\right) Q \left(\frac{d}{\sqrt{2N_0}}\right). \tag{19}$$

From (17) the above equation can be rewritten as,

$$P_{bs,\sqrt{M}-PAM} = 2 \left(1 - \frac{1}{\sqrt{M}}\right) Q \left(\frac{\sqrt{\frac{6 \log_2 M}{M-1} E_b}}{\sqrt{2N_0}}\right), \\ P_{bs,\sqrt{M}-PAM} = 2 \left(1 - \frac{1}{\sqrt{M}}\right) Q \left(\sqrt{\frac{E_b}{N_0} * \frac{3 \log_2 M}{(M-1)}}\right). \tag{20}$$

Substituting above equation in (18) results in

$$P_{bs,M-QAM} = 4 \left(1 - \frac{1}{\sqrt{M}}\right) Q \left(\sqrt{\frac{E_b}{N_0} * \frac{3 \log_2 M}{(M-1)}} \right) * \left(1 - \left(1 - \frac{1}{\sqrt{M}}\right) Q \left(\sqrt{\frac{E_b}{N_0} * \frac{3 \log_2 M}{(M-1)}} \right) \right), \quad (21)$$

$$P_{bs,M-QAM} \leq 4Q \left(\sqrt{\frac{E_b}{N_0} * \frac{3 \log_2 M}{(M-1)}} \right). \quad (22)$$

The Equation obtained above is for square constellations, for large M it gives a good approximation for general QAM constellations with $M = 2^k$ points which are either in the shape of a square (when k is even) or in the shape of a cross (when k is odd). These types of constellations are illustrated in Figure 3.

For 16-QAM with a rectangular constellation the equation for probability of error will be obtain from equation (17) and (21) as,

$$P_{bs,16-QAM} = 3Q \left(\sqrt{\frac{E_b}{N_0} * \frac{4}{5}} \right) * \left(1 - \frac{3}{4} Q \left(\sqrt{\frac{E_b}{N_0} * \frac{4}{5}} \right) \right). \quad (23)$$

$$P_{bs,16-QAM} \leq 3Q \left(\sqrt{\frac{E_b}{N_0} * \frac{4}{5}} \right). \quad (24)$$

B. Expression for the probability of bit error rate of QAM with Reed-Soloman block coding is given as,

An interesting property of Reed-Solomon codes is that their weight enumeration polynomial is known. In general, the weight distribution of a Reed-Solomon code with symbols from Galois Fields $GF(2^m)$ and with block length $n = 2^m - 1$ and minimum distance d_{min} is given by,

$$A_i = \binom{n}{i} n \sum_{j=0}^{i-d_{min}} (-1)^j \binom{i-1}{j} (n+1)^{i-j-d_{min}}, \text{ for } d_{min} \leq i \leq n. \quad (25)$$

Whereas, the performance of the hard decision decoder may be characterized by the following upper bound on the codeword error probability:

$$P_e \leq \sum_{i=i+1}^n i \binom{n}{i} P_M^i (1 - P_M)^{n-i}. \quad (26)$$

Where t is the number of errors to be corrected by the reed-soloman block Coding.

The union bound in terms of the Q function and d is given by

$$P_{bs} \leq (M - 1)Q \left(\sqrt{\frac{(d)^2}{2N_0}} \right). \quad (27)$$

Where, d is the minimum distance between any two adjacent points on the constellation diagram.

Or

$P_M \leq (M - 1)Q \left(\sqrt{\frac{(d)^2}{2N_0}} \right)$, where, P_M is the symbol error probability. When a codeword error is made, then the corresponding symbol error probability is given by,

$$P_s = \frac{1}{n} \sum_{i=i+1}^n i \binom{n}{i} P_M^i (1 - P_M)^{n-i}. \quad (28)$$

Where P_s denotes the probability of symbol error, then the probability of bit error, P_b and is given by $P_b = P_s/m$.

$$P_b = \frac{1}{m} P_s = \frac{1}{nm} \sum_{i=i+1}^n i \binom{n}{i} P_M^i (1 - P_M)^{n-i}. \quad (29)$$

C. Expression for the probability of bit error rate of QAM with two transmit and receiving antennas is given as,

The expression for the bit error probability for QAM in Rayleigh fading channel along with two transmits and L receiving antennas is given by [16].

