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PROBABILITY & IT'S DISTRIBUTION

Mrs. Anagha A. Bade

Lecturer in Bharati Vidyapeeth Institute of Technology , Navi Mumbai

Probability theory is the branch of mathematics concerned with probability. Although there are several different probability interpretations, probability theory treats the concept in a rigorous mathematical manner by expressing it through a set of axioms. Typically these axioms formalise probability in terms of a probability space, which assigns a measure taking values between 0 and 1, termed the probability measure, to a set of outcomes called the sample space.

Any specified subset of the sample space is called an event. Central subjects in probability theory include discrete and continuous random variables, probability distributions, and stochastic processes (which provide mathematical abstractions of non-deterministic or uncertain processes or measured quantities that may either be single occurrences or evolve over time in a random fashion). Although it is not possible to perfectly predict random events, much can be said about their behavior. Two major results in probability theory describing such behaviour are the law of large numbers and the central limit theorem.

As a mathematical foundation for statistics, probability theory is essential to many human activities that involve quantitative analysis of data. Methods of probability theory also apply to descriptions of complex systems given only partial knowledge of their state, as in statistical mechanics or sequential estimation. A great discovery of twentieth-century physics was the probabilistic nature of physical phenomena at atomic scales, described in quantum mechanics.

Consider an experiment that can produce a number of outcomes. The set of all outcomes is called the *sample space* of the experiment. The *power set* of the sample space (or equivalently, the event space) is formed by considering all different collections of possible results. For example, rolling an honest dice produces one of six possible results. One collection of possible results corresponds to getting an odd number. Thus, the subset {1,3,5} is an element of the power set of the sample space of dice rolls. These collections are called *events*. In this case, {1,3,5} is the event that the dice falls on some odd number. If the results that actually occur fall in a given event, that event is said to have occurred.
Probability is a way of assigning every "event" a value between zero and one, with the requirement that the event made up of all possible results (in our example, the event {1,2,3,4,5,6}) be assigned a value of one. To qualify as a probability distribution, the assignment of values must satisfy the requirement that if you look at a collection of mutually exclusive events (events that contain no common results, e.g., the events {1,6}, {3}, and {2,4} are all mutually exclusive), the probability that any of these events occurs is given by the sum of the probabilities of the events.[7]

The probability that any one of the events {1,6}, {3}, or {2,4} will occur is 5/6. This is the same as saying that the probability of event {1,2,3,4,6} is 5/6. This event encompasses the possibility of any number except five being rolled. The mutually exclusive event {5} has a probability of 1/6, and the event {1,2,3,4,5,6} has a probability of 1, that is, absolute certainty.
When doing calculations using the outcomes of an experiment, it is necessary that all those elementary events have a number assigned to them. This is done using a random variable. A random variable is a function that assigns to each elementary event in the sample space a real number. This function is usually denoted by a capital letter. In the case of a dice, the assignment of a number to certain elementary events can be done using the identity function. This does not always work.

For example, when flipping a coin the two possible outcomes are "heads" and "tails". ***Discrete probability*** *theory* deals with events that occur in countable sample spaces.

Examples: Throwing dice, experiments with decks of cards, random walk, and tossing coins.

Classical definition: Initially the probability of an event to occur was defined as the number of cases favorable for the event, over the number of total outcomes possible in an equiprobable sample space.

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Continuous probability *theory* deals with events that occur in a continuous sample space.
In probability theory and statistics, a **probability distribution** is the mathematical function that gives the probabilities of occurrence of different possible **outcomes** for an experiment. It is a mathematical description of a random phenomenon in terms of its sample space and the probabilities of events .

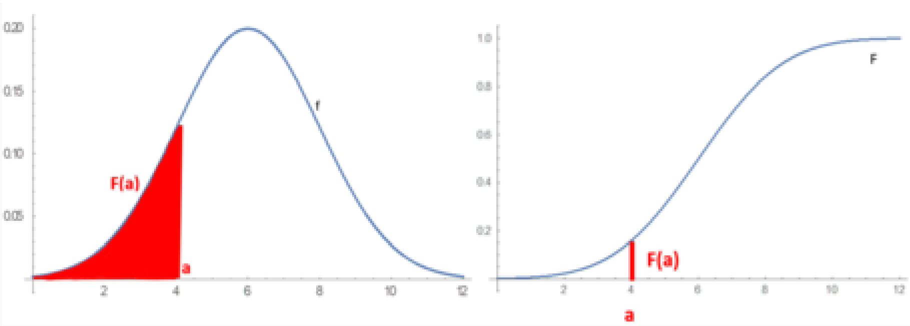
A probability distribution is a mathematical description of the probabilities of events, subsets of the sample space. The sample spaceis the set of all possible outcomes of a random phenomenon being observed; it may be any set: a set of real numbers, a set of vectors, a set of arbitrary non-numerical values, etc.

For example, the sample space of a coin flip would be Ω = {heads, tails}.

