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# Bearing Only Tracking Using Extended Kalman Filter

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**ABSTRACT**: **Extended Kalman filter (EKF) is widely used for tracking moving objects like missiles, aircrafts, robots etc. In this paper we examine the case of a single sensor or observer bearing only tracking (BOT) problem for two different models. In model 1, the target is assumed to have a constant velocity and constant course. In model 2, the target is assumed to follow a coordinated turn model with constant velocity but varying course. Extended Kalman Filter is used to track the target in both cases. The goal of this paper is to demonstrate how the performance of the filter is affected by the initial assumptions and measurement error variances in these two models. Simulation results have been presented, which demonstrate the effect of initial assumptions and measurement error covariance on the performance of the filter.**

**Keywords**: **Extended kalman filter, Bearing only tracking, manoeuvring target, coordinated turn model**

## I. INTRODUCTION

In 1960, R.E. Kalman published his famous paper**[6]** describing a recursive solution to the discrete-data linear filtering problem. Since then,the Kalman filter has been the subject of extensive research and application, particularly in the area of autonomous or assisted navigation**[2]**.The filter is said to be very powerful as it can estimate the state of a process even when the precise model of the system is unknown.

A Kalman filter is simply an optimal recursive data processing algorithm that blends all available information, including measurement outputs, prior knowledge about the system and measuring sensors, to estimate the state variables in such a manner that the error is statistically minimized In practice, linear equation system with white Gaussian noises is commonly taken as the standard model of a Kalman filter[1]. In this paper bearing only tracking is employed, which means the sensors provide the bearing angle measurements only. Bearing angle is the angle between the horizontal plane of the observer(sensor) and the line of sight between target and the observer.

## II. EXTENDED KALMAN FILTER FOR TARGET TRACKING

Kalman filter estimates a process by using a form of feedback control i.e. the filter first estimates the state using the previous state and then obtain feedback in the form of measurements. Thus the filter equations are of two groups.The time update equations that projects current state estimate ahead in time and measurement update equations that adjusts the projected estimate by an actual measurement at that time.

The system state and output equations are of the following form

$$X_k=f[X_{k-1}, k]+v_k \quad (1)$$

$$Z_k=h[X_k,k]+w_k \quad (2)$$

where $f$ and $h$ are non-linear functions depending on the system state. $X_k$ and $Z_k$ are the corresponding state vector and output vector. $v_k$ and $w_k$ are the corresponding process and measurement noises. These noises are zero mean Gaussian noises with error covariance as $Q_k$ and $R_k$ respectively.

*Prediction equation*

$$x\wedge_k^- = f(x\wedge_{k-1}, u_{k-1}, 0) \quad (3)$$

$$P_k^- = A_k P_{k-1} A_k^T + W_k Q_{k-1} W_k^T \quad (4)$$

*Update equation*

$$x\wedge_k = x\wedge_k^- + K_k(zk - h(x\wedge_k^-, 0)) \quad (5)$$

$$P_k = (I - K_k H_k) P_k^- \quad (6)$$

*Kalman gain*

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + V_k R_k V_k^T)^{-1} \quad (7)$$

Papers [2],[3] describes the above equations in detail. Here $H_{k+1}=\frac{\partial}{\partial x}h[X_k,k]$ and $P_k$ is the error covariance matrix.

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*A. Constant Velocity And Course Target*

In this model the target moves with constant velocity in a linear path. Original path of the target is modeled using kinematics equations. The position of the target in 2D is given by (x,y) and velocity is given by ($v_x$,$v_y$). Since the target moves with constant velocity the state vector at time $t_k$ is given as

$$X_k=[x_k,y_k,v_{xk},v_{yk}]^T$$

where

$x_k$=x coordinate of the target at $k^{th}$ instant

$y_k$= y coordinate of the target at $k^{th}$ instant

$v_{xk}$= velocity of the target in x direction at $k^{th}$ instant

$v_{yk}$= velocity of the target in y direction at $k^{th}$ instant

The state equation is given as

$$X_k=\mathbf{A}X_{k-1} + w_k$$

where

**A** = 1 0 t 0

0 1 0 t

0 0 1 0

0 0 0 1

$w_k$=process noise

In this model the relationship between the observations and state vector is non linear.