$$P_b = \frac{1}{2} (P_{b1} + P_{b3}). \quad (30)$$

Where, $P_{b1} = P_{b2} = \frac{1}{2} (P_1 + P_2)$ and

$$P_{b3} = P_{b4} = \frac{1}{2} (2P_1 + P_2 - P_3)$$

With $P_i, i = 1,2,3$, is given by,

$$P_i = \left[\frac{1}{2} (1 - \mu_i) \right]^{2L} \sum_{k=0}^{2L-1} \binom{2L-1+k}{k} \left[\frac{1}{2} (1 + \mu_i) \right]^k, \quad (31)$$



$$\text{where } \mu_1 = \sqrt{\frac{E_b/N_o}{5L + E_b/N_o}}, \mu_2 = \sqrt{\frac{9E_b/N_o}{5L + 9E_b/N_o}} \text{ and } \mu_3 = \sqrt{\frac{25E_b/N_o}{5L + 25E_b/N_o}}$$

Therefore the bit error probability for 16-QAM in Rayleigh fading channel with two transmit and two receiving antenna is given as,

$$P_i = \left[\frac{1}{2}(1 - \mu_i)\right]^4 \sum_{k=0}^{4-1} \binom{4-1+k}{k} \left[\frac{1}{2}(1 + \mu_i)\right]^k$$

$$P_i = \left[\frac{1}{2}(1 - \mu_i)\right]^4 \sum_{k=0}^3 \binom{3+k}{k} \left[\frac{1}{2}(1 + \mu_i)\right]^k$$

$$\text{Where } \mu_1 = \sqrt{\frac{E_b/N_o}{10 + E_b/N_o}}, \mu_2 = \sqrt{\frac{9E_b/N_o}{10 + 9E_b/N_o}} \text{ and } \mu_3 = \sqrt{\frac{25E_b/N_o}{10 + 25E_b/N_o}}$$

With, $k = \log_2 M = \log_2 16 = 4$

IV. SIMULATION & ANALYSIS

The model is developed by using one of the important tools in Matlab i.e. Simulink. A Simulink is a graphical extension to Matlab for the modeling and simulation of systems. A model is analyzed by choosing various options from the Simulink menus and by entering commands in the Matlab command window. The Performance of a model is analyzed with the help of BER Vs Eb/No curve. This is obtained by using Monte Carlo simulation in the bit error rate analysis tool of the Matlab. The final simulation results are shown in the figures below. First of all the simulation of 16-QAM in Rayleigh fading channel is performed, the model is simulated for various values of EbNo and the effect of fading is observed on the performance of 16-QAM in terms of BER Vs Eb/No curve. Figure 4 below shows the BER Vs Eb/No curve for the 16-QAM in Rayleigh fading channel. The Signal constellation Plot for 16-QAM is shown in Figure 5 when it is not subjected to Rayleigh fading which shows the symmetric distribution of signaling points in signal space. When the channel is faded the signaling points get spread in the space that is due to Rayleigh fading which is shown in Figure 6. Next, the model consisting of Alamouti 2 X 2 STBC encoder and combiner, Reed-Solomon (15,7) block encoder and decoder and Matrix Interleaver and deinterleaver under Rayleigh fading is simulated for different values of EbNo and the corresponding BER plot is obtained which is shown in Figure 7.