To define probability distributions for the specific case of random variables (so the sample space can be seen as a numeric set), it is common to distinguish between **discrete** and **absolutely continuous** random variables.

In the discrete case, it is sufficient to specify a probability mass function assigning a probability to each possible outcome: for example, when throwing a fair dice, each of the six values 1 to 6 has the probability 1/6. The probability of an event is then defined to be the sum of the probabilities of the outcomes that satisfy the event.

In contrast, when a random variable takes values from a continuum then typically, any individual outcome has probability zero and only events that include infinitely many outcomes, such as intervals, can have positive probability. For example, consider measuring the weight of a piece of ham in the supermarket, and assume the scale has many digits of precision. The probability that it weighs *exactly* 500 g is zero, as it will most likely have some non-zero decimal digits. Nevertheless, one might demand, in quality control, that a package of "500 g" of ham must weigh between 490 g and 510 g with at least 98% probability, and this demand is less sensitive to the accuracy of measurement instruments.



The left shows the probability density function. The right shows the cumulative distribution function, for which the value at **a** equals the area under the probability density curve to the left of **a**.

Absolutely continuous probability distributions can be described in several ways. The probability density function describes the infinitesimal probability of any given value, and the probability that the outcome lies in a given interval can be computed by integrating the probability density function over that interval.

An alternative description of the distribution is by means of the cumulative distribution function, which describes the probability that the random variable is no larger than a given value for some . The cumulative distribution function is the area under the probability density function .

The **philosophy of probability** presents problems chiefly in matters of epistemology and the uneasy interface between mathematical concepts and ordinary language as it is used by non-mathematicians.

Probability theory is an established field of study in mathematics. It has its origins in correspondence discussing the mathematics of games of chance between Blaise Pascal and Pierre de Fermat in the seventeenth century, and was formalized and rendered axiomatic as a distinct branch of mathematics by Andrey Kolmogorov in the twentieth century. In axiomatic form, mathematical statements about probability theory carry the same sort of epistemological confidence within the philosophy of mathematics as are shared by other mathematical statements.

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The mathematical analysis originated in observations of the behaviour of game equipment such as playing cards and dice, which are designed specifically to introduce random and equalized elements; in mathematical terms, they are subjects of indifference. This is not the only way probabilistic statements are used in ordinary human language: when people say that "*it will probably rain*", they typically do not mean that the outcome of rain versus not-rain is a random factor that the odds currently favor; instead, such statements are perhaps better understood as qualifying their expectation of rain with a degree of confidence. Likewise, when it is written that "the most probable explanation" of the name of Ludlow, Massachusetts "is that it was named after Roger Ludlow", what is meant here is not that Roger Ludlow is favored by a random factor, but rather that this is the most plausible explanation of the evidence, which admits other, less likely explanations.

Thomas Bayes attempted to provide a logic that could handle varying degrees of confidence; as such, Bayesian probability is an attempt to recast the representation of probabilistic statements as an expression of the degree of confidence by which the beliefs they express are held.

Though probability initially had somewhat mundane motivations, its modern influence and use is widespread ranging from evidence-based medicine, through six sigma, all the way to the probabilistically checkable proof and the string theory landscape.

A summary of some interpretations of probability [2]

	Classical	**Frequentist**	**Subjective**	**Propensity**
Main hypothesis	Principle of indifference	Frequency of occurrence	Degree of belief	Degree of causal connection
Conceptual basis	Hypothetical symmetry	Past data and reference class	Knowledge and intuition	Present state of system
Conceptual approach	Conjectural	Empirical	Subjective	Metaphysical
Single case possible	Yes	No	Yes	Yes
Precise	Yes	No	No	Yes
Problems	Ambiguity in principle of indifference	Circular definition	Reference class problem	Disputed concept

- The sample space is the set of all possible outcomes. An outcome is the result of a single execution of the model. Outcomes may be states of nature, possibilities, experimental results and the like. Every instance of the real-world situation (or run of the experiment) must produce exactly one outcome. If outcomes of different runs of an experiment differ in any way that matters, they are distinct outcomes. Which differences matter depends on the kind of analysis we want to do. This leads to different choices of sample space.

- The σ-algebra is a collection of all the events we would like to consider. This collection may or may not include each of the elementary events. Here, an "event" is a set of zero or more outcomes; that is, a subset of the sample space. An event is considered to have "happened" during an experiment when the outcome of the latter is an element of the event. Since the same outcome may be a member of many events, it is possible for many events to have happened given a single outcome. For example, when the trial consists of throwing two dice, the set of all outcomes with a sum of 7 pips may constitute an event, whereas outcomes with an odd number of pips may constitute another event. If the outcome is the element of the elementary event of two pips on the first die and five on the second, then both of the events, "7 pips" and "odd number of pips", are said to have happened.

