



OPERATION RESEARCH (OR)

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Operations research (OR) is an analytical method of problem-solving and decision-making that is useful in the management of organizations. In operations research, problems are broken down into basic components and then solved in defined steps by mathematical analysis.

The process of operations research can be broadly broken down into the following steps:

1. Identifying a problem that needs to be solved.
2. Constructing a model around the problem that resembles the real world and variables.
3. Using the model to derive solutions to the problem.
4. Testing each solution on the model and analyzing its success.
5. Implementing the solution to the actual problem.

Characteristics of operations research:

There are three primary characteristics of all operations research efforts:

1. **Optimization-** The purpose of operations research is to achieve the best performance under the given circumstances. Optimization also involves comparing and narrowing down potential options.
2. **Simulation-** This involves building models or replications in order to try out and test solutions before applying them.
3. **Probability and statistics-** This includes using mathematical algorithms and data to uncover helpful insights and risks, make reliable predictions and test possible solutions.

Importance of Operations research:

The field of operations research provides a more powerful approach to decision making than ordinary software and data analytics tools. Employing operations research professionals can help companies achieve more complete datasets, consider all available options, predict all possible outcomes and estimate risk.

Additionally, operations research can be tailored to specific business processes or use cases to determine which techniques are most appropriate to solve the problem.

Uses of Operations research:

Operations research can be applied to a variety of use cases, including:

- Scheduling and time management.
- Urban and agricultural planning.
- Enterprise resource planning (ERP) and supply chain management (SCM).
- Inventory management.
- Network optimization and engineering.
- Packet routing optimization.
- Risk management.

Operational research in mathematics :

- Operations Research is the branch of applied mathematics concerned with applying analytical methods to help make better management decisions. Operations research is also known as management science and industrial engineering.



Mathematical programming in operation research:

- Mathematical programming, or mathematical optimization, is a systematic approach used for optimizing (minimizing or maximizing) the value of an objective function with respect to a set of constraints.

Optimization: Optimization lies at the heart of OR, aiming to find the best possible solution from a set of feasible alternatives. Techniques such as linear programming, integer programming, dynamic programming, and simulation are commonly used to optimize processes and make optimal decisions.

Linear programming (LP) or Linear Optimisation: It is defined as the problem of maximizing or minimizing a linear function that is subjected to linear constraints. The constraints may be equalities or inequalities. The optimisation problems involve the calculation of profit and loss.

Use of linear programming in Operations management:

Linear programming (LP) is a mathematical modeling technique used by operations managers to determine optimal solutions to decision problems. Previous applications include the determination of optimal airline and trucking routes, blends of chemicals and oils, personnel schedules, and patient case mixes.

Role of Operation research in engineering:

In practice, operations research and industrial engineering address both the performance objectives and the resource constraints of an organization, working toward the establishment of policies that are most beneficial to the organization as a whole.

Applications of Operation research in information technology:

It is used in computer science applications, such as scheduling concepts and computer science applications. Additionally, operational research techniques are complementary to information technology, as they facilitate the planning and implementation of projects and activities with the help of IT.

LINEAR PROGRAMMING

In Mathematics, **linear programming** is a method of optimising operations with some constraints. The main objective of linear programming is to maximize or minimize the numerical value. It consists of linear functions which are subjected to the constraints in the form of linear equations or in the form of inequalities. Linear programming is considered an important technique that is used to find the optimum resource utilisation. The term “linear programming” consists of two words as linear and programming. The word “linear” defines the relationship between multiple variables with degree one. The word “programming” defines the process of selecting the best solution from various alternatives.

Linear Programming is widely used in Mathematics and some other fields such as economics, business, telecommunication, and manufacturing fields. In this article, let us discuss the definition of linear programming, its components, and different methods to solve linear programming problems.

Components of Linear Programming

The basic components of the LP are as follows:

- Decision Variables
- Constraints
- Data
- Objective Functions

Characteristics of Linear Programming

The following are the five characteristics of the linear programming problem:

Constraints – The limitations should be expressed in the mathematical form, regarding the resource.

Objective Function – In a problem, the objective function should be specified in a quantitative way.

Linearity – The relationship between two or more variables in the function must be linear. It means that the degree of the variable is one.



