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LOGARITHM

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Abstract: In mathematics, the logarithm is the inverse function to exponentiation. That means the logarithm of a given number x is the exponent to which another fixed number, the base b, must be raised, to produce that number x. In the simplest case, the logarithm counts the number of occurrences of the same factor in repeated multiplication.

I. INTRODUCTION

The logarithm base 10 (that is b = 10) is called the decimal or common logarithm and is commonly used in science and engineering. The natural logarithm has the number e (that is b ≈ 2.718) as its base; its use is widespread in mathematics and physics, because of its simpler integral and derivative. The binary logarithm uses base 2 (that is b = 2) and is frequently used in computer science.

Logarithms were introduced by John Napier in 1614 as a means of simplifying calculations. They were rapidly adopted by navigators, scientists, engineers, surveyors and others to perform high-accuracy computations more easily. Using logarithm tables, tedious multi-digit multiplication steps can be replaced by table look-ups and simpler addition

II. DEFINITION

The logarithm of a positive real number x with respect to base b] is the exponent by which b must be raised to yield x. In other words, the logarithm of x to base b is the unique real number y such that.
The logarithm is denoted "$\log_b x$" (pronounced as "the logarithm of x to base b", "the base-b logarithm of x", or most commonly "the log, base b, of x").
An equivalent and more succinct definition is that the function $\log_b$ is the inverse function to the function.
Example:
$\log_2 16 = 4$, since $24 = 2 \times 2 \times 2 \times 2 = 16$.

III. LOGARITHMIC IDENTITIES

Product, quotient, power, and root

The logarithm of a product is the sum of the logarithms of the numbers being multiplied; the logarithm of the ratio of two numbers is the difference of the logarithms. The logarithm of the p-th power of a number is p times the logarithm of the number itself; the logarithm of a p-th root is the logarithm of the number divided by p.

Logarithmic laws

Products:	$\log_b mn$	=	$\log_b m + \log_b n$
Ratios:	$\log_b \frac{m}{n}$	=	$\log_b m - \log_b n$
Powers:	$\log_b n^p$	=	$p \log_b n$
Roots:	$\log_b \sqrt[q]{n}$	=	$\frac{1}{q} \log_b n$
Change of bases:	$\log_b n$	=	$\log_a n \log_b a$

Change of base RULE:

The logarithm logb x can be computed from the logarithms of x and b with respect to an arbitrary base k using the following formula:

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Change of Base Formula

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$$\log_b a = \frac{\log_c a}{\log_c b}$$

(OR)

$$\log_b a . \log_c b = \log_c a$$

Typical scientific calculators calculate the logarithms to bases 10 and e
Logarithms with respect to any base b can be determined using either of these two logarithms by the previous formula.

IV. HISTORY

The history of logarithms in seventeenth-century Europe is the discovery of a new function that extended the realm of analysis beyond the scope of algebraic methods. The method of logarithms was publicly propounded by John Napier in 1614, in a book titled Mirifici Logarithmorum Canonis Descriptio (Description of the Wonderful Rule of Logarithms)

Logarithm tables, slide rules, and historical applications

By simplifying difficult calculations before calculators and computers became available, logarithms contributed to the advance of science, especially astronomy. They were critical to advances in surveying, celestial navigation, and other domains. Pierre-Simon Laplace called logarithms.

V. LOG TABLES

A key tool that enabled the practical use of logarithms was the table of logarithms.

The first such table was compiled by Henry Briggs in 1617, immediately after Napier's invention but with the innovation of using 10 as the base. Briggs' first table contained the common logarithms of all integers in the range from 1 to 1000, with a precision of 14 digits.

Subsequently, tables with increasing scope were written. These tables listed the values of log10 x for any number x in a certain range, at a certain precision. Base-10 logarithms were universally used for computation, hence the name common logarithm, since numbers that differ by factors of 10 have logarithms that differ by integers. The common logarithm of x can be separated into an integer part and a fractional part, known as the characteristic and mantissa. Tables of logarithms need only include the mantissa, as the characteristic can be easily determined by counting digits from the decimal point.