- The probability measure is a set function returning an event's probability. A probability is a real number between zero (impossible events have probability zero, though probability-zero events are not necessarily impossible) and one (the event happens almost surely, with almost total certainty). The probability measure function must satisfy two simple requirements: First, the probability of a countable union of mutually exclusive events must be equal to the countable sum of the probabilities of each of these events . Not every subset of the sample space must necessarily be considered an event: some of the subsets are simply not of interest, others cannot be "measured". This is not so obvious in a case like a coin toss.

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- In a different example, one could consider javelin throw lengths, where the events typically are intervals like "between 60 and 65 meters" and unions of such intervals, but not sets like the "irrational numbers between 60 and 65 meters".

Poisson distribution

In probability theory and statistics, the **Poisson distribution** is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event. It is named after French mathematician Siméon Denis Poisson . The Poisson distribution can also be used for the number of events in other specified interval types such as distance, area, or volume.

For instance, a call center receives an average of 180 calls per hour, 24 hours a day. The calls are independent; receiving one does not change the probability of when the next one will arrive. The number of calls received during any minute has a Poisson probability distribution with mean 3: the most likely numbers are 2 and 3 but 1 and 4 are also likely and there is a small probability of it being as low as zero and a very small probability it could be 10.

The distribution was first introduced by Siméon Denis Poisson (1781–1840) and published together with his probability theory . The work theorized about the number of wrongful convictions in a given country by focusing on certain random variables *N* that count, among other things, the number of discrete occurrences (sometimes called "events" or "arrivals") that take place during a time-interval of given length. This makes it an example of Stigler's law and it has prompted some authors to argue that the Poisson distribution should bear the name of de Moivre

In 1860, Simon Newcomb fitted the Poisson distribution to the number of stars found in a unit of space. A further practical application of this distribution was made by Ladislaus Bortkiewicz in 1898 when he was given the task of investigating the number of soldiers in the Prussian army killed accidentally by horse kicks, this experiment introduced the Poisson distribution to the field of reliability engineering.

Normal Distribution

Normal distribution, also known as the Gaussian distribution, is a probability distribution that is symmetric about the mean, showing that data near the mean are more frequent in occurrence than data far from the mean. In graphical form, the normal distribution appears as a "bell curve".

- What is the normal distribution formula used for?
- What is an example of a normal probability distribution?
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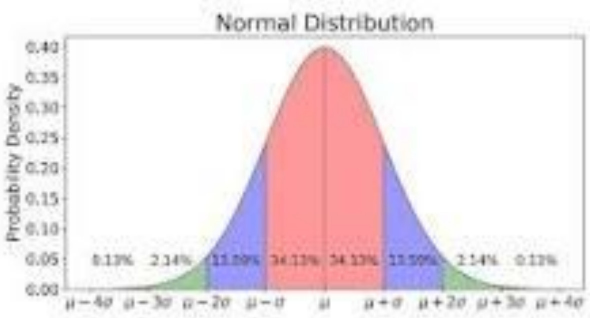

A fair rolling of dice is also a good example of normal distribution. In an experiment, it has been found that when a dice is rolled 100 times, chances to get '1' are 15-18% and if we roll the dice 1000 times, the chances to get '1' is, again, the same, which averages to 16.7% (1/6).

- What is the z-score and Z table?

Most importantly, Z-score helps to calculate how much area that specific Z-score is associated with. A Z-score table is also known as a standard normal table used to find the exact area. The Z-score table tells the total quantity of area contained on the left side of any score or value (x).

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Binomial Distribution

Binomial distribution is calculated by multiplying the probability of success raised to the power of the number of successes and the probability of failure raised to the power of the difference between the number of successes and the number of trials.

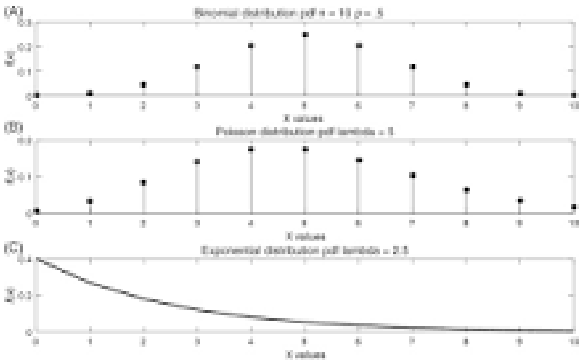
- What are the main features of binomial distribution?
- What are the main features of binomial distribution?

1) The number of trials 'n' is finite.
2) The trials are independent of each other.
3) Success probability 'p' is constant for each trial.
4) Each trial has only one of the two possible results either success or failure.

- What is application of binomial distribution?

The Binomial distribution computes the probabilities of events where only two possible outcomes can occur (success or failure), e.g. when you look at the closing price of a stock each day for one year, the outcome of interest is whether the stock price increased or not.

- What is a binomial distribution also known as?



The probability distribution of a binomial random variable is called a binomial distribution (It is also known as a Bernoulli distribution). A cumulative binomial probability refers to the probability that the binomial random variable falls within a specified range.

