

Soft Computing Technique for the Design of Band Pass FIR Digital Filter

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Abstract: This paper presents a soft computing technique for the design of band pass FIR digital filter. Nature inspired optimization technique known as predator prey optimization (PPO) has been used to design an optimal band pass FIR digital filter. PPO technique is considered as a comprehensive search technique and as a stochastic optimization process; it avoids confined stagnation as preys play the role of diversification to find best solution because of predator's fear. PPO technique enhances the ability to discover and utilize the search space locally as well as globally in order to obtain the finest filter parameters for the design of filter. Performance of PPO has been compared with craziness based Particle Swarm Optimization (CRPSO). Simulation results demonstrate that the proposed PPO algorithm gives better results for the design of band pass FIR digital filter as compared to CRPSO.

Keywords: Predator Prey Optimization, Craziness based PSO, Band-Pass FIR Digital Filter, Magnitude and ripple errors, Magnitude and Phase response.

I. INTRODUCTION

Digital filter is basically a network or a system whose main function is to add or split two or more signals, to extract meaningful information from the signal as well as to enhance the quality of a signal [6]. There is wide variety of applications in which digital filters are extensively used such as in digital signal processing, audio processing, pattern recognition, image processing, speech processing, communication system, for noise suppressions and many more. Digital filters are divided into two main classes such as finite impulse response filters (FIR) and infinite impulse response filters (IIR) [4].

FIR filters are usually known as non-recursive filters. These filters are called so, because the output of FIR filter due to a single impulse input will decay and turn to be zero in a finite time. But in case of IIR filter, feedback is present from output side of the filter to input side. Because of this feedback connection, IIR filter not only depends on current and past input values but also on current and past output values. Thus, the output (impulse response) of IIR filter never decays. For this reason, IIR filters are known to be recursive filters. FIR filters are easy to implement but when they are compared to IIR filters, they are slower. No doubt, IIR filters are faster than FIR filters but implementation of IIR filters are complicated as compared to FIR filters. Thus, because of the stability and simplicity in design, FIR filter is a smart choice [4] [10].

FIR digital filters can be designed and implemented by using different types of techniques such as window design

method, frequency sampling method and frequency sampling method. Most commonly used technique to design FIR digital filter is the windowing technique. It consists of truncating or multiplying the input sequence (ideal impulse response) by a window function. There are different types of window functions such as rectangular window, hamming window, hanning window, triangular window, blackman window and Kaiser window [12]. There are several advantages of windowing method such as (1) less computational efforts are required to design FIR digital filter. Thus, it is easy to implement window method (2) in the response of filter; the maximum value of the ripple magnitude for the given window is fixed. Thus, for the desired window, the stop band attenuation is also fixed. Along with the advantages, windowing method has some disadvantages too such as: the expression of the desired response of the filter comes out to be too complicated in a number of applications [5] [8].

Different types of evolutionary algorithms like particle swarm optimization (PSO), genetic algorithm (GA), predator prey optimization (PPO) and differential evolution (DE) has been widely used to design digital filters. When global optimization technique is considered, GA has gained the considerable attention in order to design digital filter. It is efficient to attain local optimum solution using GA but it is inefficient in order to determine the global optimum solution in terms of solution quality and convergence speed [4].

Particle swarm optimization (PSO) is a nature inspired evolutionary algorithm which was developed by Dr. Russell Eberhart and Dr. James Kennedy in 1995. It is easy to implement PSO and the convergence of swarm can be controlled by varying few parameters [1]. The main disadvantage of PSO is the premature convergence problem and local stagnation. To overcome the problems of PSO, craziness based PSO (CRPSO) has been developed, which is the modification of PSO [4]. The CRPSO technique tries to get the finest coefficients which closely resembles to the ideal frequency response of the digital filter. Thus, craziness based PSO gave better results than PSO [9].

In this paper, another nature inspired optimization technique i.e. predator prey optimization (PPO) has been proposed for the design of band pass FIR digital filter. It arbitrarily explores the search space locally as well as globally [7]. The PPO algorithm optimizes the filter coefficients in order to attain the magnitude error in pass band and stop band as well as ripple magnitude in pass band and stop band as objective functions for optimization problem.

This paper has been organized in five sections. Section II describes the problem formulation of band-pass FIR digital filter. Section III presents a brief summary of proposed PPO algorithm. Section IV describes the simulation results that have been achieved by evaluating proposed PPO algorithm and these results have been compared with the design results of CRPSO. Finally, in section V, the conclusions have been discussed.