$$z_k=h(\Theta_k+ v_k)$$

where

$\Theta_k$= bearing angle measurement

$v_k$=measurement noise vector

h(.)= non linear function that relates observations and state given by

$$h(.)=\tan^{-1}(y_k/x_k)$$

*B. Constant Velocity And Varying Course Target*

Till now we have considered a linear or a straight line motion, but in real time system the targets manoeuvre. A manoeuvre typically means a deviation from ordinary straight line path or a change in course or direction. Here we have considered manoeuvres involving turns with constant velocity and course rather than the more general case of changing velocity. We have modelled constant velocity and varying course target using coordinated turn (CT) model. In CT model the state vector is augmented with additional parameter called angular turn rate (ω, measured in radiance/sec) which is to be estimated with the other system parameter. Here the state vector is given by $X_k$=[ $x_k$ $y_k$ $v_{xk}$ $v_{yk}$ $\omega_k$ ]$^T$ where $x_k$ and $y_k$ is the Cartesian position coordinates, $v_{xk}$ and $v_{yk}$ are the velocity components and $\omega_k$ the constant turn rate[4],[5]. If we assume velocity v to be a constant, then

$$V=[v_{xk}^2+v_{yk}^2]^{1/2} \qquad (10)$$

Since we have assumed a constant velocity model, acceleration is zero. For two dimensional system, the discrete time state equation is given as

$$X_{k+1}=FX_k + BV_k \qquad (11)$$

Where F=

$$\begin{pmatrix} 1 & 0 & \frac{\sin(\omega_k \Delta t)}{\omega_k} & \frac{\cos(\omega_k \Delta t)-1}{\omega_k} & 0 \\ 0 & 1 & \frac{1-\cos(\omega_k \Delta t)}{\omega_k} & \frac{\sin(\omega_k \Delta t)}{\omega_k} & 0 \\ 0 & 0 & \cos(\omega_k \Delta t) & -\sin(\omega_k \Delta t) & 0 \\ 0 & 0 & \sin(\omega_k \Delta t) & \cos(\omega_k \Delta t) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Δt is the sampling period and $V_k=N(0,\sigma_\omega^2)$ is the uninvariant white Gaussian process noise for turn rate with zero mean and variance $\sigma_\omega^2$. Papers [4],[5] discusses these equations in detail. The rest of the equations used are same as that of extended kalman filter.

## III. SIMULATION RESULTS

*A. Constant Velocity And Course Target*

Initially the target is assumed to be at (6000,10000). The target moves with a constant velocity of 5km/hr. The observer remains stationary and is located at (1000,-500) of the Cartesian coordinate system. Now this model was evaluated for three different cases

*Case 1:* Measurement error covariance is large value and initial state estimate assumption is good. The plot of estimated target trajectory and root mean square error (rmse) for the mentioned case is shown in Fig.1 and Fig. 2 respectively.

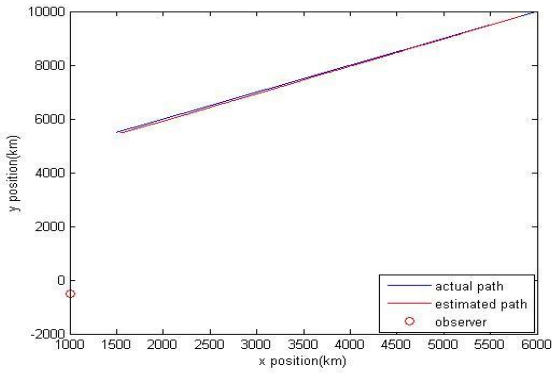


Fig.1 Target trajectory of case1

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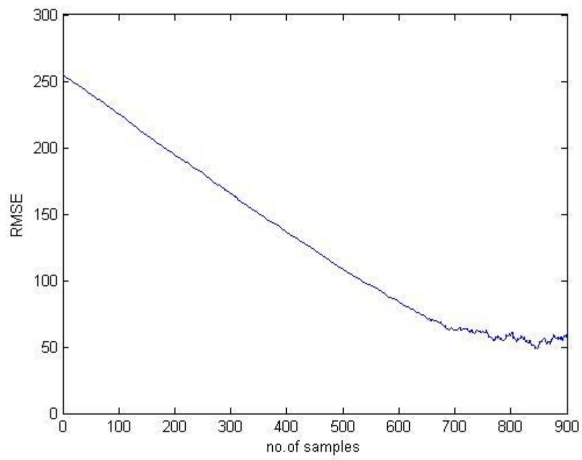


