ISSN (Print) : 2319-5940
ISSN (Online) : 2278-1021

International Journal of Advanced Research in Computer and Communication Engineering
Vol. 2, Issue 12, December 2013

Performance Evaluation of Legendre and Rudin-Shapiro Sequences using PSLR

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Abstract: In this paper, two sequences i.e., Legendre and Rudin-Shapiro are compared based on the peak side lobe ratio of an auto-correlation function. The former is applicable only for prime numbers and the latter is applicable for 2^{m-1} length sequences. The peak side lobe ratio gives an alternative to the merit factor for measuring the collective smallness of the binary sequences. A low peak side lobe ratio is the factor for good resolution and peak side lobe ratio of larger sequences can be applied in different Radar applications .The results are compare and simulated using Matlab.

Keywords: Aperiodic autocorrelation, peak side lobe level, binary sequence, merit factor, Legendre sequence, maximal length shift register sequence, Rudin-Shapiro sequence.

1. INTRODUCTION

Consider a sequence of length n is an n-tuple
$A = a_0, a_1, a_2, \ldots, a_{n-1}$

Where $a_i = 1 \text{ or } -1$

For each i = 0, 1. . . n − 1. The Aperiodic autocorrelation of A at shift u is defined as

$$c_A(u) := \sum_{i=0}^{n-u-1} (a_i a_{i+u}) \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

It is of our interest in the study of sequence design to find binary sequences whose Aperiodic autocorrelations are in some suitable sense, collectively small. Two principal measures of "smallness" have been used. One measure (surveyed in [17]) is the Merit factor, introduced by Golay in 1972 [12] which is described by

$$F(A) := \frac{n^2}{2\sum_{u=1}^{n-1}(c_A(u))^2} \quad \text{for } n>1 \quad \ldots\ldots\ldots\ldots\ldots (2)$$

The other measure, and our main interest here is the peak side lobe level (PSL) .then

$$M(A) := \max |c_A(u)| . 1 \leq u \leq n-1. \quad \ldots\ldots\ldots\ldots\ldots (3)$$

Let an denote the set of all binary sequences of length n. We would like to understand the behavior, as n → ∞, of

$$M_n = \min_{A \in A_n} M(A) \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

And to compare its asymptotic behavior with that of $1/F_n$, where $F_n := \max_{A \in A_n} F(A)$.[1]

2. FAMILIES OF SEQUENCES

The theoretical approach to the Merit factor problem includes the study of specific infinite families of sequences. We shall be concerned with the families of Legendre sequences and Rudin-Shapiro sequences. For more detailed information on these families, see [17,1].

2.1 BOUNDS ON THE PEAK SIDELOBE LEVEL OF FAMILIES OF SEQUENCES

We presented here general bounds on the PSL. In this section, we consider bounds on the PSL of specific families of binary sequences.

We begin with a connection between the merit factor and the PSL of a family of sequences. Let F be a family of binary sequences and let each $A_n \in F$ have length *n*. Suppose $\liminf_{n\to\infty} (M(A_n)/\sqrt{n}) = 0$.Then, for each *n*

$$0 < \frac{1}{\sqrt{2F(A_n)}} = \frac{\sqrt{\sum_{u=1}^{n-1}[C_A(u)]^2}}{n} \leq$$

$$\frac{\sqrt{(n-1)[M(A_n)]^2}}{2} < \frac{M(A_n)}{\sqrt{n}}. \quad \ldots\ldots\ldots\ldots (5)$$

It follows that $\liminf_{n\to\infty} (1/\sqrt{2F(A_n)}) = 0$ and therefore $\limsup_{n\to\infty} F(A_n) = \infty$.

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The PSL of any rotation of a Legendre sequence, of any *m*-sequence, and of a Rudin-Shapiro sequence all grow at least as fast as $\sqrt{n}$.