II. FIR DIGITAL FILTER DESIGN

The FIR filter is a digital filter with finite impulse response. They are also known as non-recursive digital filters as they do not have feedback from output back to input. FIR filters are implemented using a transversal filter. The transversal filter is also known as a tapped delay line filter. It consists of three basic elements: unit delay element, multiplier and adder. The difference equation of FIR filter is given below:

$$y(n) = \sum_{k=0}^{M-1} h(k)x(n-k) \quad (2.1)$$

where $h(k)$ is the set of filter coefficients. The output $y(n)$ is the function of only input signal $x(n)$. M is the order of filter.

The transfer function of FIR filter is given as:

$$H(z) = \sum_{k=0}^{M-1} h(k)z^{-k} \quad (2.2)$$

where $H(z)$ is the frequency domain representation of the impulse response and is termed as transfer function of the

digital filter, $h(k)$ is the time domain representation of the impulse response of the filter, M is the order of the filter. The total number of filter coefficients is $N+1$. But due to even symmetric property of FIR filters, we have to calculate only half number of filter coefficients i.e. $(N/2 + 1)$. So, the dimension of the problem is reduced to half.

Errors: The FIR filter is designed by optimizing the coefficients in such a way that the approximation error function in L_p -norm for magnitude is to be kept minimal. The magnitude response is specified at K equally spaced discrete frequency points in pass band and stop band.

$$e_1(x) = \sum_{i=0}^K |H_d(\omega_i) - |H(\omega_i, x)|| \quad (2.3)$$

$$e_2(x) = \left\{ \sum_{i=0}^K (|H_d(\omega_i) - |H(\omega_i, x)||)^2 \right\}^{1/2} \quad (2.4)$$

where $e_1(x)$ is absolute error L_1 -norm for magnitude response and $e_2(x)$ is squared error L_2 -norm of magnitude response.

Desired magnitude response of FIR digital filter is:

$$H_d(\omega_i) = \begin{cases} 1, & \text{for } \omega_i \in \text{passband} \\ 0, & \text{for } \omega_i \in \text{stopband} \end{cases} \quad (2.5)$$

For the design of digital FIR filters, the inclusion of stability constraints is compulsory. The ripple magnitudes of pass-band and stop-band have to be minimized which are given by $\delta_p(x)$ and $\delta_s(x)$ respectively:

$$\delta_p(x) = \max_{\omega_i} |H(\omega_i, x)| - \min_{\omega_i} |H(\omega_i, x)| \quad \dots \dots \text{for } \omega_i \in \text{passband} \quad (2.6)$$

$$\delta_s(x) = \max_{\omega_i} |H(\omega_i, x)| \quad \text{for } \omega_i \in \text{stopband} \quad (2.7)$$

Four objective functions for optimization are:

$$\text{Minimize } f_1(x) = e_1(x) \quad (2.8.1)$$

$$\text{Minimize } f_2(x) = e_2(x) \quad (2.8.2)$$

$$\text{Minimize } f_3(x) = \delta_p(x) \quad (2.8.3)$$

$$\text{Minimize } f_4(x) = \delta_s(x) \quad (2.8.4)$$

The multi-objective function is converted to single objective function:

$$\text{Minimize } f(x) = \omega_1 f_1(x) + \omega_2 f_2(x) + \omega_3 f_3(x) + \omega_4 f_4(x) \quad (2.9)$$

where ω_k are the weights.

III. PREDATOR PREY OPTIMIZATION

Predator prey optimization (PPO) model was first developed by Silva et al., A. Neves et al. and E. Costa et al. They introduced a second population of particles called predators in addition to preys so as to balance exploration and exploitation in PSO. So, PPO is basically a PSO with added predator effect. Predators have different dynamic behaviour from that of preys (swarm particles). They are attracted towards the best individuals among swarm, while repelling the other particles. Balance between exploration and exploitation can be maintained by controlling the strength and frequency of the interactions between preys and predator. It helps to maintain diversity in the population, even if it is approaching convergence to local sub-optima. Thus, PPO technique being a stochastic optimization procedure, avoids confined stagnation as preys play the role of diversification to look for optimum solution due to predator(s) fear. Exponential term will also be included in velocity vector in case if predator attacks the prey [7].

