IJARCCE

ISSN (Online) 2278-1021
ISSN (Print) 2319-5940

International Journal of Advanced Research in Computer and Communication Engineering

Vol. 10, Issue 6, June 2021

DOI 10.17148/IJARCCE.2021.10605

Fermatean Neutrosophic Sets

C. Antony Crispin Sweety[1], R. Jansi[2]

[1]Department of Mathematics, Avinashilingam Institute for Home Science and

Higher Education for Women, Coimbatore, Tamil Nadu, India.

[2]Department of Mathematics, Nirmala College for Women, Coimbatore, Tamil Nadu, India.

Abstract: To deal with the unpredictability of real time challenges, many tools and techniques has been proposed. One of the tools in dealing with imprecision is neutrosophic sets and its combinations. These sets generalize fuzzy sets, intuitionistic fuzzy sets and their extensions with a wider scope of application and thus a motivation for developing various theories. In this paper, the new concept of Fermatean Neutrosophic Sets (FNS) is defined. Further the algebraic properties and set theoretical of the Fermatean Neutrosophic Set is studied.

Keywords: Fermatean fuzzy set, neutrosophic set, Fermatean neutrosophic set.

I. INTRODUCTION

In 1965, A. Zadeh [8] invented fuzzy sets. Only the membership function was addressed in Zadeh's fuzzy set notion, which evolved as a novel tool capable of dealing with uncertainties in real-world scenarios. Following on from the expansions of fuzzy set theory, Atanassov [1] generalised this concept and introduced a new set called intuitionistic fuzzy set (IFS) that can be used to describe the non-membership grade of an imprecise event in addition to its membership grade, with the restriction that the sum of both membership and non-membership grades does not exceed 1. Yager [6] made idea of Pythagorean fuzzy sets. Yager [7] introduced a general class of these sets called q-rung ortho pair fuzzy sets in which the sum of the qth power of the support for and the qth power of the support against is bonded by one. He noted that as q increases the space of acceptable ortho pairs increases and thus gives the user more freedom in expressing their belief about membership grade. When q = 2, Yager have considered q-rung ortho pair fuzzy sets as Pythagorean fuzzy sets. When q = 3, we consider q-rung orthopair fuzzy sets as Fermatean fuzzy sets (FFSs). It was introduced Senapati, Yager [5]. Later Pythagoren fuzzy set was extended to neutrosophic environment by Jansi et.al [3]. In this paper, we propose Fermatean neutrosophic sets and focus on algebraic properties of Fermatean neutrosophic sets.

II. PRELIMINARIES

Definition 2.1 [1]

Let X be a universe. An intuitionistic fuzzy set A on E can be defined as follows:

$$A = \{(x, u_A(x), v_A(x))/x \in X\}$$

Where $u_A: E \to [0,1]$ *and* $v_A: E \to [0,1]$ such that $0 \le u_A(x) + v_A(x) \le 1$ for any $x \in E$.

Here, $u_A(x)$ and $v_A(x)$ is the degree of membership and degree of non-membership of the element x, respectively.

Definition 2.2 [6]

Let X be a non-empty set and I the unit interval [0,1]. A Pythagorean fuzzy set S is an object having the form $A = \{(x, u_A(x), v_A(x))/x \in X\}$ where the functions $u_A: X \to [0,1]$ *and* $v_A: X \to [0,1]$ denote respectively the degree of membership and degree of non-membership of each element $x \in X$ to the set P, and $0 \le (u_A(x))^2 + (v_A(x))^2 \le 1$ for each $x \in X$.

Definition 2.3 [5]

Let X be a non-empty set and I the unit interval [0,1]. A Fermatean fuzzy set S is an object having the form $A = \{(x, u_A(x), v_A(x))/x \in X\}$ where the functions $u_A: X \to [0,1]$ *and* $v_A: X \to [0,1]$ denote respectively the degree of membership and degree of non-membership of each element $x \in X$ to the set P, and $0 \le (u_A(x))^3 + (v_A(x))^3 \le 1$ for each $x \in X$.