Finiteness – There should be finite and infinite input and output numbers. In case, if the function has infinite factors, the optimal solution is not feasible.

Non-negativity – The variable value should be positive or zero. It should not be a negative value.

Decision Variables – The decision variable will decide the output. It gives the ultimate solution of the problem. For any problem, the first step is to identify the decision variables.

Linear Programming Problems

The Linear Programming Problems (LPP) is a problem that is concerned with finding the optimal value of the given linear function. The optimal value can be either maximum value or minimum value. Here, the given linear function is considered an objective function.

The objective function can contain several variables, which are subjected to the conditions and it has to satisfy the set of linear inequalities called linear constraints. The linear programming problems can be used to get the optimal solution for the following scenarios, such as manufacturing problems, diet problems, transportation problems, allocation problems and so on.

Methods to Solve Linear Programming Problems

The linear programming problem can be solved using different methods, such as the graphical method, simplex method, or by using tools such as R, open solver etc. Here, we will discuss the two most important techniques called the simplex method and graphical method in detail.

Linear Programming Simplex Method

The simplex method is one of the most popular methods to solve linear programming problems. It is an iterative process to get the feasible optimal solution. In this method, the value of the basic variable keeps transforming to obtain the maximum value for the objective function. The algorithm for linear programming simplex method is provided below:

Step 1: Establish a given problem. (i.e.,) write the inequality constraints and objective function.

Step 2: Convert the given inequalities to equations by adding the slack variable to each inequality expression.

Step 3: Create the initial simplex tableau. Write the objective function at the bottom row. Here, each inequality constraint appears in its own row. Now, we can represent the problem in the form of an augmented matrix, which is called the initial simplex tableau.

Step 4: Identify the greatest negative entry in the bottom row, which helps to identify the pivot column. The greatest negative entry in the bottom row defines the largest coefficient in the objective function, which will help us to increase the value of the objective function as fast as possible.

Step 5: Compute the quotients. To calculate the quotient, we need to divide the entries in the far right column by the entries in the first column, excluding the bottom row. The smallest quotient identifies the row. The row identified in this step and the element identified in the step will be taken as the pivot element.

Step 6: Carry out pivoting to make all other entries in column is zero.

Step 7: If there are no negative entries in the bottom row, end the process. Otherwise, start from step 4.

Step 8: Finally, determine the solution associated with the final simplex tableau.

Graphical Method

The graphical method is used to optimize the two-variable linear programming. If the problem has two decision variables, a graphical method is the best method to find the optimal solution.

In this method, the set of inequalities are subjected to constraints. Then the inequalities are plotted in the XY plane. Once, all the inequalities are plotted in the XY graph, the intersecting region will help to decide the feasible region. The feasible region will provide the optimal solution as well as explains what all values our model can take. Let us see an example here and understand the concept of linear programming in a better way.

**Example:**

Calculate the maximal and minimal value of $z = 5x + 3y$ for the following constraints.

$$x + 2y \leq 14$$

$$3x - y \geq 0$$

$$x - y \leq 2$$

Solution:

The three inequalities indicate the constraints. The area of the plane that will be marked is the feasible region.

The optimisation equation (z) = $5x + 3y$.

You have to find the (x,y) corner points that give the largest and smallest values of z.