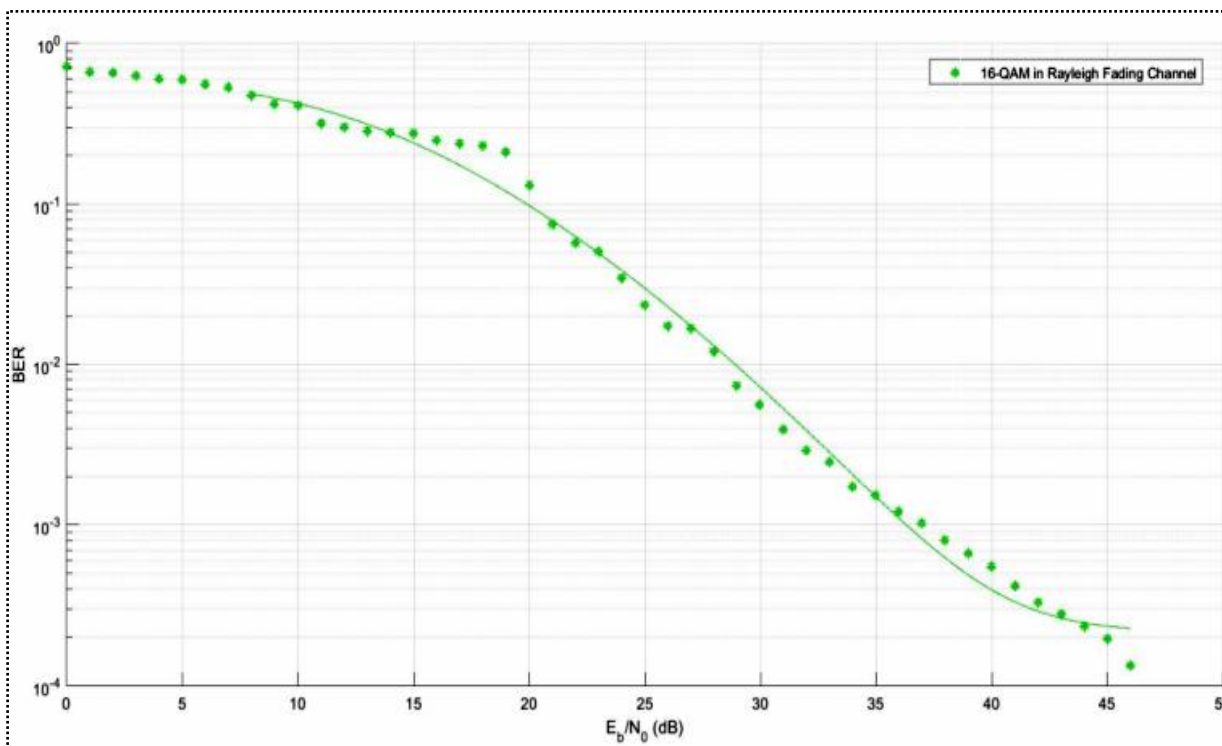


Fig. 4. Plot of BER Vs Eb/No for 16-QAM under fading channel



From the BER performance curves of the models it is very clear that the performance of 16-QAM is improved and it can also be observed from the Signal constellation (Scattered) Plot for the 16-QAM in Rayleigh fading channel with Reed-Solomon block coding, interleaver and Alamouti 2 X 2 STBC shown in Figure 8.

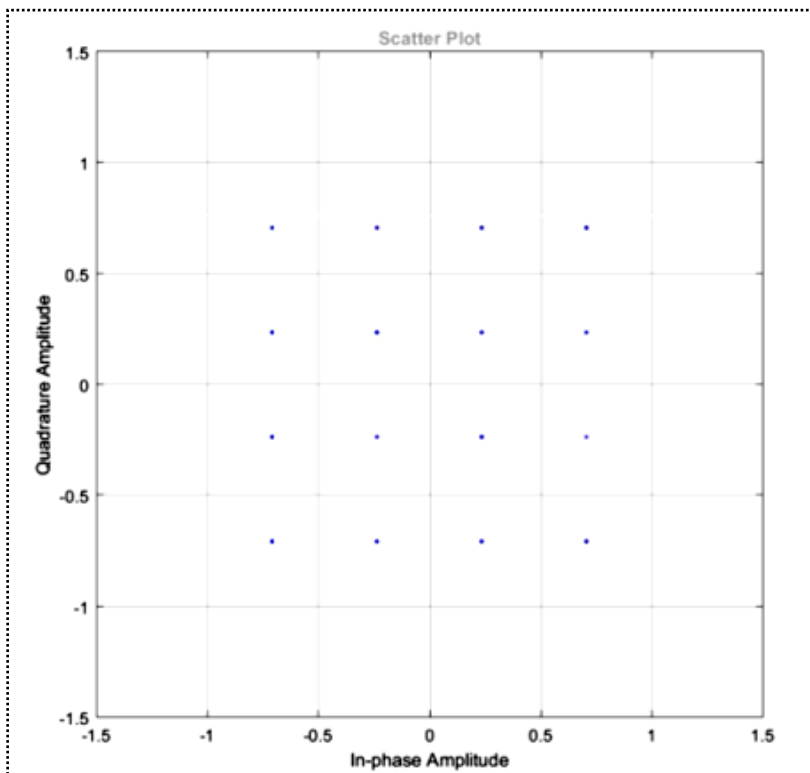


Fig. 5. Signal constellation plot for 16-QAM modulation technique.

