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A Study of Vectors and Link Utilization of Hypercube

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Abstract: This paper presents an efficient analytical approach to study the performance of interconnection networks namely, Hypercube and Perfect Difference Network. The performance measure has been defined as Link utilization. As the number of processors increases in a system, the processing speed increases. A threshold is reached after which the increase in the number of processors decrease the utilization of the processors as they spend most of their time in communicating the messages. We have compared the Link utilization and topological properties of hypercube and perfect difference network with some simplifying assumptions. The assumptions include that the links are overlap (1-2,2-1) at which a processor can communicate parallel with adjacent processor. Both hypercube and perfect difference network are regular, vertex symmetric and edge symmetric.

Keywords- Interconnection Network, Multiprocessors, Hypercube, Perfect Difference Network, Link Utilization.

INTRODUCTION:

Hypercube [1] and PDN [1,2] based interconnection network have been utilized extensively in the design of parallel computer in recent years. Link utilization has always strived to increase the performance of their system. High performance may come from fast dense circuitry, parallelism and some technology [2,3]. Connectivity between processors is defined by the interconnection network used to communicate each other. As the density of processor package increase, the length of the links connecting a certain number of processors decreased. Some of the more common interconnection networks are: two dimensional mesh, ring, tree, perfect difference network and hypercube. The first three are intuitive while the Fourth and fifth needs some elaboration[1,3,4].

HYPERCUBE:

In a hypercube of dimension d, there are 2d processors. Assume that these are labeled 0 , 1, . . . , 2d - 1. Two processors i and j are directly connected iff the binary representations of i and j differ in exactly one bit. Thus in a hypercube of dimension d, each processor is connected to d others[1,4,5]. If the direct connection between a pair of processors i and j is unidirectional, then at any given time messages can flow from either i to j or from j to i. In the case of bidirectional connections, it is possible for i to send a message to j and for j to simultaneously send one to i. The popularity of the hypercube network may be attributed to the following [1,6]:

(1) Using d connections per processor, 2d processors may be interconnected such that the maximum distance between any two processors is d. While meshes, rings, and binary trees use a smaller number of connections per processor, the maximum distance between processors is larger. It is interesting to note that other networks such as the star graph (Akers, Harel, and Krishnamurthy 1987) do better than a hypercube in this regard. A star graph connects (d +1)! Processors using d connections per processor. The hypercube has the advantage of being a well studied network while the star graph is relatively new and few algorithms have been developed for it.

2) Most other popular networks are easily mapped into a hypercube. For example a 2 × 4 mesh, 8 node ring, and a 7 node full binary tree may be mapped into an 8 node hypercube.

(3) A hypercube is completely symmetric. Every processor's interconnection pattern is like that of every other processor. Furthermore, a hypercube is completely decomposable into sub-hypercube (i.e., hypercube of smaller dimension). This property makes it relatively easy to implement recursive divide-and-conquer algorithms on the hypercube [1,6,7].

STUDY OF LINK UTILIZATION OF HYPERCUBE AND PDN

The hypercube graph Qq is an undirected regular graph with 2^q vertices, where each vertex corresponds to a binary string of length q. Two vertices labeled by strings x and y are joined by an edge if and only if x can be obtained from y by changing a single bit. As usual, the number of nodes is denoted $n = 2^q$[1,7].