Definition 2.4 (Pythagorean Neutrosophic Set) [3]

Let X be a non-empty set (universe). A Pythagorean neutrosophic set [PN Set] A on X is an object of the form:

$$A = \{(x, u_M(x), \zeta_M(x), v_M(x))/x \in X\},$$

Where $u_M(x), \zeta_M(x), v_M(x) \in [0,1]$,

ISSN (Online) 2278-1021
ISSN (Print) 2319-5940

International Journal of Advanced Research in Computer and Communication Engineering

Vol. 10, Issue 6, June 2021

DOI 10.17148/IJARCCE.2021.10605

Then

$$0 \le (u_M(x))^2 + (\zeta_M(x))^2 + (v_M(x))^2 \le 2, \quad for\ all\ x\ in\ X.$$

$u_M(x)$ is the degree of membership, $\zeta_M(x)$ is the degree of inderminancy and $v_M(x)$ is the degree of non-membership .Here $u_M(x)$ *and* $v_M(x)$ are dependent components and $\zeta_M(x)$ is an independent components.

Fermatean Neutrosophic Set and Algebraic properties of Fermatean Neutrosophic set operations

Definition 3.1 (Fermatean Neutrosophic Set)

Let X be a non-empty set (universe). A Fermatean neutrosophic set [FN Set] A on X is an object of the form:

$$A = \{(x, u_M(x), \zeta_M(x), v_M(x))/x \in X\},$$

Where $u_M(x), \zeta_M(x), v_M(x) \in [0,1]$, $0 \le (u_M(x))^3 + (v_M(x))^3 \le 1$ and $0 \le (\zeta_M(x))^3 \le 1$.

Then

$$0 \le (u_M(x))^3 + (\zeta_M(x))^3 + (v_M(x))^3 \le 2, \quad for\ all\ x\ in\ X.$$

$u_M(x)$ is the degree of membership, $\zeta_M(x)$ is the degree of inderminancy and $v_M(x)$ is the degree of non-membership .Here $u_M(x)$ *and* $v_M(x)$ are dependent components and $\zeta_M(x)$ is an independent components.

For understanding the FN Set better, we give to illuminate the understandability of the FN Set: For example, if Truth membership =0.8 and false membership=0.7.We can definitely get $0.8^2 + 0.7^2 > 1$, and therefore, it does not follow the condition of Pythagorean neutrosophic set because their conditions that $0 \le (u_M(x))^2 + (v_M(x))^2 \le 1$ and $0 \le (\zeta_M(x))^2 \le 1$. However, we can get $0.8^3 + 0.7^3 = 0.512 + 0.343 = 0.855 \le 1$, which is good enough to apply the FN Set to control it.

Definition 3.2

Let X be a nonempty set and I the unit interval [0,1]. A Fermatean neutrosophic sets M and N of the form $M = \{(x, u_M(x), \zeta_M(x), v_M(x)) : x \in X\}$ and $N = \{(x, u_N(x), \zeta_N(x), v_N(x)) : x \in X\}$.

Then

1) $M^C = \{(x, v_M(x), 1 - \zeta_M(x), u_M(x)) : x \in X\}$
2) $M \cup N = \{(x, \max(u_M(x), u_N(x)), \min(\zeta_M(x), \zeta_N(x)), \min(v_M(x), v_N(x))) : x \in X\}$
3) $M \cap N = \{(x, \min(u_M(x), u_N(x)), \max(\zeta_M(x), \zeta_N(x)), \max(v_M(x), v_N(x))) : x \in X\}$

Proposition 3.3

(Identity Law)

For any FN set A defined on the absolute FN set X.

(1) $M \cup 0_X = M$
(2) $M \cap 1_X = M$

Proof

(1) Let M be the two FN set.

$$M = \{(x, u_M(x), \zeta_M(x), v_M(x))/ x \in X\}$$

0_X is defined as follows

$$0_X = \{(x, 0,1,1)/ x \in X\}$$

So, $M \cup 0_X = \{(x, \max\{u_M(x), 0), \min\{\zeta_M(x), 1\}, \min\{v_M(x), 1)/x \in X\}$

Therefore, $M \cup 0_X = \{(x, u_M(x), \zeta_M(x), v_M(x))/ x \in X\}$

(2) Proof is similar to (1).

Proposition 3.4

(Domination Law)

For any FN set M defined on the absolute FN set X.