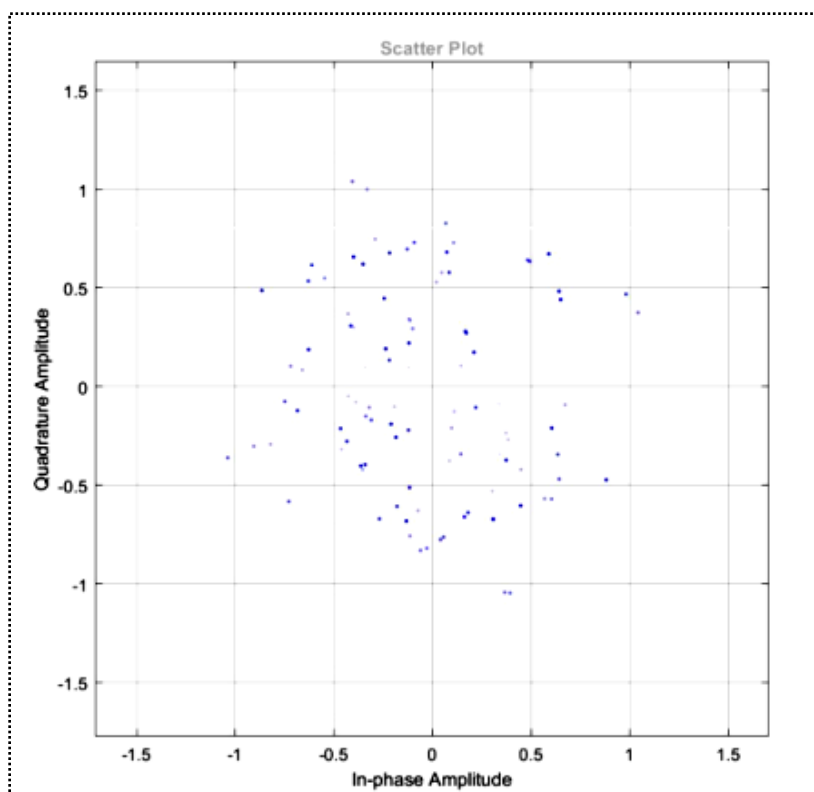


Fig. 6. Signal constellation (highly Scattered forming cluster of points) Plot for 16-QAM after Rayleigh fading.