3.1 INITIALIZATION OF POSITION AND VELOCITY OF POPULATION

Prey and predator's starting positions are randomly initialized within their upper and lower limits.

$$x_{ik}^0 = x_i^{\min} + R_{ik}^1 (x_i^{\max} - x_i^{\min}) \quad (3.1)$$

$$x_{pi}^0 = x_i^{\min} + R_i^2 (x_i^{\max} - x_i^{\min}) \quad (3.2)$$

where $i = 1, 2, \dots, S$ and $k = 1, 2, \dots, S_p$.

Prey and predator's position is denoted by x_{ik}^0 and x_{pi}^0 respectively. S_p is the total prey population. x_i^{\min} and x_i^{\max} is the range of i^{th} decision variable. R_{ik}^1 and R_i^2 are random numbers having values within the range of 0 and 1.

After positions, velocities of prey i.e. V_{ik}^0 and predator i.e. V_{pi}^0 are randomly initialized within their predefined range.

$$V_{ik}^0 = V_i^{\min} + R_{ik}^1 (V_i^{\max} - V_i^{\min}) \quad (3.3)$$

$$V_{pi}^0 = V_i^{\min} + R_{pi}^1 (V_i^{\max} - V_i^{\min}) \quad (3.4)$$

where $i = 1, 2, \dots, S$ and $k = 1, 2, \dots, S_p$. Minimum and maximum prey velocities are set by using the following relation:

$$V_i^{\min} = -\alpha (x_i^{\max} - x_i^{\min}) \quad (3.5)$$

$$V_i^{\max} = +\alpha (x_i^{\max} - x_i^{\min}) \quad (3.6)$$

where $i = 1, 2, \dots, S$. Minimum and maximum velocities for preys are obtained by varying the value of α . α is taken as 0.25.

3.2 PREDATOR VELOCITY AND POSITION EVALUATION

The velocity and position of predator at the end of each iteration is updated by using Equ. 3.7 and Equ. 3.8:

$$V_{pi}^{t+1} = C_4 (GPbest_i^t - V_{pi}^t) \quad (i=1, 2, \dots, S) \quad (3.7)$$

$$x_{pi}^{t+1} = x_{pi}^t + V_{pi}^{t+1} \quad (i=1, 2, \dots, S) \quad (3.8)$$

where $GPbest_i^t$ is global best position of prey of i^{th} variable, C_4 is the random number whose value lies between 0 and its upper limit.

3.3 PREY VELOCITY AND POSITION EVALUATION

Preys velocity and position are updated at the end of each iteration by using the Equ.3.9 and Equ.3.10:

$$V_{ik}^{t+1} = \begin{cases} wV_{ik}^t + AC_1 R_1 (xbest_{ik}^t - x_{ik}^t) + AC_2 R_2 (GPbest_{ik}^t - x_{ik}^t) & ; P_f \leq P_f^{\max} \\ wV_{ik}^t + AC_1 R_1 (xbest_{ik}^t - x_{ik}^t) + AC_2 R_2 (GPbest_{ik}^t - x_{ik}^t) + C_3 a(e^{-b \cdot e_k}) & ; P_f > P_f^{\max} \end{cases} \dots (i=1, 2, \dots, S; k=1, 2, \dots, S_p) \quad (3.9)$$

$$x_{ik}^{t+1} = x_{ik}^t + c_{cf} V_{ik}^{t+1} \quad (i=1, 2, \dots, S; k=1, 2, \dots, S_p) \quad (3.10)$$

where AC_1 and AC_2 are the acceleration constants, w is inertia weight, $xbest_{ik}^t$ is local best position and $GPbest_{ik}^t$ is the global best position of prey, R_1 and R_2 are the random numbers having values between 0 and 1, C_3 is a random number lies in the range 0 and 1 and it influences the effect of predator on prey, the term $a(e^{-b \cdot e_k})$ introduces the predator effect that increases exponentially. Every time predator goes closer to prey, this exponential term introduces disturbance in the prey population, constant 'a' represents the maximum amplitude of the predator effect over a prey and 'b' allows controlling the effect. The distance between predator and prey position is defined by Euclidean distance i.e. e_k for k^{th} population which is given as:

$$e_k = \sqrt{\sum_{i=1}^S (x_{ik} - x_{pi})^2} \quad (3.11)$$

The inertia weight is calculated by using Equ.3.12:

$$w = [w^{\max} - (w^{\max} - w^{\min}) (t/t_{\max})] \quad (3.12)$$

C_{cf} is the constriction factor and is defined by the following equation:

$$C_{cf} = \begin{cases} 2 - \phi - \sqrt{\phi^2 - 4\phi} & \text{if } \phi \geq 4 \\ 1 & \text{if } \phi < 4 \end{cases} \quad (3.13)$$

