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Using Generalized Interpolation Formulae to Scale a Digital Image

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Abstract: An important place in the problems of digital image analysis is occupied by the task of enlarging the resolution of an image by scaling it. Such tasks include, in particular: obtaining more detailed information from a fragment of an image because of its enlargement; image magnification for object identification; obtaining a high-resolution image from a low-resolution image to facilitate its further detailed analysis, etc. Each of the existing technics is characterized by both the positive and negative sides. In particular, the negative side is the distortion of the geometric shape of small parts and damage to the texture of the image. Interpolation algorithms are used to reduce these disadvantages. One approach to solve this problem is to use interpolation techniques. The presented article attempts to use the generalized interpolation formulae (Piranashvili's formulae) with a high-speed convergence for the task of enlarging image size. The results of using Whittaker-Kotelnikov-Shannon and Piranashvili interpolation formulae for enlarging digital images are shown. To estimate the accuracy and quality of approximation of images obtained after interpolation, the remainder terms and signal-to-noise ratio (SNR) values are calculated and compared.

Keywords: Image interpolation, Digital image enlarging, Whittaker-Kotelnikov-Shannon interpolation, Generalized interpolation formulae, Remainder term, Signal-to-noise ratio.

I. INTRODUCTION

One way to get as much information as possible from a digital image is to enlarge its scale and resolution. As a result, we receive an image more suitable than initial image for a further application. Such images are widely applied in many application fields like computer vision, biomedical image analysis, criminology, remote sensing, geology, agriculture etc. Among the technics for solving these problems, an important place is occupied using interpolation technics of stochastic processing. Such tasks include, in particular: obtaining more detailed information from a fragment of an image by enlarging it; enlarging of an image to identify objects; obtaining a high-resolution image from a low-resolution image to facilitate its further detailed analysis, etc. Each of the existing techniques is characterized by both positive and negative sides. In particular: when an image is enlarged, the image quality getting worse, resulting in various types of defects such as staircase effect, smoothing, blurring, distortion of the geometric shape of small details, damage to the image texture, etc. Interpolation algorithms are used to reduce these defects [1].

Interpolation allows to convert an image from one resolution to another without compromising the visual quality of the image. Various interpolation methods are used to scale an image. For example, the easiest way to double the size of a digital image is to simply copy its pixels [2,3]. In general, interpolation techniques are divided into 2 groups: adaptive and non-adaptive. Consequently, there are many image interpolation algorithms. Each of them gives different results regarding the quality of the resulting enlarged image. Thus, the algorithm that preserves image quality is considered the best.

Non-adaptive digital image interpolation techniques include nearest neighbour, bilinear and bicubic algorithms. These are conventional techniques used to enlarge images. When using non-adaptive interpolation algorithms, the intensity value at each new pixel is determined based on the average intensity value of neighboring pixels. This operation is repeated for all new pixels in the image. Accordingly, such algorithms are characterized by simplicity of calculations and low computational cost.

The calculated average intensity values usually do not exactly match the intensity values of the original image. The change in the average intensity value at each new pixel depends on the resizing ratio and the interpolation algorithm used. Non-adaptive interpolation behaves quite well when the resizing ratio is not too large. But when the resizing ratio increases, all algorithms start to perform poorly. In particular, the average value changes very slowly as the intensity values begin to repeat. As a result, the quality of the enlarged image is getting worse.

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Adaptive algorithms are more complex ones, because to calculate the intensity value in new pixels, in addition to the intensity value, information about edges, texture, and other data must be considered. Therefore, the computational costs of such algorithms are high, but the result is better than when using non-adaptive algorithms [2,3].

By enlarging size of the digital image while maintaining the same value of resolution, new pixels are added to the digital image in which different algorithms of interpolation are used to calculate the brightness values. Accordingly, different are the computational cost and the visual quality of the obtained images. Therefore, the task is to achieve a certain compromise between computational costs and the quality of the resulting image. Obviously, the best option would be to obtain the highest quality image from the minimum number of samples. To do this, it is proposed to use generalized, with a high rate of convergence, interpolation formulae of the Whitaker-Kotelnikov-Shannon type for interpolation of images, which, with a smaller number of samples, unlike the Whitaker-Kotelnikov-Shannon formula, allow obtaining images of better quality. The application of these formulae gave a good results for a compact description of the closed contour of a segment on a segmented image [4]. This article is an attempt to use generalized interpolation formulae (Piranashvili's formulae) for the problem of enlarging the image size while maintaining its quality. The presented approach refers to the method of non-adaptive interpolation.