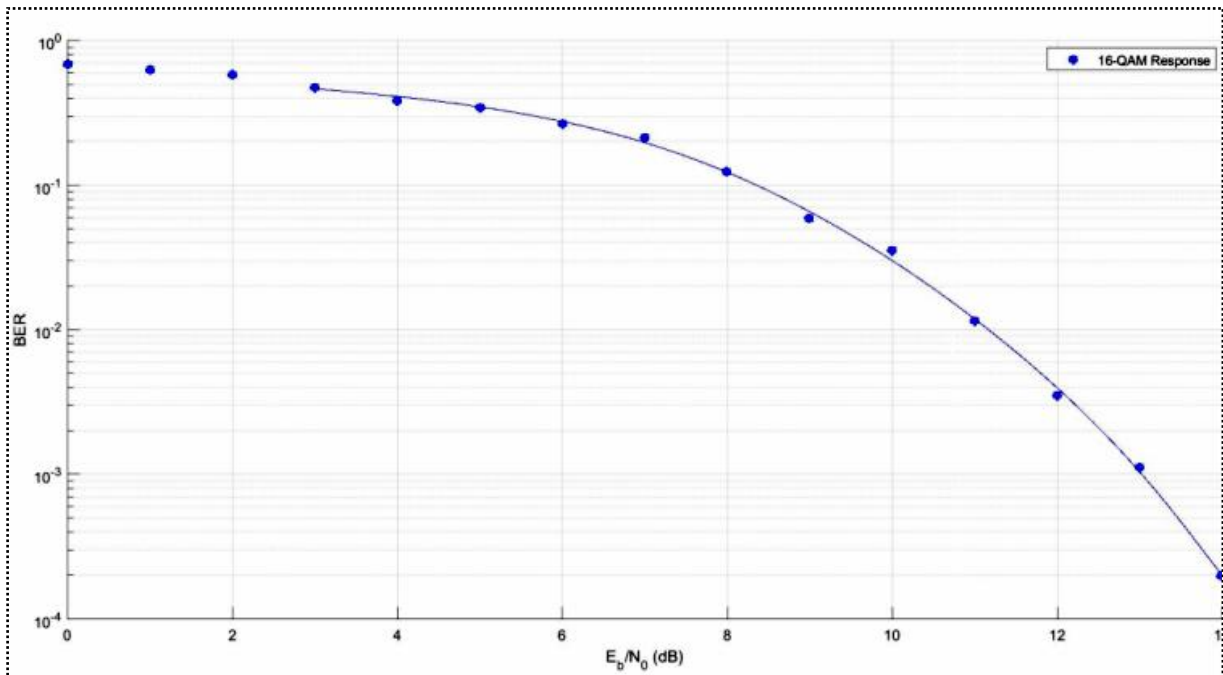


Fig. 7. BER Vs Eb/No Plot for 16-QAM in Rayleigh fading channel with Reed-Solomon block encoder and decoder, Matrix Interleaver and deinterleaver and Alamouti 2 X 2 STBC encoder and combiner.

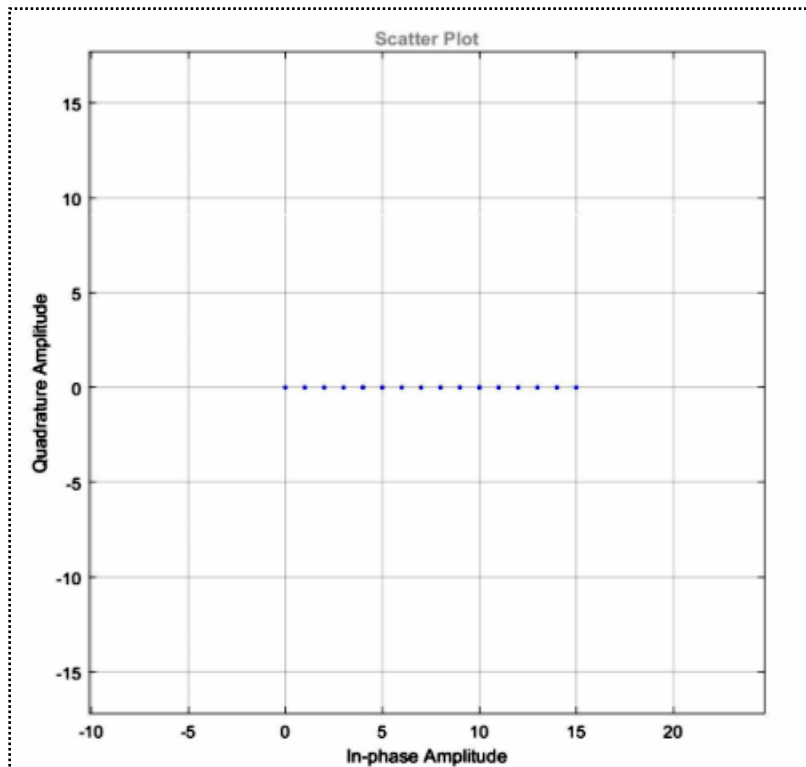


Fig. 8. Signal constellation (Scattered) plot for the 16-QAM under Rayleigh fading channel with Reed-Solomon block coding encoder and decoder, Interleaver and deinterleaver and Alamouti 2 X 2 STBC encoder and combiner blocks.

V. CONCLUSIONS

From this analysis, it is evident that Quadrature Amplitude Modulation is seriously affected by the Rayleigh fading. To know this we analyzed the BER performance of QAM schemes in Rayleigh fading channels without and with Reed-Solomon block coding encoder and decoder, Interleaver and deinterleaver and Alamouti 2 X 2 STBC encoder and

combiner blocks. After the implementation and simulation of the systems, the results were successfully obtained in two models. Comparing the performance of the two in terms of error rate for various values of the E_b/N_0 shows that the 16-QAM system performance can be improved to a much greater extent under fading channel, this allows utilising a lower order QAM techniques for transmission in wireless communication applications as higher order QAM are prone to increase in bit error rate with noise and also the transmission time.

Therefore in this work, it can conclude that the BER for the 16-QAM with Reed-Solomon block coding encoder and decoder, Interleaver and deinterleaver and Alamouti 2 X 2 STBC encoder and combiner blocks combats the detrimental effects under Rayleigh fading channel and thus can also be used for improving the performance of other modulation techniques.

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