

Fundamental study of a non-linear capacitor to use it in non-linear transmission line models

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Abstract: The theme of this paper is to analyze a non linear capacitor so that this non linear device can be used in realizing the non linear transmission line to observe the dispersion characteristics. In a linear capacitor, the charge stored on the electrodes and the potential difference between them is a linear function. In this paper, a quadratic relation is considered between charge and the potential difference. The relevance of this analysis can be attributed to the study of non linear transmission lines. This device behavior is observed in both time and frequency domains. The impulse response can be used to find the frequency response for a linear circuit but this is not standard to find the frequency response of a non-linear system. The frequency response for this non linear capacitor is obtained for different inputs by taking the ratio of output Fourier transform to input Fourier transform. After analysis, it is observed that non linearity is giving some odd results indicating that the control on the nonlinearity is of great importance in circuits and systems and if this non linearity can be controlled somehow, these elements are going to give a new class of devices with good applications.

Keywords: non linear circuits, capacitor, admittance, impulse response, Fourier Transform, Frequency response and phase variation.

I. INTRODUCTION

Linear circuit theory has been in study for a long time and it is well established [1]. The problems with the non-linear circuits are that the analysis is very specific depending on the type of non linearity [2-3]. Another way to analyse the non linear circuits is by linearising that circuit element using Taylor series expansion [4] about some operating point for small signals and uses the linear circuit theory. This gives better approximations to the requirements depending on the problem and in some cases these results are not so close to the exact expected quantities. This report gives the analysis without any assumptions and approximations. In this paper a non linear capacitor is analysed. The non linearity in the capacitor can come from the dielectric material filling the space between the electrodes or from the fringing fields (depending on geometry of electrodes). A capacitor with no fringing field can be taken as an ideal capacitor. But all practical capacitors have the fringing fields making it a non linear behaviour to some extent over some band of frequencies or by operating the capacitor beyond its operating ranges. Very frequently used non linear capacitor in microwaves is varactor diode which changes its capacitance with the voltage applied across its terminals. Again this model is specific only to that diode and need a complete analysis if that is to be used with slight modifications with other circuit elements. Section II describes the non-linear model and procedure to find the frequency characteristics, while section III gives the discussion on the analysis and results. Section IV gives the conclusions.

II. NON-LINEAR MODEL

In linear circuit theory, every energy storage element (capacitor and inductor) and energy dissipative elements are related with a linear relation between current and

voltage. It is a well known fact that most of the active devices are non linear in nature and they are also of great importance in electronics. In a linear capacitor, charge on electrodes and potential difference between the electrodes are related with a linear equation with the proportional constant being defined as the capacitance.

In this paper, this relation is taken as a second order or quadratic form. Even thou this type of exact non linear devices are not readily available as of now, it is of some importance to start the study with this relation (for that matter non liner inductor) which can be extended further to other non linearities. For a linear capacitor the relation is given as $q \propto v \Rightarrow q = c v \Rightarrow i = \frac{dq}{dt} = c \dot{v}$. The non linearity considered in this paper is given as $q = c_1 v + c_2 v^2 \Rightarrow i = \frac{dq}{dt} = c_1 \dot{v} + 2 c_2 v \dot{v} = (c_1 + 2 c_2 v) \dot{v}$.

By using the Continuous Time Fourier Transform (CTFT) [5], this time domain equation is converted in to frequency domain as $I(j\omega) = c_1 j\omega V(j\omega) + 2 c_2 V(j\omega) * j\omega V(j\omega) \Rightarrow Y(j\omega) = c_1 j\omega + 2 c_2 \left\{ \frac{V(j\omega) * j\omega V(j\omega)}{V(j\omega)} \right\}$.

Here multiplication in time domain and differentiation in time domain are related as convolution and multiplication by $j\omega$ in the frequency domain. For a non linear system it is not possible to find a unique transfer function as the ratio of frequency transform of output to the frequency transform of input. This ratio will be different for different types of inputs.