VI. COMPUTATIONS

The product and quotient of two positive numbers c and d were routinely calculated as the sum and difference of their logarithms.

For manual calculations that demand any appreciable precision, performing the lookups of the two logarithms, calculating their sum or difference, and looking up the antilogarithm is much faster than performing the multiplication by earlier methods such as prosthaphaeresis, which relies on trigonometric identities.
Calculations of powers and roots are reduced to multiplications or divisions
Trigonometric calculations were facilitated by tables that contained the common logarithms of trigonometric functions.

VII. SLIDE RULES

Another critical application was the slide rule, a pair of logarithmically divided scales used for calculation. The non-sliding logarithmic scale, Gunter's rule, was invented shortly after Napier's invention. William Oughtred enhanced it to create the slide rule—a pair of logarithmic scales movable with respect to each other.

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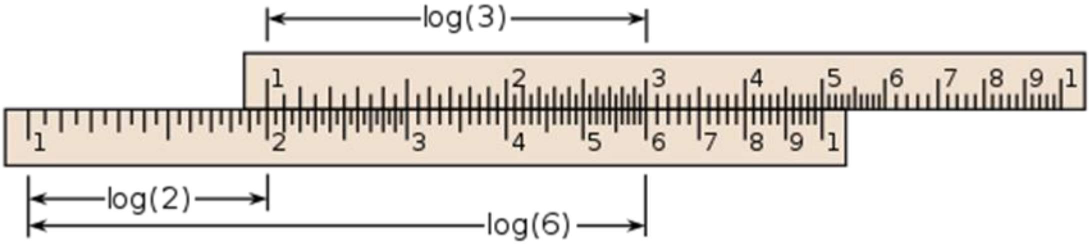
Numbers are placed on sliding scales at distances proportional to the differences between their logarithms. Sliding the upper scale appropriately amounts to mechanically adding logarithms, as illustrated here:

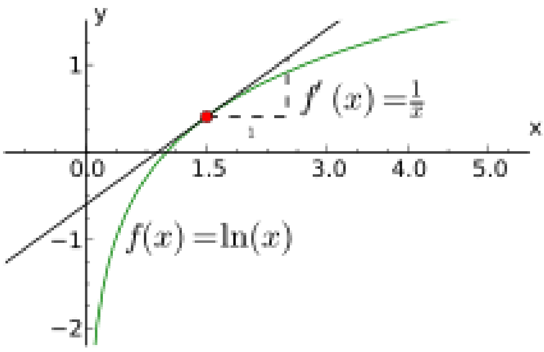


Schematic depiction of a slide rule. Starting from 2 on the lower scale, add the distance to 3 on the upper scale to reach the product 6. The slide rule works because it is marked such that the distance from 1 to x is proportional to the logarithm of x.
For example, adding the distance from 1 to 2 on the lower scale to the distance from 1 to 3 on the upper scale yields a product of 6, which is read off at the lower part. The slide rule was an essential calculating tool for engineers and scientists until the 1970s, because it allows, at the expense of precision, much faster computation than techniques based on tables.

VIII. CHARACTERIZATION BY THE PRODUCT FORMULA

The function $\log_b x$ can also be essentially characterized by the product formula
Log (xy)= log x + log y

IX. DERIVATIVE AND ANTIDERIVATIVE



The graph of the natural logarithm (green) and its tangent at x = 1.5

Analytic properties of functions pass to their inverses. Thus, as $f(x) = b^x$ is a continuous and differentiable function, so is $\log_b y$. Roughly, a continuous function is differentiable if its graph has no sharp "corners". Moreover, as the derivative of f(x) evaluates to $\ln(b)\, b^x$ by the properties of the exponential function, the chain rule implies that the derivative of $\log_b x$