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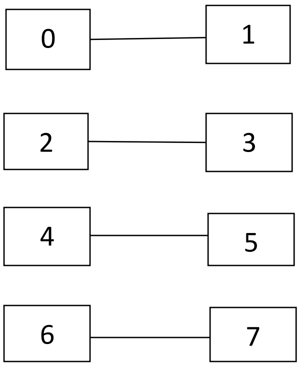
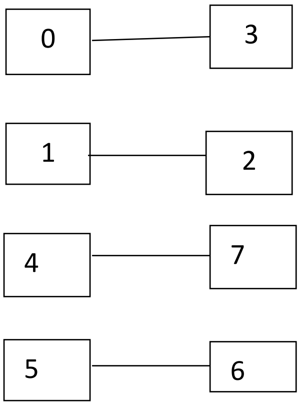
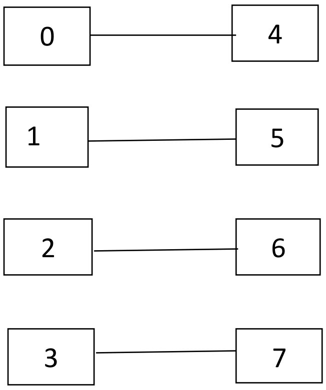
Horizontal connectivity of nodes:



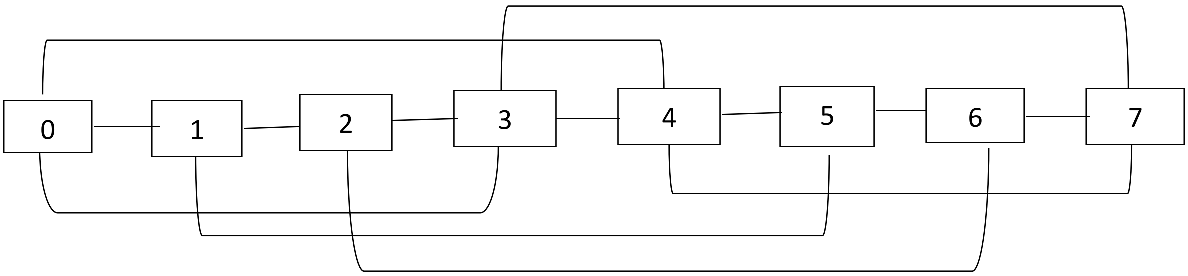
Vertical connectivity of nodes:



Diagonal connectivity of nodes:



Overall connectivity of nodes:



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PERFECT DIFFERENCE NETWORK

The Perfect Difference set[4,7,8] of each node of the PDN can be evaluated by the remainder theorem i.e.(N= R+ D * Q)
Where N= Numerator, R=Remainder, D= Denominator and Q=Quotient
The above equation can be written as Integer = (Si-Sj) +(δ^2+ δ+ 1)*1
Where integer is a member of the set (1, 2,…, δ^2+ δ) and Si-Sj is numerator or the difference set.So we can write as-
(Si-Sj) = (integer) mod δ^2+ δ+ 1[2,5,6]
In the due case of study we are assuming that a node is connected to itself therefore the node is self connected in PDN.
The following is the connectivity relation[6,8] between nodes of a PDN.

□ i ± 1 (0<i< $\delta^2 + \delta$)
□ i ± Sj (mod n) for 2 ≤ j ≤ δ

	No of Nodes	No of links	Degree	Diameter
Hypercube	2^n	$n.2^{n-1}$	N	n
PDN	$\delta^2+ \delta + 1$	$2\delta(\delta^2+ \delta + 1)$	2δ	2

Lemma 1: Half of Hypercube is mirror image of the remaining half of the hypercube.
Proof:
In order to prove half of the hypercube is mirror image of the second half of the hypercube. Let n the total number of nodes in hypercube. We divided into two parts
i.e. I = int(n/2)
where 0 ≤ I ≤ n and n is the total number of nodes in hypercube.
پ 2 i = n
پ i+I = n
= i = n – i
= n – i = i - n + n
= (n-i) = (i-n) + n
= (i-n) = (i-n) mod n => According to the remainder theorem
But here

- The value of quotient will be 1.
- The value of quotient will depend upon the value of i.