II. INTERPOLATION TECHNIQUE

In the task of enlarging the size of the digital image by introducing additional new pixels, we use the well-known the Whitaker-Kotelnikov-Shannon interpolation formula as well as generalized interpolation formulae with high-speed convergence [5]. Both approaches are compared in terms of approximation accuracy and visual quality of the resulting images by calculating the remainder term of the series and the signal-to-noise ratio (SNR).
It is known that the image can be described by intensity function $x(t)$. To calculate the values of the intensity function (brightness) in each new pixel obtained as a result of image enlarging by several times, let's consider the finite sum $x_n(t)$ of Piranashvili's interpolation formulae [5]:

$$x_n(t) = \left(a \cdot e^{\delta t} + b \cdot e^{-\delta t}\right) \cdot \sum_{k=-n}^{n} \frac{x\left(\frac{k\pi}{\alpha}\right)}{\left(a \cdot e^{\delta \frac{k\pi}{\alpha}} + b \cdot e^{-\delta \frac{k\pi}{\alpha}}\right)} \cdot \frac{\sin \alpha\left(t - \frac{k\pi}{\alpha}\right)}{\alpha\left(t - \frac{k\pi}{\alpha}\right)} \cdot \left(\frac{\sin \beta\left(t - \frac{k\pi}{\alpha}\right)}{\beta\left(t - \frac{k\pi}{\alpha}\right)}\right)^{q} \quad (1),$$

where $\alpha > \sigma$, $0 < \beta < (\alpha - \sigma)/q$, $0 < \delta < \alpha - \sigma - q\beta$, while q – some non-negative integer and $\alpha > 0$, b> 0 – the real numbers.

For further calculations, it is more convenient to write formula (1) as follows (due to the simple transformation of the summation variable):

$$x_n(t) = \left(ae^{\delta i} + be^{-\delta i}\right) \sum_{k=1}^{2n+1} \frac{x\left(\frac{(k-n-1)\pi}{\alpha}\right)}{\left(ae^{\delta \frac{(k-n-1)\pi}{\alpha}} + be^{-\delta \frac{(k-n-1)\pi}{\alpha}}\right)} \cdot \frac{\sin \alpha\left(t - \frac{(k-n-1)\pi}{\alpha}\right)}{\alpha\left(t - \frac{(k-n-1)\pi}{\alpha}\right)} \cdot \left(\frac{\sin \beta\left(t - \frac{(k-n-1)\pi}{\alpha}\right)}{\beta\left(t - \frac{(k-n-1)\pi}{\alpha}\right)}\right)^{q} \quad (2).$$

As for the accuracy of the approximation, we calculated the remainder term using the following formula [5]:

$$|x(t) - x_n(t)| \leq L_x \cdot \frac{2^{q+2} \cdot D_1(a,b)}{D_0(a,b) \cdot (1 - e^{-\pi})} \cdot |\sin(\alpha t)| \cdot \left[e^{-(1-\theta)\delta\left(n+\frac{1}{2}\right)\frac{\pi}{\alpha}} + e^{-\delta\left(n+\frac{1}{2}\right)\frac{\pi}{\alpha}}\right] \cdot$$
$$\cdot \left[\left(\frac{\alpha}{\pi\left(n+\frac{1}{2}\right)}\right)^{q+1} + \left(\frac{\alpha}{\pi\left(n+\frac{1}{2}\right)}\right)^{q+1-m}\right] \quad (3),$$

where $|t| \leq \theta\left(n + \frac{1}{2}\right)\frac{\pi}{2}$; $0 < \theta < 1$, $D_1(a,b) = D_1(b,a) = max\{a,b\}$, $L_x = sup|x(t)|, -\infty < t < \infty$, $D_0(a,b) = min\{a, b, |a-b|\}$. In the Z. Piranashvili's article, it is recommended to take:

$b = \frac{\alpha}{2}$ or $b = 2\alpha$, and particularly, to take $a = 1$, $b = 2$, or $a = 2$, $b = 1$.

If in (1) $\delta = 0$ and $q = 0$, then we get the finite sum of the well-known Whittaker-Kotelnikov-Shannon series [6]:

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$$x_n(t) = \sum_{k=-n}^{n} x\left(\frac{k\pi}{\alpha}\right) \cdot \frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)} \quad (4).$$

To estimate the remainder term for the Whittaker-Kotelnikov-Shannon interpolation formula, we use the following formula [5]:

$$|x(t) - x_n(t)| \leq \frac{4L_x\alpha}{(\alpha-\sigma)n\pi(1-e^{-\pi})} \quad (5).$$

The presented formulae (2) and (4) were used to solve both approaches. In particular, calculate the brightness values obtained by interpolation in the new pixels using Piranashvili's and Whittaker-Kotelnikov-Shannon interpolation formulae.

III. EXPERIMENTAL RESULTS

To show the results of applying the presented approach with an example, we considered the task of image scaling enlarging the image size (width and height) for two cases:

1. Enlarging the image size by 4 times.
2. Enlarging the image fragment size by 4 times.

To test our interpolation technique, we select 2 images. To test our interpolation technique for the experiment we selected 2 images, namely an image and the image fragment presented on Fig. 1-2 respectively. The results of application of the (2) and (4) interpolation formulae (Piranashvili and Whittaker-Kotelnikov-Shannon respectively) for original image and its fragment presented on Fig. 1-2. At the same time, all images obtained because of interpolation were not subjected to further processing to improve their quality.