As described we would like to identify a family of sequences whose PSL grows like o ($\sqrt{n \ln n}$).Among these families of sequences discussed earlier the largest asymptotic merit factor is achieved by rotated Legendre sequences. We might therefore expect that, if any of these families has a PSL that grows like o ($\sqrt{n \ln n}$), the family of Legendre sequences (and their rotations) is the most likely candidate; we might even hope that the PSL of some rotation grows like O ($\sqrt{n}$). This is under investigation.

The PSL of *m*-sequences Y of length *n* has been much discussed in the literature. In 1980, McEliece [23] showed that $\sqrt{n+1}$ ln (*en*) is an upper bound for M (Y). In 1984, Sarwate improved this bound as follows.

Let Y be an *m*-sequence of length *n*. Then

$$M(Y) < 1+\frac{2}{\pi}\sqrt{n+1}\,\ln\left(\frac{4n}{\pi}\right). \quad \ldots\ldots\ldots\ldots\ (6)$$

It does not tell us whether the PSL of (some or all) *m*-sequences grows like o ($\sqrt{n \ln n}$). However, Cohen, Baden and Cohen [5] state, without reference, that *m*-sequences "can achieve peak side lobe levels (PSLs) on the order of $N^{1/2}$"! This is the most modest growth of the PSL that an *m*-sequence could possibly achieve It is not clear whether the statement in [6] is intended to apply to any rotation of an *m*-sequence generated by any primitive polynomial, or only to some (infinite) subset of *m*-sequences. But even if it held for some infinite subset, this would imply that M_n (grows like O ($\sqrt{n}$). This would greatly improve on Moon and Moser's sayings, and indeed would render Mercer's improvement of little value.[21,1]

However we were unable to find a proof or supporting numerical evidence for this claim.

Farnett and Stevens [11] state that the PSL of *m*-sequences is approximately $\sqrt{n}$ for large n makes the same statement, adding that "as N increases, the rule-of-thumb approximation improves". Likewise Vakman claims that the PSL of *m*-sequences grows like O($\sqrt{n}$), and further states: "It has been noted repeatedly that either by empirical methods, by combining several M-sequences, or, finally, by constructing other types of sequences, it is possible to find [other long sequences for which the PSL grows like O($\sqrt{n}$)]".[1].

Once again, however, The PSL of families of *m*-sequences numerically tested its growth against the claimed bounding function $\sqrt{n}$ and also against the function $\sqrt{n \ln n}$.

We study the PSL of Rudin-Shapiro sequences and their rotations, as an example of a sequence family with no known periodic property. Although an upper bound for the PSL of unrotated Rudin-Shapiro sequences is known, it is weak in comparison with the function $\sqrt{n \ln n}$:

The PSL of both sequences $X^{(m)}$ and $Y^{(m)}$ of a Rudin-Shapiro sequence pair of length $n = 2^m$ grows like O ($n^{0.9}$).

2.1.1 LEGENDRE SEQUENCES

The Legendre sequence (also called a quadratic residue sequence)

$A = a_0, a_1, a_2, \ldots, a_{n-1}$ Of prime length n is defined so that

$$A_i = \begin{cases} 1 & \text{if I is a quadratic residue mod n} \\ -1 & \text{otherwise} \end{cases}$$

By convention, we take $a_0 = 1$. A Legendre sequence is equivalent to a cyclic difference set with parameters from the Hadamard family for n ≡ 3 (mod 4) and to a partial difference set for n ≡ 1 (mod 4) [13]. When a sequence A = ($a_0, a_1 \ldots a_{n-1}$) of length n is rotated by a rotational fraction r, we obtain a new sequence $A_r = (b_0, b_1, \ldots, b_{n-1})$ such that

$$b_i := a_{(i+[rn]) \bmod n} \quad \ldots\ldots\ldots\ldots\ (7)$$

In 1988, Hoholdt and Jensen [15], building on earlier work of Turyn and Golay [13], established a Legendre sequence i.e,