That is, the slope of the tangent touching the graph of the base-b logarithm at the point (x, logb (x)) equals 1/(x ln(b)).
The derivative of ln(x) is 1/x; this implies that ln(x) is the unique antiderivative of 1/x that has the value 0 for x = 1. It is this very simple formula that motivated to qualify as "natural" the natural logarithm; this is also one of the main reasons of the importance of the constant e.
The derivative with a generalized functional argument f(x) is
The quotient at the right hand side is called the logarithmic derivative of f. Computing f'(x) by means of the derivative of ln(f(x)) is known as logarithmic differentiation.[39] The antiderivative of the natural logarithm ln(x) is:

Related formulas, such as antiderivatives of logarithms to other bases can be derived from this equation using the change of bases.

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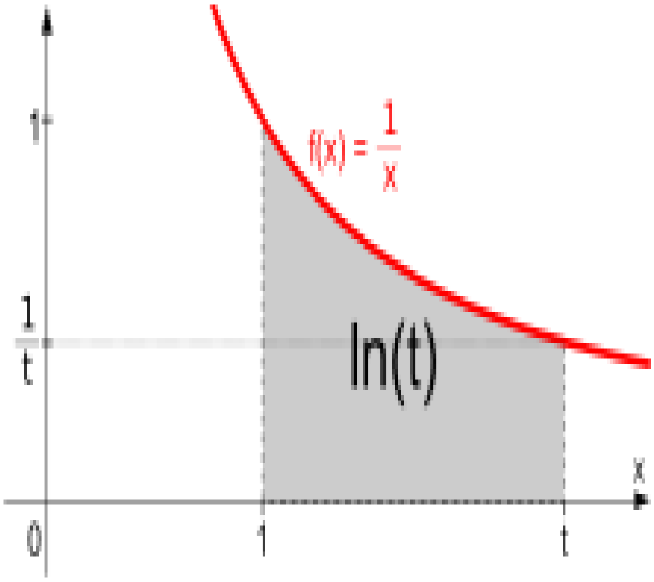
X. INTEGRAL REPRESENTATION OF THE NATURAL LOGARITHM

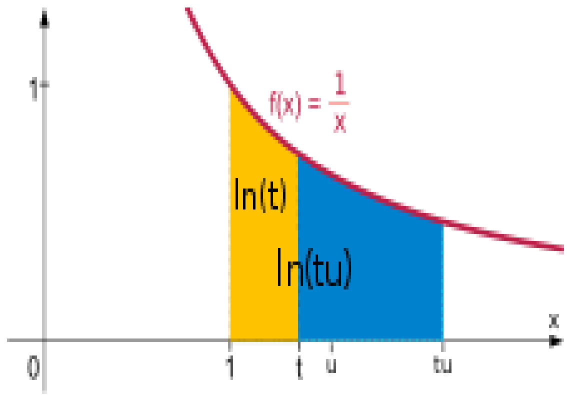


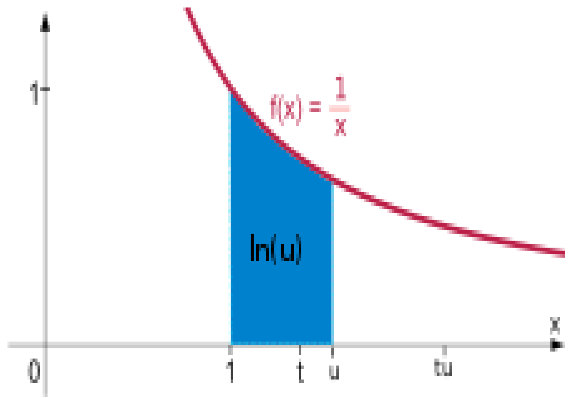
The natural logarithm of t is the shaded area underneath the graph of the function f(x) = 1/x (reciprocal of x).

