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A Comparative Study of Stability of Fixed Point State – Space Digital Filters using Saturation Arithmetic

Shiva Jaiswal[1], Pritee Verma[2]

Assistant Professor, Electronics and Communication Engineering Department, SRMSWCET, Bareilly, India[1, 2]

Abstract: The problem concerning the elimination of overflow oscillation in fixed-point state-space digital filter employing saturation arithmetic is considered by various researchers. In this paper a comparative study of the stability of digital filters under saturation arithmetic is done.

Keywords: Digital filters, Finite word length, asymptotic stability, nonlinear system.

I. INTRODUCTION

When a digital filter is implemented on a digital computer or on special-purpose digital hardware, the filter coefficients are stored in binary registers. These registers can accommodate only a finite number of bits and hence the filter coefficients have to be truncated or rounded-off in order to fit into these register. Whether the computations are performed in fixed-point or floating-point arithmetic is another consideration. All these problems are usually called finite –word-length effect and results in degradation of system performance.
The finite-word length in recursive digital filter produces non-linearities, namely quantization and overflow. The presence of such non-linearities may result in the instability of the designed system. The zero input limit cycle, which are undesirable, may possibly occur due to such non-linearities. The quantization and overflow non-linearities may interact with each other. However, if total number of quantization steps is large or, in other words, the internal wordlength is sufficiently long, then the effect of these non-linearities can be regarded as decoupled or no interacting and can be investigated separately. Under this decoupling approximation, quantization effects may be neglected when studying the effect of overflow. The problem concerning the elimination of overflow oscillation in fixed-point state-space digital filter employing saturation arithmetic is considered here.

II. SATURATION ARITHMETIC IN DIGITAL FILTER AND SYSTEM DESCRIPTION

The digital filter is described by the state space model implemented in fixed-point arithmetic as

$$x(r+1) = f(y(r)) = [f_1(y_1(r)) \quad f_2(y_2(r))......f_n(y_n(r))]^T \qquad (1a)$$

$$y(r) = [y_1(r) \quad y_2(r) \quad ....y_n(r)]^T = Ax(r), \qquad (1b)$$

where **x**(r) is an n-vector state, **A** = $[a_{i\ j}]$ is the n × n coefficient matrix, and T denotes the transpose. The saturation nonlinearities given by

$$f_i(y_i(r)) = \begin{cases} 1 & y_i(r) > 1 \\ y_i(r) & |y_i(r)| \le 1 \\ -1 & y_i(r) < -1 \end{cases} \qquad (1c)$$

i = 1, 2, . . . ,n, are under consideration.
Eq(1) may be used to describe a class of discrete-time dynamical system with symmetric state saturation which include digital filters implemented in finite register length under zero external inputs, digital control system with saturation nonlinearities on the state, recurrent neural networks and many other engineering problem.
The system to be studied in this paper is described by eq(1) and is used to describe digital filters with symmetric saturation implemented with finite register length under zero external inputs

III. STABILITY CRITERIA FOR FIXED-POINT STATE-SPACE DIGITAL FILTERS WITH SATURATION ARITHMETIC

A criterion for the global asymptotic stability of system (1) is reported by Liu.D and Michel.N [1].According to [1], the zero solution of system (1) is globally asymptotically stable if there exist a positive definite symmetric matrix $P = [p_{ij}] \in R^{nxn}$ satisfying

$$P - A^T PA > 0 \qquad (2a)$$

$$p_{ii} \ge \sum_{j=1,\, j\ne i}^{n} |p_{ij}| \quad i=1, 2.......n \qquad (2b)$$

Where >0 denotes the matrix is positive definite
Now define

$$k_i \ge \sum_{j=1}^{n} |a_{ij}| \quad i=1,2....n \qquad (2c)$$

And assume that the elements of matrix a satisfy

$$k_i > 1 \ , i=1,2,........, m \qquad (2d)$$

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$$k_i \le 1 \text{, i=m+1,m+2,.........n} \tag{2e}$$

Pertaining to the global asymptotic stability of system described by (1), ref [2] brings out the following result.

