

# REGULA-FALSI METHOD

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**ABSTRACT:** This research paper gives us the information about ‘Regula-Falsi Method’. This method helps us to find the roots of transcendental and polynomial equations. It is closed bracket method and closely resembles the bisection method. The method of false position provides an exact solutions for linear functions.

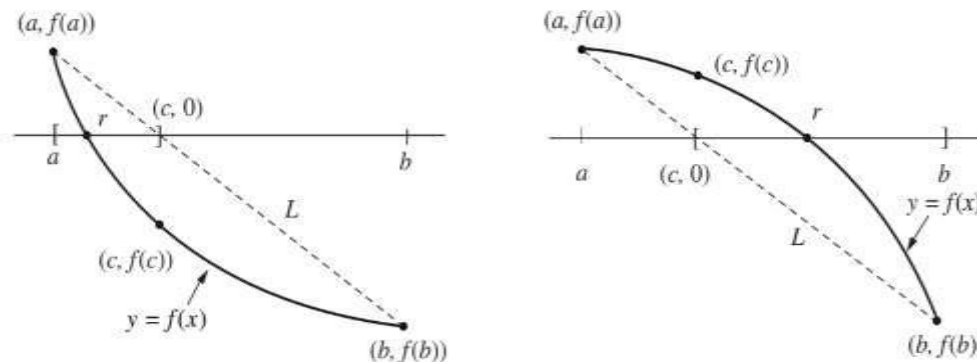
## INTRODUCTION

Regula Falsi method or the method of false position is a numerical method for solving an equation in one unknown. It is quite similar to bisection method algorithm and is one of the oldest approaches. The convergence process in the bisection method is very slow. It depends only on the choice of end points of the interval  $[a, b]$ . The function  $f(x)$  does not have any role in finding the point  $c$  (which is just the midpoint of  $a$  and  $b$ ). It is used only to decide the next smaller interval  $[a, c]$  or  $[c, b]$ . A better approximation to  $c$  can be obtained by taking the straight line  $L$  joining the points  $(a, f(a))$  and  $(b, f(b))$  intersecting the  $x$ -axis. To obtain the value of  $c$  we can equate the two expressions of the slope  $m$  of the line  $L$ .

## REGULA FALSI METHOD –

### THEORY

As before (in bisection method), for a given continuous function  $f(x)$  we assume that  $f(a)$  and  $f(b)$  have opposite signs ( $a$  = lower guess,  $b$  = upper guess). This condition satisfies the Balzano’s Theorem for continuous function. Now after this bisection method used the midpoint of the interval  $[a, b]$  as the next iterate to converge



(a) If  $f(a)$  and  $f(c)$  have opposite signs, then squeeze from the right.

(b) If  $f(c)$  and  $f(b)$  have opposite signs, then squeeze from the left.

towards the root of  $f(x)$ . A better approximation is obtained if we find the point  $(c, 0)$  where the secant line  $L$  joining the points  $(a, f(a))$  and  $(b, f(b))$  crosses the  $x$ -axis. To find the value  $c$ , we

Using points  $(a, f(a))$  and  $(b, f(b))$  :

$$m = \frac{f(b) - f(a)}{b - a} \quad (1)$$

and Using points  $(c, 0)$  and  $(b, f(b))$  :

$$m = \frac{0 - f(b)}{c - b} \quad (2)$$

Equating equation (1) and (2) we get:

$$\frac{f(b) - f(a)}{b - a} = \frac{0 - f(b)}{c - b} \quad (3)$$

Solving for  $c$  we get:

$$c = b - \frac{f(b)(b - a)}{f(b) - f(a)} = \frac{af(b) - bf(a)}{f(b) - f(a)} \quad (4)$$

write down two versions of the slope  $m$  of the line  $L$ :

Now the next smaller interval which brackets the root can be obtained by checking

$$\begin{aligned} f(a) * f(b) < 0 \text{ then } b = c \\ > 0 \text{ then } a = c \end{aligned}$$

= 0 then  $c$  is the root.

Selecting  $c$  by the above expression is called Regula-Falsi method or False position method.

#### ALGORITHM

\*Given a continuous function  $f(x)$

1. Find points  $a$  and  $b$  such that  $a < b$  and  $f(a) * f(b) < 0$ .
2. Take the interval  $[a, b]$  and determine the next value of  $x_1$ .
3. If  $f(x_1) = 0$  then  $x_1$  is an exact root, else if  $f(x_1) * f(b) < 0$  then let  $a = x_1$ , else if  $f(a) * f(x_1) < 0$  then let  $b = x_1$ .
4. Repeat steps 2 & 3 until  $f(x_i) = 0$  or  $|f(x_i)| \square DOA$ , where **DOA** stands for **degree of accuracy**.