The elements of prey positions x_{ik}^t and velocities V_{ik}^t may violate their limits. This violation is set by updating their values on violation either at lower or upper limits.

$$V_{ik}^t = \begin{cases} V_{ik}^t + R_3 V_i^{max} & ; \text{if } V_{ik}^t < V_i^{min} \\ V_{ik}^t + R_3 V_i^{max} & ; \text{if } V_{ik}^t > V_i^{max} \\ V_{ik}^t & ; \text{no violation of limits} \end{cases} \quad (3.14)$$

where R_3 is any uniform random number having value between 0 and 1. The process is repeated until the limits are satisfied.

3.4 OPPOSITON BASED STRATEGY

Evolutionary optimization techniques begin with a few initial solutions and then attempt to improve them towards some optimal solutions. The procedure of searching stops as soon as some predefined criteria are fulfilled. It is generally started with a random guesses, in case, if the former information about the solution is absent. The possibility to begin with a better solution can be improved by checking the opposite solution simultaneously. Thus, the better one (either random guess or opposite guess) can be selected as an initial solution by using opposition based learning. According to probability hypothesis, about 50% of time, a guess is farther away from the solution than its opposed guess. Thus, to begin with the closer of the two guesses (as judged by its objective function) has the potential to accelerate convergence [12]. The same approach can be applied not only to initial solutions but also continuously to each solution in the current population.

$$x_{i+Sp,j}^t = x_j^{min} + x_j^{max} - x_{ij}^t \quad (3.15)$$

where $(j = 1, 2, \dots, S; i = 1, 2, \dots, S_p)$. x_j^{min} and x_j^{max} are the lower and upper limits of filter coefficients [11].

Algorithm: Predator Prey Optimization

1. Initialize the parameters of PPO such as population size (S_p), acceleration constants (AC_1/AC_2), maximum and minimum limit of position and velocity of prey and predator, maximum probability fear (P_f^{max}) etc.
2. Initialize the prey and predator positions and velocities randomly.
3. Apply opposition based strategy.
4. Calculate objective function.
5. Select S_p best preys from total $2S_p$ preys.
6. Calculate the personal best position (pbest) of each prey and then select best value among all pbest values of prey and assign that pbest position to all preys.
7. Calculate global best position (gbest) among local best position of prey.
8. Update predator velocity and position by using Equ.3.7 and Equ.3.8.
9. Generate the probability fear factor between 0 and 1 randomly.
10. IF (probability fear > maximum probability fear)
THEN

Update prey velocity and position with predator affect by using Equ. 3.9 and Equ. 3.10.

ELSE

Update prey velocity and position without predator affect by using Equ. 3.9 and Equ. 3.10.

ENDIF.

11. Calculate objective function again for all prey population.
12. Then update local best positions of prey particles.
13. Calculate global best position of prey particles based on fitness.
14. Check stopping criteria, if not met, repeat step 8.
15. Stop.

IV. RESULTS AND DISCUSSIONS

In this section, FIR band-pass digital filter has been designed by using Predator Prey Optimization (PPO) technique. The prescribed design conditions for band pass filter are shown below in the Table 4.1.

Table 4.1: Design conditions for band-pass FIR filter

Filter Type	Pass Band	Stop Band	Max. value of $ H(w,x) $
Band Pass	$0.4\pi \leq \omega \leq 0.6\pi$	$0 \leq \omega \leq 0.25\pi$ $0.75 \pi \leq \omega \leq \pi$	1

The PPO algorithm has been implemented by varying the filter order along with PPO parameters. MATLAB 2013a software has used to perform simulation results for band-pass FIR digital filter. The magnitude response and phase response graphs have been plotted. The initial values of parameters that have taken for PPO algorithm are given below in Table 4.2.