IV. THEOREM 1

The zero solution of the system described by (1) is globally asymptotically stable if there exist a positive definite symmetric matrix $P = [p_{ij}] \in R^{nxn}$ satisfying

$$P - A^T PA > 0 \tag{3a}$$

$$p_{ii} \ge \sum_{j=1, j \ne i}^{n} | p_{ij} | \text{, i=1,2,................m} \tag{3b}$$

From ref [2], Theorem 1 presents a modified form of criterion due to Liu.D and Michel.N [1].Now with m=n,(3b) is same as (2b).For the situation where m≠n, (3b) is more relaxed than (2b). Therefore, in case where m≠n, (3b) is less stringent than (2b).A noticeably improved version of Theorem 1 is presented by Kar. H [3] by imposing less restriction on the elements of matrix P than (2b), and brings out the following result.

V. THEOREM 2

The zero solution of the system described by (1) is globally asymptotically stable if there exist a positive definite symmetric matrix $P = [p_{ij}] \in R^{n \times n}$ satisfying

$$P - A^T PA > 0 \tag{4a}$$

$$p_{ii} \ge \sum_{j=1, j \ne 1}^{m} |p_{ij}| + \sum_{j=m+1, j \ne 1}^{n} k_j \, | p_{ij} | \text{i=1,2,3,...m} \tag{4b}$$

Theorem 2 is applicable to one combination of elements of the matrix A, i.e. where the elements of first m rows of A satisfy (2d) and those of the remaining (n-m) satisfy (2e).The result pertaining to other possible combinations of the elements of matrix A can be easily conceived.

Condition (4) implies global asymptotic stability of the null solution, which assures not only the absence of limit cycle oscillations, but also elimination of any other kind of instability in the system under consideration

The stability criterion of fixed-point state-space digital filters proposed by Ooba.T is improved from some of the earlier existing criteria [1]-[3].A linear algebra given by [7] broadens the scope of stability test. Pertaining to the global asymptotic stability of system described by (1), ref [7] brings out following result.

VI. THEOREM 3

The system described in (1) is asymptotically stable if there exists a positive definite matrix P satisfying

$$(P)_{i,i} - \sum_{j \ne i} \left(w_{|A|}\right)_j \left(| P |\right)_{i,j} > 0 \quad for \quad all \quad i \in J_{|A|}^c \tag{5a}$$

such that $P - A^T PA$ is positive definite.

There are some prerequisite which are to be known before stating the algorithm to calculate $w_{|A|}$ for (5a), they are

a) Stability test is to be done on matrix A, where $A \in R^{nxn}$,
b) The order of the matrix A is n.
c) The matrix $B = | a_{ij} |$, i, j=1 2….n
d) J_0=ϕ ; J_k contains coordinates indices
e) n_0 =0 ; n_k contains the number of indices of J_k
f) J_k^c contains the complement indices of J_k
g)

$$w_o = 1 \in R^n; \qquad w_o = \begin{bmatrix} 1 \\ 1 \\ 1 \\ . \\ . \\ . \\ n \end{bmatrix}$$

VII. ALGORITHM

The following procedure is proposed by ref [7] with k=1, to obtain J_B, n_B and $w_B = R^n$

i. Let J_k denotes the list of coordinate indices i's satisfying $(Bw_{k-1})_i < 1$,and let n_k denotes the number of the indices in J_k ;

ii. If $J_k = J_{k-1}$, or if $n_k = n$, then define $J_B = J_k$, $n_B = n_k$,and

$$w_B = \begin{cases} w_{k-1} & if \quad n_B < n \\ 0 & if \quad n_B = n \end{cases} \tag{5b}$$

and then exit the loop;

iii. Define $w_k \in R^n$ such that

$$\begin{cases} (w_k)_{J_k} = \left(I_{n_k} - B_{J_k, J_k}\right)^{-1} B_{J_k, J_k^c} I_{n-n_k} \\ (w_k)_{J_k^c} = I_{n-n_k} \end{cases} \tag{5c}$$

and return to step (i) with k=k+1.