Fig.2 RMSE of case 1

*Case2:*Measurement error covariance is large and initialization is poor. Fig. 3 and Fig.4 respectively represent the estimated trajectory and rmse plot for the case

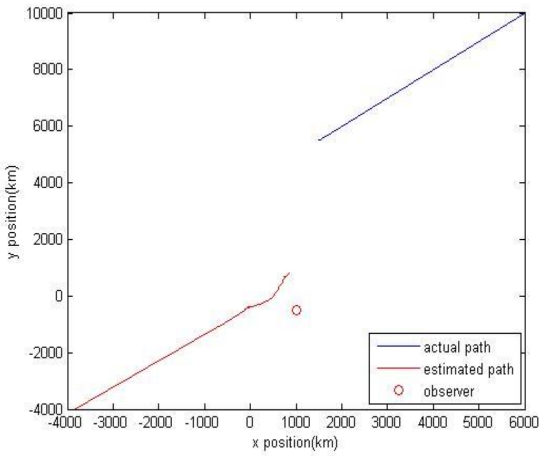


Fig.3 Target tragectory of case 2

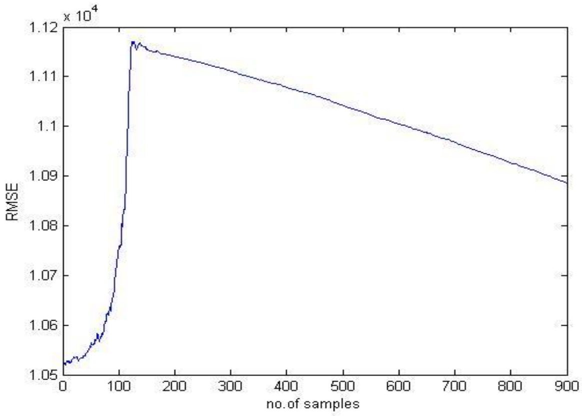


Fig. 4 RMSE of case 2

*Case3:* Measurement error covariance is very small but initial assumption is good..Figure 5 and 6 shows the target trajectory and rmse plot for this case