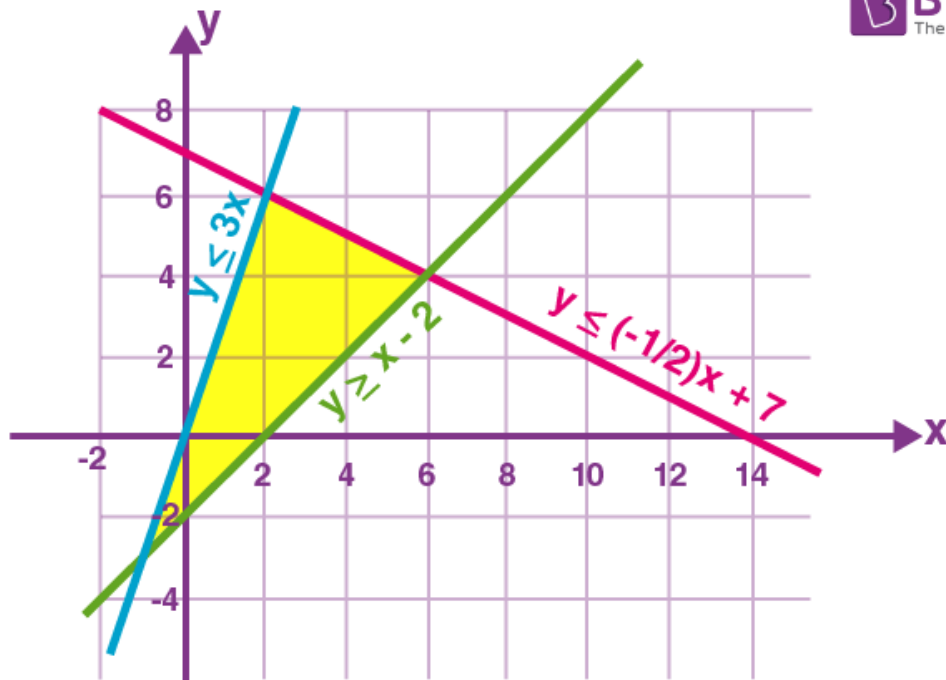
To begin with, first solve each inequality.

$$x + 2y \leq 14 \Rightarrow y \leq -(1/2)x + 7$$

$$3x - y \geq 0 \Rightarrow y \leq 3x$$

$$x - y \leq 2 \Rightarrow y \geq x - 2$$

Here is the graph for the above equations.



Now pair the lines to form a system of linear equations to find the corner points.

$$y = -(1/2)x + 7$$

$$y = 3x$$



Solving the above equations, we get the corner points as (2, 6)

$$y = -1/2 x + 7$$

$$y = x - 2$$

Solving the above equations, we get the corner points as (6, 4)

$$y = 3x$$

$$y = x - 2$$

Solving the above equations, we get the corner points as (-1, -3)

For linear systems, the maximum and minimum values of the optimisation equation lie on the corners of the feasibility region.

Therefore, to find the optimum solution, you only need to plug these three points in $z = 3x + 4y$

(2, 6) :

$$z = 5(2) + 3(6) = 10 + 18 = 28$$

(6, 4):

$$z = 5(6) + 3(4) = 30 + 12 = 42$$

(-1, -3):

$$z = 5(-1) + 3(-3) = -5 - 9 = -14$$

Hence, the maximum of $z = 42$ lies at (6, 4) and the minimum of $z = -14$ lies at (-1, -3)

Linear Programming Applications

A real-time example would be considering the limitations of labours and materials and finding the best production levels for maximum profit in particular circumstances. It is part of a vital area of mathematics known as optimisation techniques. The applications of LP in some other fields are

- Engineering – It solves design and manufacturing problems as it is helpful for doing shape optimization
- Efficient Manufacturing – To maximise profit, companies use linear expressions
- Energy Industry – It provides methods to optimise the electric power system.
- Transportation Optimisation – For cost and time efficiency.

Importance of Linear Programming

Linear programming is broadly applied in the field of optimisation for many reasons. Many functional problems in operations analysis can be represented as linear programming problems.

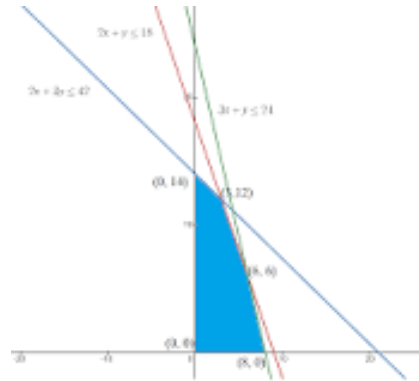
Some special problems of linear programming are such as network flow queries and multi-commodity flow queries are deemed to be important to have produced much research on functional algorithms for their solution.

LINEAR PROGRAMMING & OPERATION RESEARCH

Technique of choosing the shortest route is called linear programming. In this case, the objective of the delivery person is to deliver the parcel on time at all 6 destinations. The process of choosing the best route is called Operation Research.



What are the uses of LPP in operation research?



Manufacturing Industries: Industries use lpp models to maximize efficiency with minimum operational cost. Transport Company: Company like ola, uber, and Rapido uses LPP to optimize delivery routes. The objective of these companies is to reduce time and operational costs and increase revenue.