This is due to the reason that non linear circuits produces harmonics and power shifts from one frequency to another frequency due to non linearity. It is also clearly evident

from the equation that the ratio is not independent of $V(j\omega)$. This is verified with four different cases analytically and results are given in the Table 1.

Table 1

S. No	$x(t)$	$\frac{\{x(t) * t x(t)\}}{x(t)}$ $0 < t < 1$
1	$U(t) - U(t - 1)$	$t^2/2$
2	$e^{-at}U(t)$	$t^2/2$
3	$t \{U(t) - U(t - 1)\}$	$t^3/12$
4	$t^2 \{U(t) - U(t - 1)\}$	$t^4/60$

From this it is obvious now that the expression $\{x(t) * t x(t) / x(t)\}$ can be used to completely change the device properties by some proper selection of input. At the same time two completely different signals are giving the same ratio over certain interval. This is further explored in the next section with different inputs such as Gaussian and modulated Gaussian pulse.

III. ANALYSIS AND RESULTS

To validate the above observation from another angle, two different types of excitations are applied for the non linear capacitor. Simulations are carried out in MATLAB [6].

Firstly, a Gaussian pulse, $x(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{t-\mu}{\sigma})^2}$, with different σ . For another observation the non linearity is changed with same input. Secondly, a modulated Gaussian pulse, $x_m(t) = \frac{1}{\sigma\sqrt{2\pi}} \sin(2\pi ft) e^{-\frac{1}{2}(\frac{t-\mu}{\sigma})^2}$ with different carrier frequencies (f) with fixed non linearity.

Fig.1, Fig.2 and Fig.3 represents the signals for different inputs. Different inputs are given by changing the standard deviation σ in the Gaussian pulse with keeping the non linearity as $c_1 = 1$ and $c_2 = -0.001$. These values are taken as normalized values for simplicity. These can be scaled according to frequency content in the input signal. From the results it is observed that there is a peculiar behaviour for the capacitor for different inputs as expected from the above mentioned theory. It is observed that the non-linearity in the system is going to change the phase from positive side to negative side and the magnitude is passing through a minimum. This property can be used in transmission lines [3] for realizing in some important dispersion properties which will be communicated in next article.