Table 4.2: PPO parameters

Parameters	Value
Run	100
Iterations	100
AC_1, AC_2	2.0
W_{max}	0.4
W_{min}	0.1
Population Size	100
W_3	9.0
W_4	8.0
P_f	0.7
P_f^{max}	1.0

A. SELECTION OF FILTER ORDER

Firstly, the filter order has been varied from 20 to 32. The achieved objective function with respect to filter order has been shown in Table 4.3.

Table 4.3: Filter Order v/s Objective Function

Sr. No.	Filter Order	Objective Function
1	20	1.928457
2	21	2.103815
3	22	1.938703
4	23	1.872286
5	24	1.966658
6	25	1.889127
7	26	2.020063
8	27	1.25581
9	28	0.785962
10	29	0.791115
11	30	10.62864
12	31	7.934611
13	32	23.46085

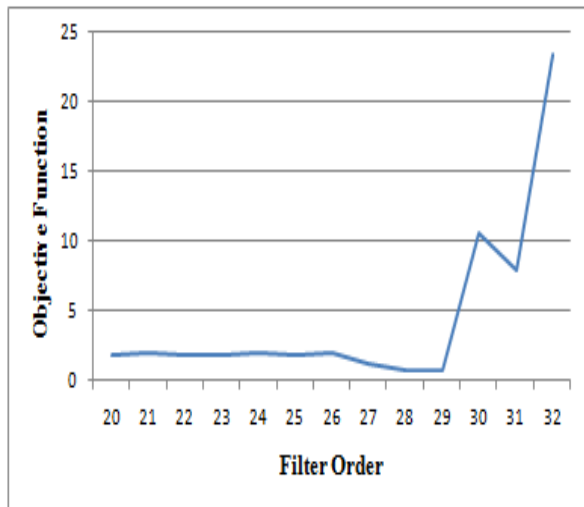


Fig. 4.1: Filter Order versus Objective Function of band pass FIR digital filter at filter order 28

Fig. 4.1 shows the graph of objective function versus filter order of PPO. The objective function of CRPSO [9] is minimum at order 28 where as objective function of PPO is also minimum at filter order 28 as seen from graph. So, PPO has been compared with CRPSO at filter order 28 in terms objective function.

Table 4.4: Comparison of PPO and CRPSO of band-pass FIR digital filter at filter order 28 in terms of objective function

Algorithm	Filter Order	Objective Function
PPO	28	0.785962
CRPSO [9]	28	0.832798

From Table 4.4, it is observed that PPO has minimum objective function as compared to CRPSO at filter order 28.

B. PARAMETER TUNING

In order to get more optimum results by using PPO algorithm, control parameters like population size and acceleration constants (AC_1/AC_2) have been varied. So firstly, the population size of PPO algorithm has been varied in the range of 20 – 140.

Table 4.5: Population size v/s Objective function of band pass FIR digital filter at filter order 28

Sr. No.	Population Size	Objective Function
1	20	0.796704
2	40	0.785576
3	60	0.786769
4	80	0.78586
5	100	0.803882
6	120	1.08431
7	140	1.21754

From the above table, it is observed that population 40 has the minimum value of objective function which is even better than population 100 that was used initially.

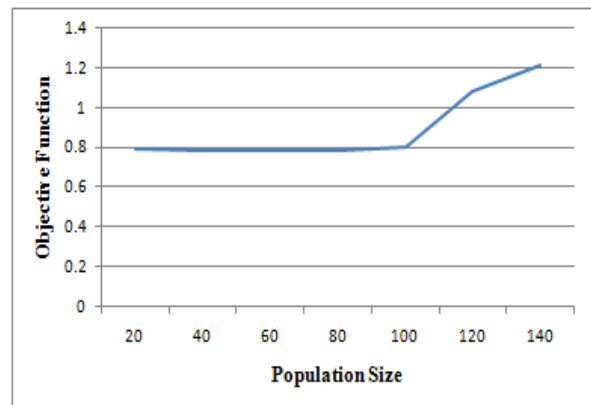


Fig. 4.2: Population versus Objective Function at filter order 28

From Fig 4.2, it is seen that the value of objective function starts decreasing from population 20 and at population 40, objective function is minimum. From population 40 to 100, objective function remains almost constant. After 100, it starts increasing. Furthermore, at this population i.e. at 40, the acceleration constants i.e. AC_1 and AC_2 have been varied from 1 to 3.5.