NUMERICAL –

**Example-**  $f(x)=2x^3-2x-5$

**Find a root of an equation  $f(x)=2x^3-2x-5$  using False Position method (regula falsi method)**

**Solution:**

Here  $2x^3-2x-5=0$

Let  $f(x)=2x^3-2x-5$

Here

|        |    |    |   |
|--------|----|----|---|
| $x$    | 0  | 1  | 2 |
| $f(x)$ | -5 | -5 | 7 |

1st iteration :

Here  $f(1)=-5<0$  and  $f(2)=7>0$

$\therefore$  Now, Root lies between  $x_0=1$  and  $x_1=2$

$$x_2 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x_2 = 1 - (-5) \cdot \frac{2 - 1}{7 - (-5)}$$

$$x_2 = 1.41667$$

$$f(x_2) = f(1.41667) = 2 \cdot 1.41667^3 - 2 \cdot 1.41667 - 5 = -2.14699 < 0$$

2nd iteration :

Here  $f(1.41667) = -2.14699 < 0$  and  $f(2) = 7 > 0$

$\therefore$  Now, Root lies between  $x_0=1.41667$  and  $x_1=2$

$$x_3 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x_3 = 1.41667 - (-2.14699) \cdot \frac{2 - 1.41667}{7 - (-2.14699)}$$

$$x_3 = 1.55359$$

$$f(x_3) = f(1.55359) = 2 \cdot 1.55359^3 - 2 \cdot 1.55359 - 5 = -0.60759 < 0$$

3rd iteration :

Here  $f(1.55359) = -0.60759 < 0$  and  $f(2) = 7 > 0$

$\therefore$  Now, Root lies between  $x_0=1.55359$  and  $x_1=2$

$$x_4 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x_4 = 1.55359 - (-0.60759) \cdot \frac{2 - 1.55359}{7 - (-0.60759)}$$

$$x_4 = 1.58924$$



$$f(x_4)=f(1.58924)=2 \cdot 1.58924^3 - 2 \cdot 1.58924 - 5 = -0.15063 < 0$$

4th iteration :

$$\text{Here } f(1.58924) = -0.15063 < 0 \text{ and } f(2) = 7 > 0$$

$\therefore$  Now, Root lies between  $x_0 = 1.58924$  and  $x_1 = 2$

$$x_5 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x_5 = 1.58924 - (-0.15063) \cdot \frac{2 - 1.58924}{7 - (-0.15063)}$$

$$x_5 = 1.59789$$

$$f(x_5)=f(1.59789)=2 \cdot 1.59789^3 - 2 \cdot 1.59789 - 5 = -0.0361 < 0 \text{ 5th iteration :}$$

$$\text{Here } f(1.59789) = -0.0361 < 0 \text{ and } f(2) = 7 > 0$$

$\therefore$  Now, Root lies between  $x_0 = 1.59789$  and  $x_1 = 2$

$$x_6 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x_6 = 1.59789 - (-0.0361) \cdot \frac{2 - 1.59789}{7 - (-0.0361)}$$

$$x_6 = 1.59996$$

$$f(x_6)=f(1.59996)=2 \cdot 1.59996^3 - 2 \cdot 1.59996 - 5 = -0.00858 < 0$$

6th iteration :

$$\text{Here } f(1.59996) = -0.00858 < 0 \text{ and } f(2) = 7 > 0$$

$\therefore$  Now, Root lies between  $x_0 = 1.59996$  and  $x_1 = 2$

$$x_7 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x_7 = 1.59996 - (-0.00858) \cdot \frac{2 - 1.59996}{7 - (-0.00858)}$$

$$x_7 = 1.60045$$

$$f(x_7)=f(1.60045)=2 \cdot 1.60045^3 - 2 \cdot 1.60045 - 5 = -0.00203 < 0$$



7th iteration :

Here  $f(1.60045)=-0.00203<0$  and  $f(2)=7>0$

$\therefore$  Now, Root lies between  $x_0=1.60045$  and  $x_1=2$

$$x_8 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x_8 = 1.60045 - (-0.00203) \cdot \frac{2 - 1.60045}{7 - (-0.00203)}$$

$$x_8 = 1.60056$$

$$f(x_8) = f(1.60056) = 2 \cdot 1.60056^3 - 2 \cdot 1.60056 - 5 = -0.00048 < 0$$

Approximate root of the equation  $2x^3 - 2x - 5 = 0$  using False Position method is 1.60056

| $n$ | $x_0$   | $f(x_0)$ | $x_1$ | $f(x_1)$ | $x_2$   | $f(x_2)$ | Update      |
|-----|---------|----------|-------|----------|---------|----------|-------------|
| 1   | 1       | -5       | 2     | 7        | 1.41667 | -2.14699 | $x_0 = x_2$ |
| 2   | 1.41667 | -2.14699 | 2     | 7        | 1.55359 | -0.60759 | $x_0 = x_2$ |
| 3   | 1.55359 | -0.60759 | 2     | 7        | 1.58924 | -0.15063 | $x_0 = x_2$ |
| 4   | 1.58924 | -0.15063 | 2     | 7        | 1.59789 | -0.0361  | $x_0 = x_2$ |
| 5   | 1.59789 | -0.0361  | 2     | 7        | 1.59996 | -0.00858 | $x_0 = x_2$ |
| 6   | 1.59996 | -0.00858 | 2     | 7        | 1.60045 | -0.00203 | $x_0 = x_2$ |
| 7   | 1.60045 | -0.00203 | 2     | 7        | 1.60056 | -0.00048 | $x_0 = x_2$ |

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