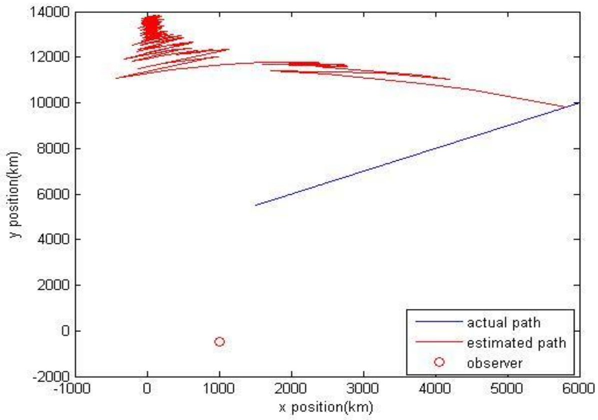


Fig.5 Target trajectory of case3

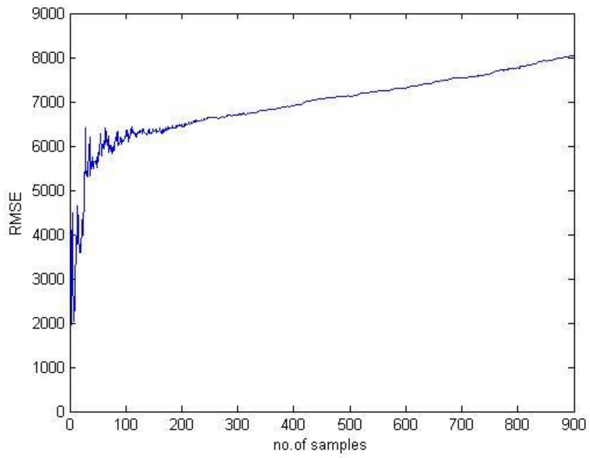


Fig.6 RMSE of case 3

*B. Constant Velocity And Varying Course Target*

For the simulation algorithm we have considered a body initially at (100,5) moving with a constant velocity of 12km/sec on a course of 0.1rad/sec. The observer is stationary and is at (-200,200). The system is simulated for 100 times. Now this model is evaluated for three different cases.

*Case1:* Measurement error covariance is a large value and initial state estimate assumption is good. The plot of estimated target trajectory and root mean square error (rmse) for the mentioned case is shown in Fig.7 and Fig.8 respectively.

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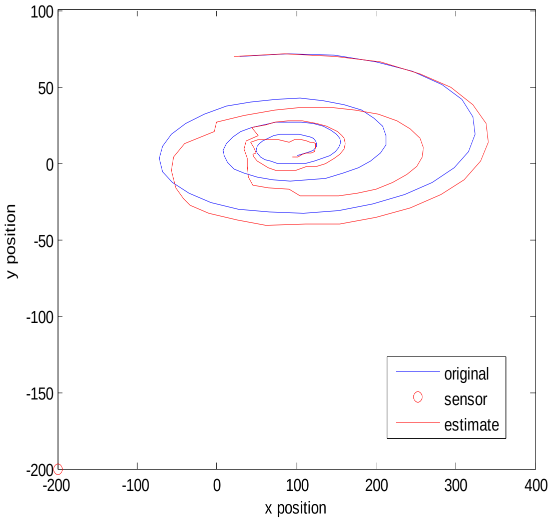