Table 4.6: Acceleration constants v/s Objective function at filter order 28

Sr. No.	Acceleration Constants (AC_1/AC_2)	Objective Function
1	1.0	0.879877
2	1.5	0.835606
3	2.0	0.785576
4	2.5	0.789105

5	3.0	0.810199
6	3.5	0.844495

From Fig 4.3, it is observed that the value of acceleration constants that yields the best results is 2.0.

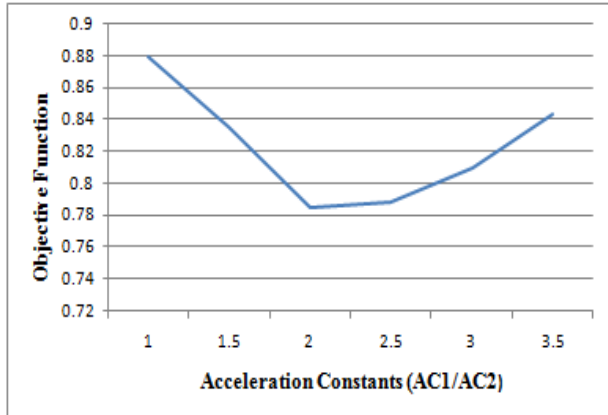


Fig. 4.3: Acceleration constants versus Objective Function at order 28

Fig 4.4 shows the graph that how the value of objective function varies at different iterations at population size 40.

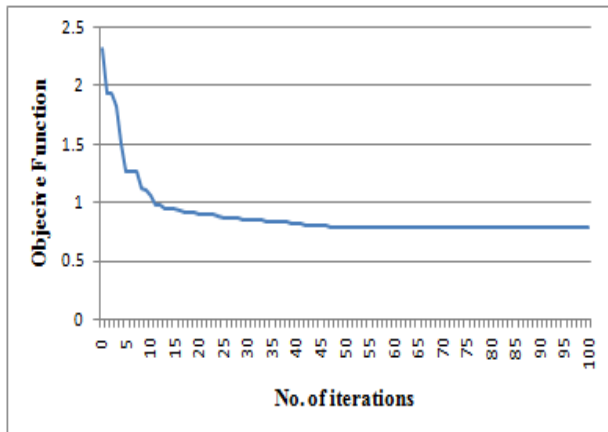


Fig. 4.4: Plot of Iterations versus Objective function at filter order 28

For filter 28, the best results have been achieved when population size is 40 and acceleration constants AC_1 and AC_2 are 2.0 each.

Table 4.7: Design results for band pass FIR digital filter at filter order 28

Objective Function	0.785576
Magnitude Error $e_1(x)$	0.506234
Magnitude Error $e_2(x)$	0.05931
Pass band ripples (δ_p)	0.009402
Stop band ripples (δ_s)	0.016715

C. ANALYSIS OF MAGNITUDE AND PHASE RESPONSE OF BAND PASS FIR DIGITAL FILTER

The obtained results have been performed in MATLAB software. The resulting optimized coefficients at filter order 28 are given in Table 4.7.

Table 4.7: Optimized coefficients of band pass FIR digital filter at filter order 28

Sr. No.	Coefficients	Value of coefficients
1	$A(0) = A(28)$	-.010020
2	$A(1) = A(27)$	-.001817
3	$A(2) = A(26)$.011117
4	$A(3) = A(25)$.000388
5	$A(4) = A(24)$.017710
6	$A(5) = A(23)$	-.002704
7	$A(6) = A(22)$	-.052492
8	$A(7) = A(21)$	-.002577
9	$A(8) = A(20)$.022697
10	$A(9) = A(19)$	-.001493
11	$A(10) = A(18)$.110535
12	$A(11) = A(17)$	-.003963
13	$A(12) = A(16)$	-.281599
14	$A(13) = A(15)$	-.001992
15	$A(14)$.360892

By using these filter coefficients, magnitude and phase response graphs have been plotted. Fig 4.5 shows the plot for variation in magnitude response with variation in normalized frequency. The ideal range of pass-band in band pass FIR filter varies from $0.4\pi \leq \omega \leq 0.6\pi$ and that of stop band varies from $0 \leq \omega \leq 0.25\pi$ to $0.75\pi \leq \omega \leq \pi$ which is shown respectively in Fig 4.5, 4.6 and 4.7.