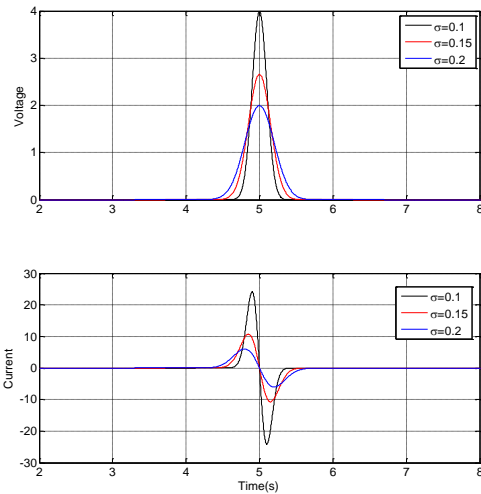


Fig.1

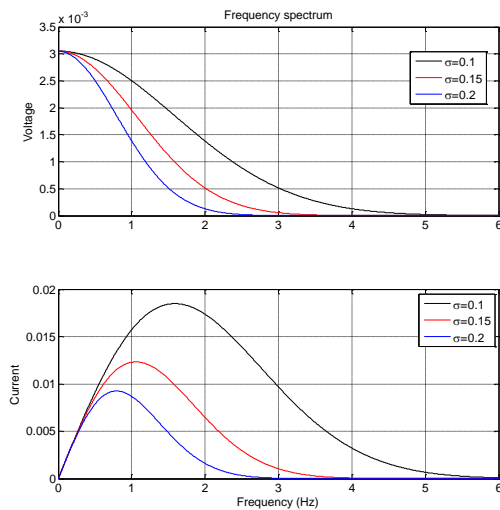


Fig.2

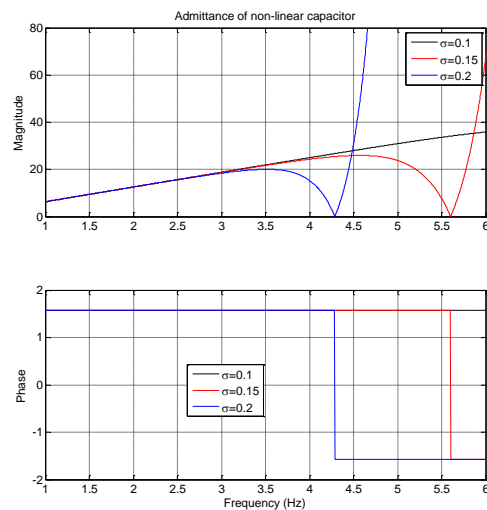


Fig.3

Fig.4, Fig.5 and Fig.6 represents the signals for different values of non-linearity (c_2 is a variable) by keeping the

input signal same for all cases. c_2 has taken the values as -0.2, 0 and 0.2 with $c_1 = 1$. Here also, it is observed that the same kind of observations is coming from this as before.

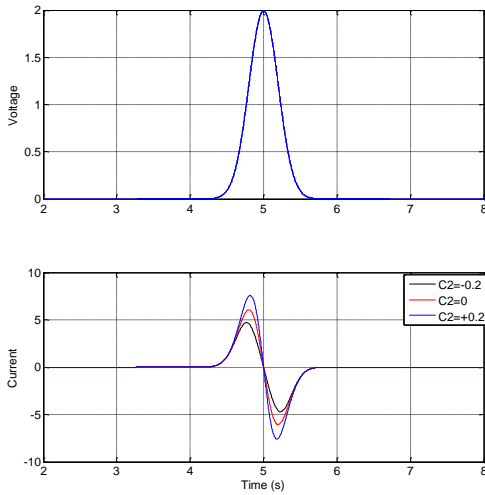


Fig.4

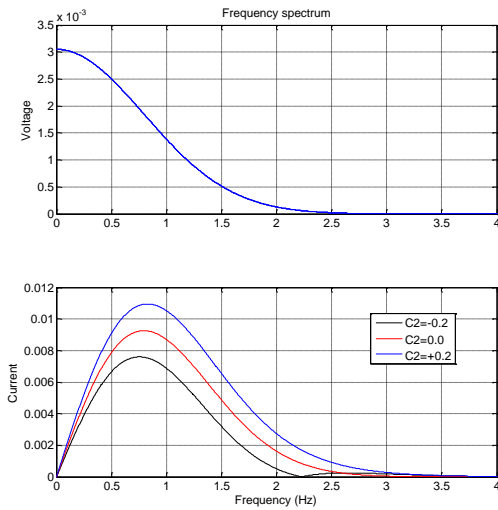


Fig.5

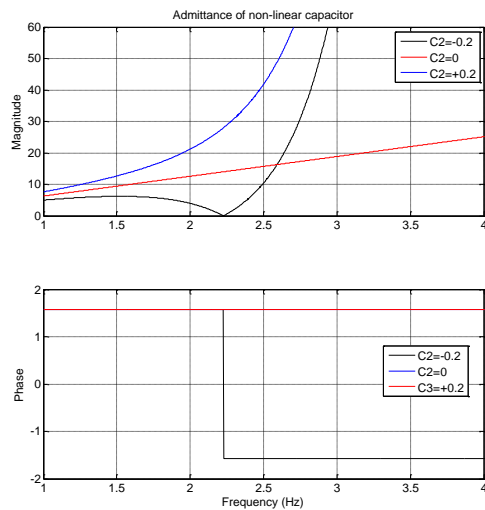


Fig.6

Fig.7, Fig.8 and Fig.9 represents the signals for a modulated Gaussian pulse with carrier frequencies being changed. Here, the phase of impedance is changing gradually from positive to negatives side depending on the input. An abrupt phase difference of 180° represents the change in the reactive component only. But any gradual change in the phase represents some fictitious resistance in the element. This is entirely a different behavior from the above mentioned results.