IJARCCE

ISSN (Online) 2278-1021
ISSN (Print) 2319-5940

International Journal of Advanced Research in Computer and Communication Engineering

Vol. 10, Issue 6, June 2021

DOI 10.17148/IJARCCE.2021.10605

(1) $M \cap 0_X = 0_X$
(2) $M \cup 1_X = 1_X$

Proof

(1)Let M be the two FN set.

$$M = \{(x, u_M(x), \zeta_M(x), v_M(x)) / x \in X\}$$

0_X is defined as follows

$$0_X = \{(x, 0,1,1) / x \in X\}$$

So, $M \cap 0_X = \{(x, \min\{u_M(x), 0), \max\{\zeta_M(x), 1\}, \max\{v_M(x), 1)/x \in X\}$

Therefore, $M \cap 0_X = \{(x, 0,1,1) / x \in X\}$

(2) Proof is similar to (1).

Proposition 3.5

(Idempotent Law)
For any FN set M defined on absolute FN set X.

(1) $M \cap M = M$
(2) $M \cup 0_X = M$

Proof

The proof is obvious.

Proposition 3.6

(Commutative Law)
For any FN set M defined on absolute FN set X

(1) $M \cup N = N \cup M$
(2) $M \cap N = N \cap M$

Proof: Proof is obvious.

Proposition 3.7

(Associative Law)
For any FN sets M, N and W defined on absolute FN set X.

(1) $(M \cup N) \cup W = M \cup (N \cup W)$
(2) $(M \cap N) \cap W = M \cap (N \cap W)$

Proof: Proof is obvious

Proposition 3.8

(Distributive Law)
For any FN sets U, N and W defined on absolute FN set X.

ISSN (Online) 2278-1021
ISSN (Print) 2319-5940

International Journal of Advanced Research in Computer and Communication Engineering

Vol. 10, Issue 6, June 2021

DOI 10.17148/IJARCCE.2021.10605

(1) $U \cup (N \cap W) = (U \cup N) \cap (U \cup W)$
(2) $U \cap (N \cup W) = (U \cap N) \cup (U \cap W)$

Proof

Proof is obvious

Proposition 3.9

(Double Complement Law)
For any FN set M defined on absolute FN set X

$(M^c)^c = M$

Proof

Let M be FN set defined as follows:

$$M = \{(x, u_M(x), \zeta_M(x), v_M(x)) / x \in X\}$$

So,

$$M^c = \{(x, v_M(x), 1 - \zeta_M(x), u_M(x)) / x \in X\}$$

$$(M^c)^c = \{(x, u_M(x), 1 - (1 - \zeta_M(x)), v_M(x)) / x \in X\}$$

$$(M^c)^c = \{(x, u_M(x), \zeta_M(x), v_M(x)) / x \in X\}$$

Therefore, $(M^c)^c = M$

Proposition 3.10

(Absorption Law)
For any FN set M and N defined on absolute FN set X

(1) $M \cup (M \cap N) = M$
(2) $M \cap (M \cup N) = M$

Proof

Proof is obvious.

REFERENCES

[1]. K.Atanassov,Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, 20(1986) 87-96.
[2]. K.Atanassov and G.Gargov, Interval-Valued Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems,31 (1989) 343-349.
[3]. R.Jansi, K.Mohana,F.Smarandache, Correlation Measure for Pythagorean Neutrosophic Sets with T and F as Dependent Neutrosophic Components, Neutrosophic sets and system, 30 (2019),202-212.
[4]. R.Jansi, K.Mohana, Tangent and Cotangent Similarity Measures of Pythagorean Neutrosophic Sets with T and F are Dependent Neutrosophic Components, Infokara Research ,8 (2019),657-671.
[5]. Senapati T, Yager RR (2019a) Fermatean fuzzy sets. J Ambient Intell Hum Computi. https://doi.org/10.1007/s12652-019-01377-0
[6]. Yager RR (2014) Pythagorean membership grades in multicriteria decision making. IEEE Trans Fuzzy Syst 22:958–965
[7]. Yager RR (2017) Generalized orthopair fuzzy sets. IEEE Trans Fuzzy Syst 25(5):1222–1230
[8]. L. A. Zadeh, Fuzzy Sets, Inform. and Control, 8(1965), 338- 353.