Fig. 7 Target trajectory of case 1 of manoeuvring target

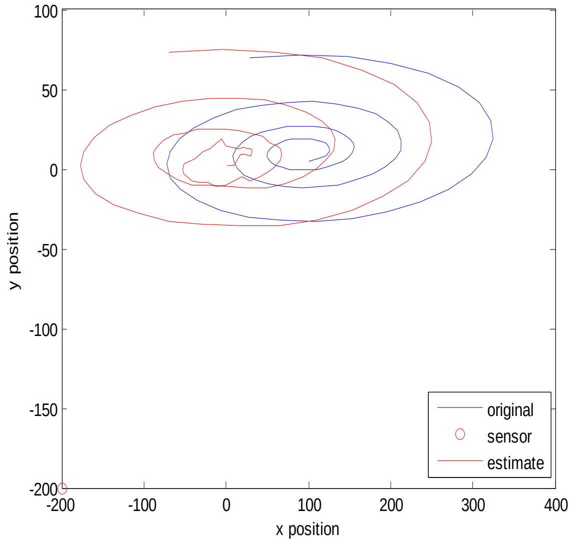


Fig. 9 Target trajectory of case 2 of manoeuvring target

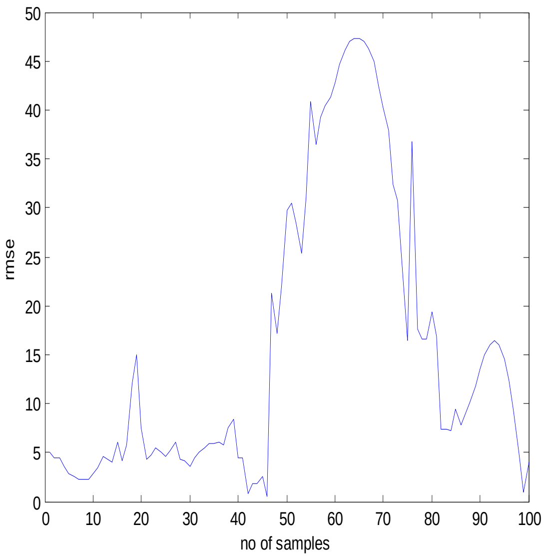


Fig. 8 RMSE of case 1 of manoeuvring target

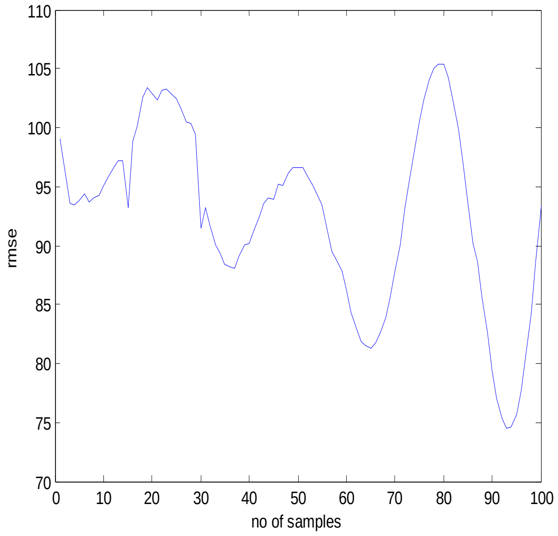


Fig. 10 RMSE of case 2 of manoeuvring target

*Case2:*Measurement error covariance is large but initialization is poor. Fig.9 and Fig.10 respectively represent the estimated trajectory and rmse plot for the case.

*Case3:* Measurement error covariance is very small but initial assumption is good.Figure 11 and Figure 12 shows the target trajectory and root mean square error (rmse) plot for this case.

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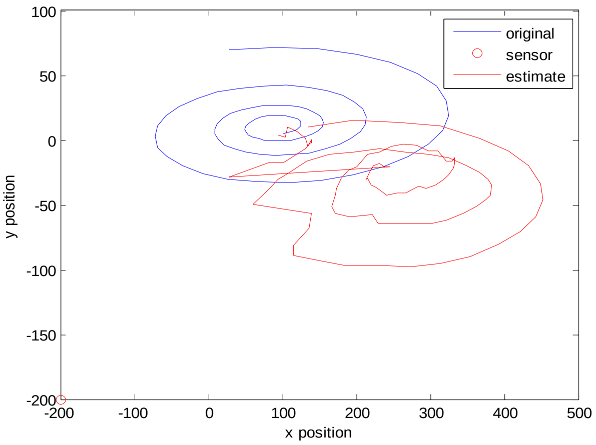


Fig. 11 Target trajectory of case 3 of manoeuvring target

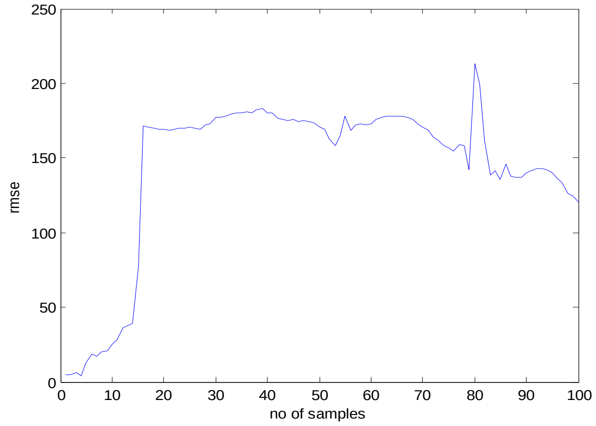


Fig. 12 RMSE of case 3 of manoeuvring target

## IV. CONCLUSION

In this paper we have analyzed various scenarios of target tracking using extended kalman filter. Based on the simulation results obtained we have arrived at three major conclusions. Firstly if the initialization or initial state assumption is poor it would result in track loss, even for most robust filter. Secondly if the initialization is good or close to the real values, then however large be the value of measurement error co-variance (R), the estimated trajectory will be optimal. Lastly, for smaller values of measurement error covariance (i.e. more accurate the measurement) though we expect a good performance for a properly designed filter, it is not true. This is because for smaller values of measurement error covariance, the measurement non linearity would be more significant that deviates the filter performance.

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