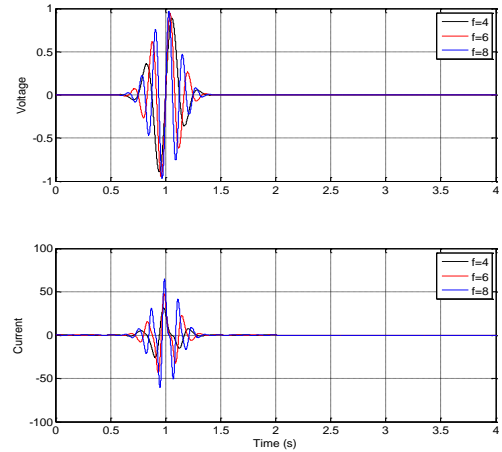


Fig.7

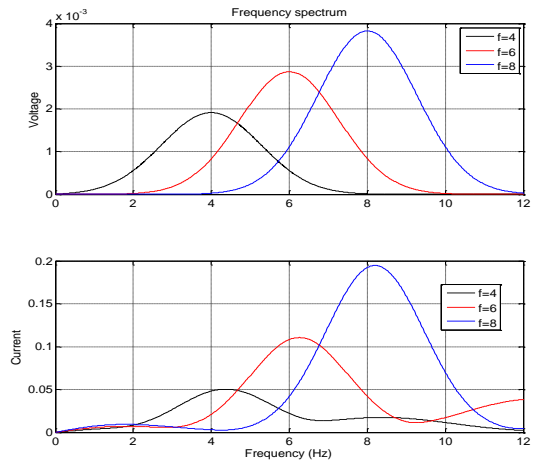


Fig.8

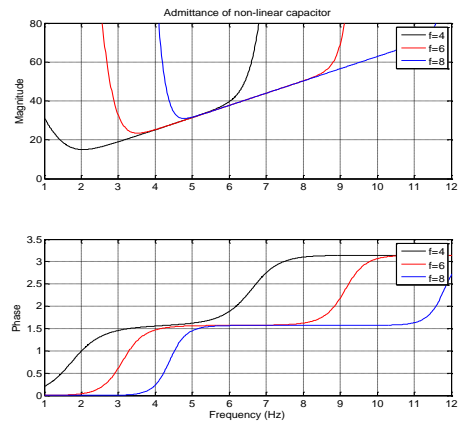


Fig.9

IV. CONCLUSION

From mathematical expressions and simulations, it is possible to come to the conclusion that even a small non-linearity in the system is going to change the systems properties drastically. Non linearity produces undesired responses depending on the inputs. However, if it is possible to control this non-linearity, these non-linear devices may be used in different applications. Along with this it is also understood that even the second order non linearity itself is a very complicated issue but if it can be controlled somehow, it can be of great importance of practical use in the coming days.

REFERENCES

- [1] William Hart Hayt, William Hayt, Jack E.Kimmerly and Steven M.Durbin, *Enginnering Circuit Analysis*. New York: MCGRAW-HILL higher Education, 2012.
- [2] Halliday, Resnick and Walker, *Fundamentals of Physics Electricity and Magnetism*, Wiley India Private Limited, 2011
- [3] William Hart Hayt and John A. Buck, *Engineering Electriomagnetcs*. New York: MCGRAW-HILL Higher Education, 2012
- [4] Erwin Kreyszig, *Advanced Engineering Mathematics*, Wiley India Private Limited, 2006
- [5] Alan Victor Oppenheim, Alan S. Willsky, with S. Hamid Nawab, *Signals and Systems*, Prentice-Hall International, 1997.
- [6] MATLAB version 7.10.0.499 (R2010a).Natick, Massachusetts: The Math Works Inc., 2010