VIII. NUMERICAL EXAMPLE

To illustrate the algorithm for the stability test of fixed-point state-space digital filter with saturation arithmetic, a specific example of a third-order digital filter is considered with

$$A = \frac{1}{10}\begin{bmatrix} 10 & 50 & 0 \\ -1 & 1 & -15 \\ 0 & 1 & -5 \end{bmatrix}$$

According to the prerequisite of the algorithm Order of the matrix A is 3

$$B = \frac{1}{10}\begin{bmatrix} 10 & 50 & 0 \\ 1 & 1 & 15 \\ 0 & 1 & 5 \end{bmatrix}$$

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$J_0 = \phi$

$n_0 = 0$, and

$$w_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Iteration1

Step (i)

$$Bw_0 = \frac{1}{10}\begin{bmatrix} 10 & 50 & 0 \\ 1 & 1 & 15 \\ 0 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 1.7 \\ 0.6 \end{bmatrix}$$

$(Bw_0)_i < 1$, where 'i' is the indices values satisfying the given condition, in this step it is {3} Therefore J_1= {3} and n_1=1(number of indexes in J_1)

Step (ii)

$J_0 \neq J_1$ and $n_1 \neq n$

Step (iii)

$$\begin{cases} (w_1)_{J_1} = \left(I_{n_1} - B_{J_1,J_1}\right)^{-1} B_{J_1,J_1^c} I_{n-n_1} \\ (w_1)_{J_1^c} = I_{n-n_1} \end{cases}$$

Where J_1^c contains the complement indexes of J_1 i.e. $J_1^c = \{1,2\}$

Now

$$(w_1)_{J_1} = \left[1 - \frac{5}{10}\right]^{-1}\left[0 \quad \frac{1}{10}\right]\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} = \frac{1}{5} = 0.2$$

$$(w_1)_{J_1^c} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now return to step (i) of the algorithm, with k=k+1 i.e. k=2

Iteration 2

Step (i)

$w_1 = \begin{bmatrix} 1 \\ 1 \\ 0.2 \end{bmatrix}$; since $J_1 = \{3\}$ therefore $w_1(3) = (w_1)_{J_1}$

$$Bw_1 = \frac{1}{10}\begin{bmatrix} 10 & 50 & 0 \\ 1 & 1 & 15 \\ 0 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 6 \\ 0.5 \\ 0.2 \end{bmatrix}$$

$(Bw_1)_i < 1$, where 'i' is the indices values satisfying the given condition, in this step it is {2,3} Therefore $J_2 = \{2,3\}$ and $n_2 = 2$ (the number of indices in J_2)

Step (ii)

$J_1 \neq J_2$ and $n_2 \neq n$

Step (iii)

$$\begin{cases} (w_2)_{J_2} = \left(I_{n_2} - B_{J_2,J_2}\right)^{-1} B_{J_2,J_2^c} I_{n-n_2} \\ (w_2)_{J_2^c} = I_{n-n_2} \end{cases}$$

Where J_2^c contains the complement indexes of J_2 i.e. $J_2^c = \{1\}$

No

$$(w_2)_{J_2} = \left[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{1}{10}\begin{bmatrix} 1 & 15 \\ 1 & 5 \end{bmatrix}\right]^{-1}\begin{bmatrix} 1/10 \\ 0 \end{bmatrix}[1] = \begin{bmatrix} 0.1666 \\ 0.0333 \end{bmatrix}$$

$$(w_2)_{J_2^c} = [1]$$

Return to step (i) of the algorithm with k=k+1, i. e k=3

Iteration 3

Step (i)