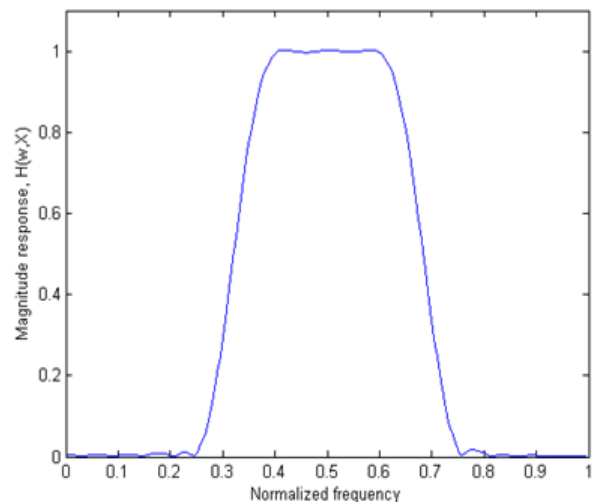


Fig. 4.5: Magnitude response v/s Normalized frequency at filter order 28

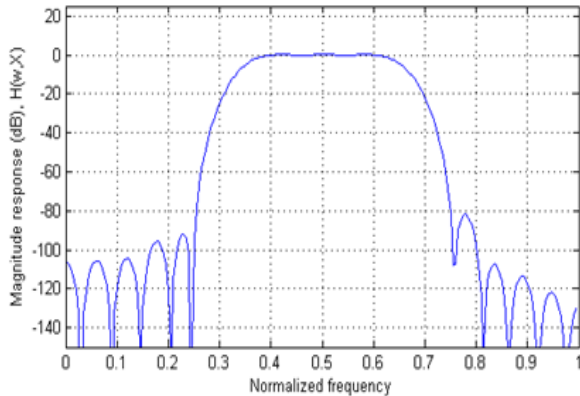


Fig. 4.6: Magnitude response v/s Normalized frequency in db at filter order 28

Fig. 4.6 shows the graph of Magnitude Response with variation in Normalized Frequency in db for Order 28 to design band pass FIR digital filter.

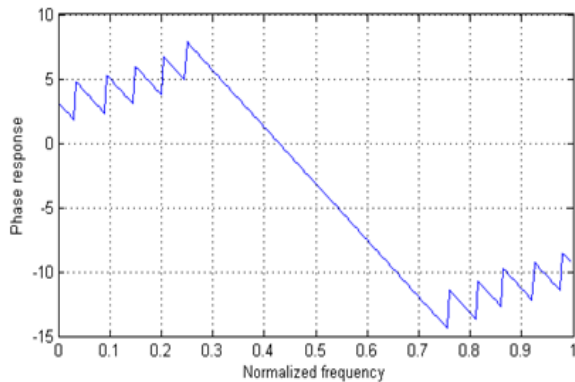


Fig. 4.7: Phase response v/s Normalized frequency at filter order 28

The average and standard deviation values of filter order 28 have been calculated to be:

Table 4.5: Statistical calculation for band-pass FIR digital filter at order 28

Algorithm	PPO	CRPSO [9]
Maximum value of objective function	1.018802	0.877318
Minimum value of objective function	0.785576	0.832798
Average value of objective function	0.83994	0.840409
Standard deviation	0.049757	0.08201

From the above table, it is clear that PPO has less standard deviation as compared to CRPSO. Lower the value of standard deviation, more robust will be the filter.

V. CONCLUSION

In this paper, predator prey optimization technique has been used in order to design band pass FIR digital filter.

Firstly, the filter order for PPO has been varied from 20 - 32 and it is observed that the minimum objective function has been achieved at filter order 28. Thus, band pass FIR digital filter has been designed at filter order 28. In order to get more optimum results at filter order 28, the algorithm parameters such as population size and acceleration constants have been tuned. Population size has been varied between 20- 140 and the best results have been noted at population 40. Then at this population size of 40, acceleration constants (AC_1/AC_2) has been varied between 1.0 - 3.5 and more optimum results have been achieved at $AC_1=AC_2=2.0$ each. The value of standard deviation obtained by choosing these parameters is 0.049757 which is less than 1 that authenticates that the robustness and stability of band pass FIR digital filter. From the simulation results, it is clear that the PPO gives better results in terms of objective function and standard deviation as compared to CRPSO [9] at filter order 28. PPO technique can also be used to design band stop, low pass, high pass FIR digital filters.

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