Let X be a Legendre sequence of prime length n. Then

$$\frac{1}{\lim_{n\to\infty} F(X_r)} = \begin{cases} \frac{1}{6} + 8\left(r-\frac{1}{4}\right)^2 & \text{for } 0 \le r \le \frac{1}{2} \\ \frac{1}{6} + 8\left(r-\frac{3}{4}\right)^2 & \text{for } \frac{1}{2} \le r \le 1 \end{cases} \quad \ldots\ldots\ldots\ldots\ (8)$$

It follows that the maximum asymptotic merit factor of any rotation of a Legendre sequence is 6, and is achieved when the rotational fraction r is 1/4 and 3/4. Although this value 6 is the greatest proven asymptotic result for the merit factor of binary sequences, Bor-wein, Choi and Jedwab [4] gave strong numerical evidence that there are binary sequences whose asymptotic merit factor exceeds 6.34. Their construction involves sequences given by appending the initial elements of some rotation of a Legendre sequence to itself. [1,8]

ISSN (Print) : 2319-5940
ISSN (Online) : 2278-1021

International Journal of Advanced Research in Computer and Communication Engineering
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2.1.2 THE PEAK SIDE LOBE LEVEL OF LEGENDRE SEQUENCES

In this section we compare the growth of the PSL of Legendre sequences with the functions $\sqrt{n}$ and $\sqrt{n \ln n}$. Write R = $\{0, \frac{1}{n} . . . n\text{-}1/n\}$ and let X be a Legendre sequence of prime length n. We calculated M (X_r) for all r € R for various values of n, using similar strategies to those described in [17] for efficiency.

For N=127, the Auto correlation function of Legendre sequence of length 127 is obtained and the Main lobe level =21, first side lobe level =10, second side lobe level=18 and Peak side lobe ratio=0.476 is observed as shown in the figure 1. Similarly PSLR of 0.715 and 0.612 is obtained for sequence length of 8191 and 131071 respectively i.e., for N=8191, the Auto correlation function of Legendre sequence of length 8191 is obtained and the Main lobe level =169, first side lobe level =121, second side lobe level=109 and Peak side lobe ratio=0.715 is observed as shown in the figure 2.

For N=131071, the Auto correlation function of Legendre sequence of length 131071 is obtained and the Main lobe level =763, first side lobe level =467, second side lobe level=674 and Peak side lobe ratio=0.612 is observed as shown in the figure 3.

The PSLR obtained here are better than the PSLR obtained in the former [4], for the growth of the PSL of Legendre sequences.

TABLE.1

Legendre	**MLL**	**FSL**	**SSL**	**PSLR**
N=127	21	10	18	0.476
N=8191	169	121	109	0.715
N=131071	763	467	674	0.612

Table.1 describes about the Legendre sequences of different lengths and corresponding PSLR are obtained respectively.

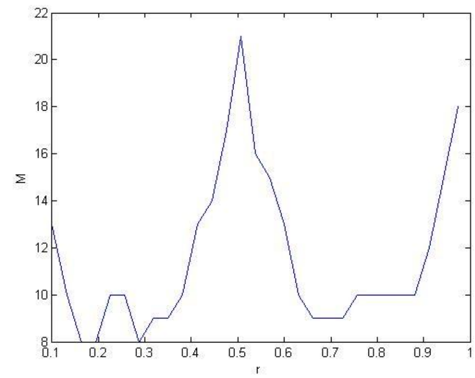

Figure 1: Auto correlation function of Legendre sequence of length N=127, PSLR =0.476

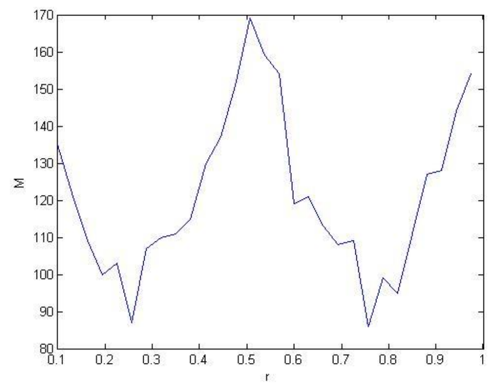

Figure 2: Auto correlation function of Legendre sequence of length N=8191, PSLR =0.715