$w_2 = \begin{bmatrix} 1 \\ 0.1666 \\ 0.0333 \end{bmatrix}$; $J_2 = \{2,3\}$ therefore $w_2\begin{pmatrix} 2 \\ 3 \end{pmatrix} = (w_2)_{J_2}$

$$Bw_2 = \frac{1}{10}\begin{bmatrix} 10 & 50 & 0 \\ 1 & 1 & 15 \\ 0 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 0.1666 \\ 0.0333 \end{bmatrix} = \begin{bmatrix} 1.8333 \\ 0.1667 \\ 0.0333 \end{bmatrix} \quad (Bw_2)_i < 1$$

where 'i' is the indices values satisfying the given condition, in this step, it is again {2, 3}

Therefore $J_3 = \{2,3\}$ and $n_3 = 2$ (the number of indices in J_3)

Step (ii)

$J_2 = J_3$, and $n_3 \neq n$

In step (ii) of iteration 3, one of the conditions stated in step (ii) of the algorithm is satisfied. Therefore we will define

$J_B = J_3 = \{2,3\}$

$n_B = n_3 = 2$ and

$w_B = w_2$, since $n_B < n$

$$(w_B) = (w)_2 = \begin{bmatrix} 1 \\ 0.1667 \\ 0.0333 \end{bmatrix}$$

Exit the loop.

To calculate the value of P for the given A in numerical example , we will use MATLAB LMI tool box. The matrix P for given A in numerical example comes out to be

$$P = \begin{bmatrix} 0.1085 & 0.3915 & -0.1064 \\ 0.3915 & 4.2761 & -3.4778 \\ -0.1064 & -3.4778 & 10.3273 \end{bmatrix}$$

Following the algorithm stated in VII, for the A given in numerical example we have $J_{|A|} = \{2,3\}$ and $J_{|A|}^c = \{1\}$.

Considering $J_{|A|}$, $J_{|A|}^c$ and P, for the given A in numerical example, we will check whether Theorem 3 is satisfied, i.e. in our case.

$$(P)_{i,i} - \sum_{j \neq i} \left(w_{|A|}\right)_j \left(|P|\right)_{i,j} > 0 \quad for \quad all \quad i \in J_{|A|}^c = \{1\} \qquad (5d)$$

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$$(P)_{1,1} - (w_{|A|})_2 (|\,P\,|)_{1,2} - (w_{|A|})_3 (|\,P\,|)_{1,3}$$

(0.1085)-(0.1667)*(|0.3915|)-(0.0333)*(|-0.1064|) = 0.03969383

Thus the value of (5d) comes out to be greater than zero. Hence the system considered in the numerical example is judged to be asymptotically stable according to Theorem 3.The same can also be verified by plotting the state trajectories of the numerical example. The figure1 shows that the system under consideration is stable, as the next state of the system reaches zero with increasing iterations i.e. the output reaches zero with zero input condition.

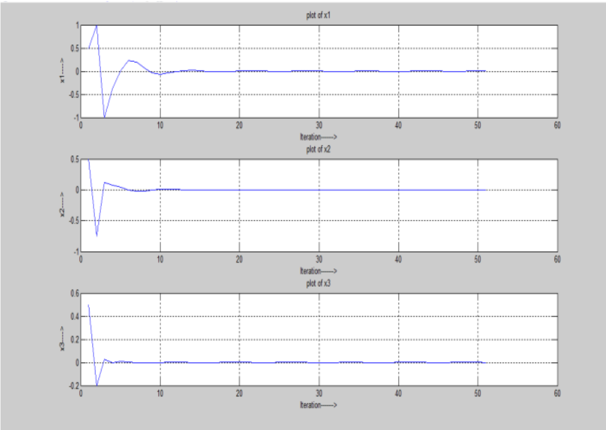

Fig.1 Dynamical behavior of the system considered in numerical example

IX. CONCLUSION

The criteria for the global asymptotic stability of fixed-point state-space digital filters with saturation nonlinearity have been given by several researchers. Theorem1 and Theorem 2 both fails in case of the system considered in the numerical example. But a finite procedure proposed by Ooba.T [7] ascertains the global asymptotic stability of the system considered in the numerical example. A little linear algebra [7] before searching for a Lyapunov solution is reasonably inexpensive way to broaden the scope of stability test from those of earlier results.

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