Hence the above equation can be written as
(n-i) mod n ……………………………(1)
Eqn 1 is called the mirror image equation of the hypercube

Let d=3, total nodes = 8 naming 0 to 7,and The relation matrix of 3D hypercube is represent as follows:

Node "0" ->	0	1	1	0	1	0	0	0
Node "1" ->	1	0	0	1	0	1	0	0
Node "2" ->	1	0	0	1	0	0	1	0
Node "3" ->	0	1	1	0	0	0	0	1
Node "4" ->	1	0	1	0	0	1	0	0
Node "5" ->	0	1	0	0	1	0	0	1
Node "6" ->	0	0	1	0	1	0	0	1
Node "7" ->	0	0	0	1	0	1	1	0

Let i=0
Vector of n_0 -> 0 1 1 0 1 0 0 0

= (n-i) mod n
= (7-0) mod 7
=7
Vector of n_7
0 0 0 1 0 1 1 0

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i.e. Vector 7 is the mirror image of vector 0 in hypercube
Now its reciprocal , when i=7 and 8 nodes then we have
Let i=7
= (n-i) mod n
= (7-7) mod 7
=0

Let i=1
= (n-i) mod n
= (7-1) mod 7
=6
i.e. Vector 6 is the mirror image of vector 1 in hypercube
Now its reciprocal, when i=6 and 8 nodes then we have
Let i=6
= (n-i) mod n
= (7-6) mod 7
=1.
Hence it is proved.

STUDY OF VECTORS OPERATIONS BETWEEN NODES USING LOGICAL OPERATIONS

In this section we are exploring the bitwise connection between the nodes of a interconnection network. We are taking hypercube as a model; First of all we are converting the interconnection network into its bitwise representation. [6, 7, 8] Then bitwise representation of node is used to present the value of a particular node of interconnection network. Each bitwise vector shows connectivity with another node in position of bit 1.The bitwise representation also shows the mathematical property of hypercube, it means the value of bitwise representation of a node in the data structure preserves the mathematical property of the topology [5,8]. Exclusive-or property between nodes is shown below:

000(0)	000(0)	000(0)	100(4)	100(4)	100(4)
001(1)	011(3)	100(4)	000(0)	101(5)	111(7)
001(1)	011(3)	100(4)	100(4)	001(1)	011(3)
001(1)	001(1)	001(1)	101(5)	101(5)	101(5)
000(0)	010(2)	101(5)	001(1)	100(4)	110(6)
001(1)	011(3)	100(4)	100(4)	001(1)	011(3)
010(2)	010(2)	010(2)	110(6)	110(6)	110(6)
011(1)	011(3)	110(6)	010(2)	101(5)	111(7)
011(3)	001(1)	100(4)	100(4)	011(3)	011(1)
011(3)	011(3)	011(3)	111(7)	111(7)	111(70
000(0)	010(2)	111(7)	011(3)	100(4)	110(6)
011(3)	001(1)	100(4)	100(4)	011(3)	001(1)

Applying ex-or property parallelly

011(3)	000(0)	111(7)	100(4)
010(2)	001(1)	110(6)	101(5)
001(1)	001(1)	001(1)	001(1)
000(0)	010(2)	111(7)	110(6)
011(3)	001(1)	100(4)	101(5)
011(3)	011(3)	011(3)	011(3)
110(6)	111(7)	101(5)	000(0)

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101(2)	011(3)	001(1)	100(4)
100(4)	100(4)	100(4)	100(4)

CONCLUSION AND FUTURE WORK:

Link utilization of hypercube is elaborated for interconnection network. We have also discussed the communication pattern and logical operations and have derived a new relation between nodes. It has been proved that the half of the hypercube is mirror image of other remaining node of hypercube. Our research aims at delineate link utilization using topological properties so as to elucidate the connectivity aspect of hypercube. In prefatory illustration of topological properties of PDN and hypercube through vectors and binary representation gives a blueprint for modeling the nodes relationship and algorithmic development for the same. Topological properties is fundamental pillar of interconnection network.

Presently we are trying to derived some relations between nodes to improve architecture further we may develop communication algorithm and to make it more suitable for highly parallel system.

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