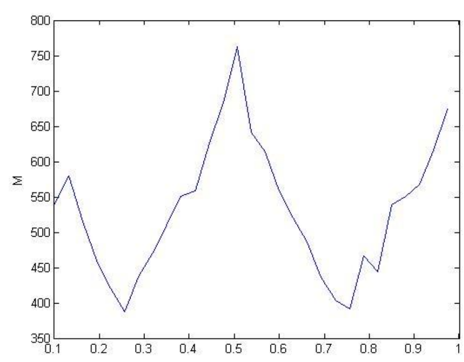

Figure 3: Auto correlation function of Legendre sequence of length N=131071, PSLR =0.612

2.2 Rudin-Shapiro Sequences

Given sequences A = (a_0, a_1. . . a_{n-1}) of length n and A' = (a'_0, a'_1 , . . . , $a'_{n'-1}$) of length n', let A; A' denote the sequence (b_0, b_1, . . . , $b_{n+n'-1}$) of length $n + n'$ given by appending A' to A

$$b_i := \begin{cases} a_i, \; for\, 0 \le i < n \\ a'_{i-n}, \; for\, n \le i < n+n' \end{cases} \quad \text{.......... (9)}$$

The *Rudin-Shapiro sequence* pair X $^{(m)}$ and Y $^{(m)}$ of length 2^m is defined recursively so that X $^{(0)}$ = Y $^{(0)}$: = (1) , and

$$\left.\begin{array}{l} X^{(m)} := X^{(m-1)}; Y^{(m-1)} \\ Y^{(m)} := X^{(m-1)}; -Y^{(m-1)}, \end{array}\right\} for\; m > 0. \quad \text{. ………….. (10)}$$

In 1968, Littlewood determined the exact merit factor of a Rudin-Shapiro sequence of any length 2^m in the following. The merit factor of both sequences X $^{(m)}$ and Y $^{(m)}$ of a Rudin-Shapiro sequence pair of length 2^m is $\frac{3}{1-(-1/2)^m}$.

Consequently, the asymptotic merit factor of both sequences of a Rudin-Shapiro sequence pair is 3. Rudin-Shapiro sequences differ from Legendre sequences and *m*-sequences in that they have no known periodic property (under sequence rotations), such as equivalence to a difference set or partial difference set.

2.2.1The Peak Side lobe Level of Rudin-Shapiro Sequences

In this section we pursue the apparent similarity between the shapes of the graphs as the rotational fraction r varies, as observed in the case of Legendre sequences. We assumed that this similarity depends on an underlying periodic property. We tested this assumption using the Rudin-Shapiro sequences [1], which have no known periodic property.
To our knowledge the merit factor of Rudin-Shapiro sequence pair of length n = 2^m. With the rotational fraction r ∈ R, for M = 7, M = 13 and M=17. Similar shapes of graph were obtained for all values of m.

The PSL of the unrotated sequence $X^{(m)}$ grows like $O(n^{0.9})$. The shape of the graphs becomes more regular as m increases, apparently approaching a piecewise linear function composed of 12 pieces with minima at r = 0, 1/4, 3/8, 1/2, 3/4, and 7/8. Unlike the case of Legendre sequences and m-sequences, there appears to be no "fuzziness" in the graph of M at large lengths. Perhaps more surprisingly, there is still a considerable similarity between the graphs of M and 1/F as r varies we conclude that this phenomenon is not restricted to sequences having an underlying periodic property.

We performed the same calculations for the Rudin-Shapiro pair of same sequence length and compared with Legendre Sequence. The corresponding graphs, both the sequences are plotted.