The natural logarithm of t can be defined as the definite integral:

This definition has the advantage that it does not rely on the exponential function or any trigonometric functions; the definition is in terms of an integral of a simple reciprocal. As an integral, ln(t) equals the area between the x-axis and the graph of the function 1/x, ranging from x = 1 to x = t. This is a consequence of the fundamental theorem of calculus and the fact that the derivative of ln(x) is 1/x. Product and power logarithm formulas can be derived from this definition.[42] For example, the product formula ln(tu) = ln(t) + ln(u) is deduced as:

The equality (1) splits the integral into two parts, while the equality (2) is a change of variable (w = x/t). In the illustration below, the splitting corresponds to dividing the area into the yellow and blue parts. Rescaling the left hand blue area vertically by the factor t and shrinking it by the same factor horizontally does not change its size. Moving it appropriately, the area fits the graph of the function f(x) = 1/x again. Therefore, the left hand blue area, which is the integral of f(x) from t to tu is the same as the integral from 1 to u. This justifies the equality (2) with a more geometric proof.





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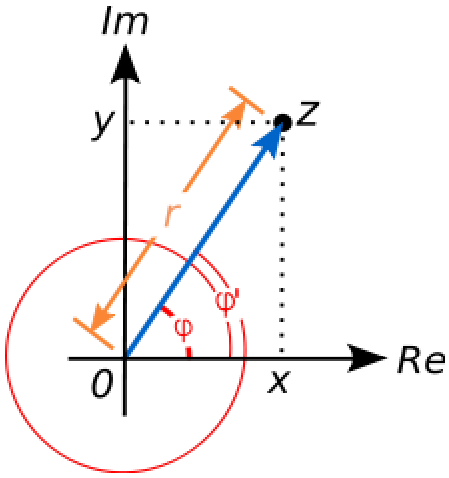
XI. COMPLEX LOGARITHM

Main article: Complex logarithm:



Polar form of z = x + iy. Both φ and φ' are arguments of z.

All the complex numbers a that solve the equation

$e^a = z$

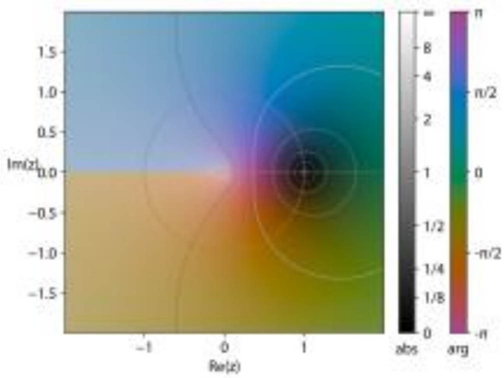
are called complex logarithms of z, when z is (considered as) a complex number. A complex number is commonly represented as z = x + iy, where x and y are real numbers and i is an imaginary unit, the square of which is −1. Such a number can be visualized by a point in the complex plane, as shown at the right. The polar form encodes a non-zero complex number z by its absolute value, that is, the (positive, real) distance r to the origin, and an angle between the real (x) axis Re and the line passing through both the origin and z. This angle is called the argument of z.
The absolute value r of z is given by

$r = \sqrt{(x^2\ ) + y^2}$

Using the geometrical interpretation of sine and cosine and their periodicity in 2π, any complex number z may be denoted as

Z = x + iy = r (cos α + sin α)

for any integer number k. Evidently the argument of z is not uniquely specified: both φ and φ' = φ + 2kπ are valid arguments of z for all integers k, because adding 2kπ radians or k⋅360° to φ corresponds to "winding" around the origin counter-clock-wise by k turns. The resulting complex number is always z, as illustrated at the right for k = 1. One may select exactly one of the possible arguments of z as the so-called principal argument, denoted Arg(z), with a capital A, by requiring φ to belong to one, conveniently selected turn, e.g. −π < φ ≤ π or 0 ≤ φ < 2π. These regions, where the argument of z is uniquely determined are called branches of the argument function.



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The principal branch (-π, π) of the complex logarithm, Log(z). The black point at z = 1 corresponds to absolute value zero and brighter colors refer to bigger absolute values. The hue of the color encodes the argument of Log(z).

Euler's formula connects the trigonometric functions sine and cosine to the complex exponential:

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