RS	N=2^{m-1}	MLL	FSL	SSL	PSLR
M=7	127	33	23	23	0.696
M=13	8191	1968	984	581	0.500
M=17	131071	31458	15730	15728	0.502

Table.2 describes about the Rudin-Shapiro sequences of different lengths and corresponding PSLR are obtained respectively

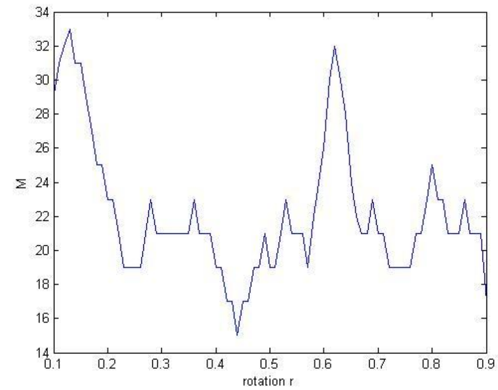

Figure 4: Auto correlation function of Rudin-Shapiro of length N=127, PSLR =0.696, M=7, $N = 2^{m-1}$ = **127**

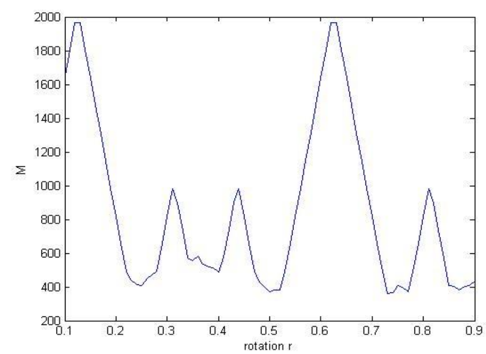

Figure 5: Auto correlation function of Rudin-Shapiro of length N=8191, PSLR =0.500, M=13, $N = 2^{m-1}$ = **8191**

For N=127, M=7 the Auto correlation function of Rudin-Shapiro sequence of length 127 is obtained and the Main lobe level =33, first side lobe level =23, second side lobe level=23 and Peak side lobe ratio=0.696 is observed as shown in the figure 4. Similarly PSLR of 0.50 and 0.502 is obtained for sequence length of 8191, M=13 and 131071, M=17 respectively i.e., for N=8191 the Auto correlation function of Rudin-Shapiro sequences of length 8191, M=13 is obtained and the Main lobe level =1968, first side lobe level =984, second side lobe level=581 and Peak side lobe ratio=0.50 is observed as shown in the figure 5.
For N=131071, M=17 the Auto correlation function of Rudin-Shapiro sequence of length 131071 is obtained and the Main lobe level =31458, first side lobe level =15730, second side lobe level=15728 and Peak side lobe ratio=0.502 is observed as shown in the figure 6.

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ISSN (Online) : 2278-1021

International Journal of Advanced Research in Computer and Communication Engineering
Vol. 2, Issue 12, December 2013

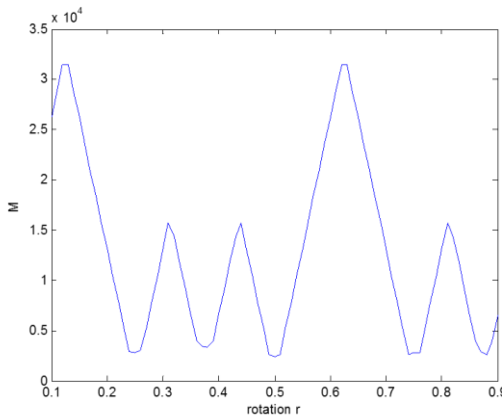

Figure 6: Auto correlation function of Rudin-Shapiro of length N=131071, PSLR =0.502, M=17, $N = 2^{m-1}$ = **131071**

3. CONCLUSION

PSLR is the ratio of energy in the side lobe to that of energy in the main lobe, for a better sequence the PSLR should be as low as possible to get desired results. Here, the Legendre Sequence and Rudin-Shapiro sequences are compared with the same sequence length based on the Auto Correlation using pslr as shown in the tables 1 and 2. It can be summarized that in practical applications it is better to consider Rudin-Shapiro for larger number of sequences and Legendre Sequence for small number of sequences.

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