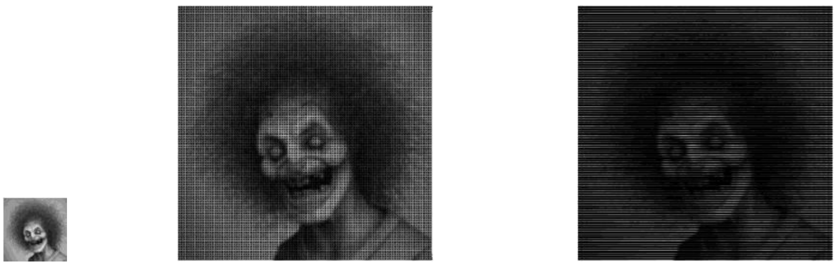

Fig. 1 The original image and the images obtained after enlargement using Piranashvili and Whittaker-Kotelnikov-Shannon formulae by 4 times are shown sequentially.

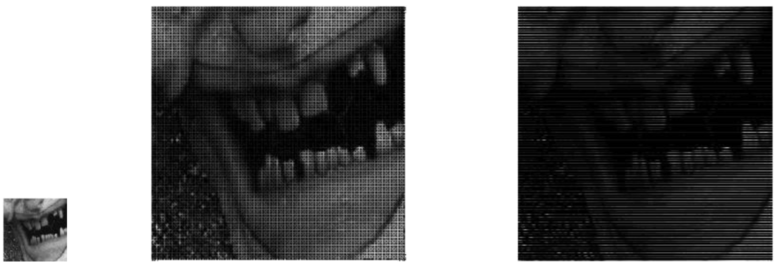

Fig. 2 The original image fragment and the images obtained after enlargement using Piranashvili and Whittaker-Kotelnikov-Shannon formulae by 4 times are shown sequentially.

To estimate the accuracy of approximation of images enlarged by 4 times using interpolation formulae (2) and (4), the estimate of the remainder terms of interpolation was calculated using formulae (3) and (5), respectively. The results of calculating the estimates of the remainder terms of the Piranashvili and Whittaker-Kotelnikov-Shannon interpolation formulae for the obtained images presented in Fig. 1-2 are shown in Table 1.

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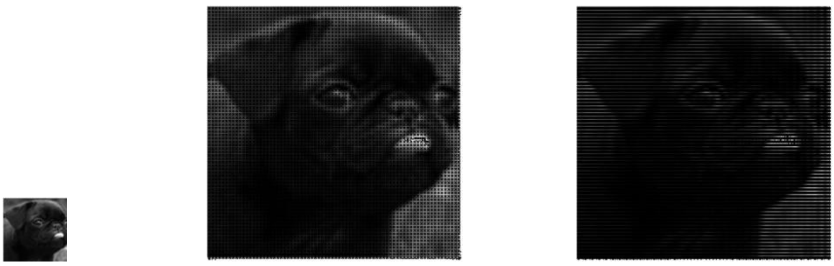
TABLE I THE REMAINDER TERMS OF PIRANASHVILI AND WHITTAKER--KOTELNIKOV-SHANNON FORMULAE FOR IMAGES PRESENTED IN FIG. 1-2

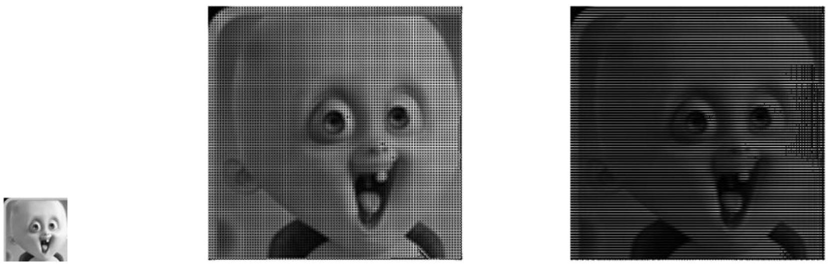
Figure #	**Whittaker-Kotelni-kov-Shannon**	**Piranashvili**
1	4.011	0.0022
2	3.9722	0.0072

Table 1 shows that in both of the above cases presented in Fig. 1-2 the accuracy of the approximation obtained as a result of calculating the values of the remainder term for the Piranashvili's interpolation formulae gives a much more accuracy then the value of the remainder term of the Whittaker-Kotelnikov-Shannon interpolation formula. Thus, using the generalized interpolation technique of Z. Piranashvili, we obtain images enlarged by 4 times with better visual quality, which also coincides with the visual representation given above.

To quantitatively analyse the quality of 4 times magnified images using the Whittaker-Kotelnikov-Shannon and the generalized Piranashvili interpolation formulae, we tested 20 digital images and calculated signal-to-noise ratio (SNR) values, where SNR is defined as the ratio of the average brightness of the image (signal) μ_{sig} to the standard deviation σ_{bg} of the background (noise) [7]. The calculation results for five digital images (Fig. 1-3) are presented in Table 2.



(a)



(b)

(c)

Fig. 3 The original images and the images obtained after enlargement using Piranashvili and Whittaker-Kotelnikov-Shannon formulae by 4 times are shown sequentially

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TABLE II THE SNR VALUES FOR IMAGES IN FIG.1-3 OBTAINED BY PIRANASHVILI AND WHITTAKER--KOTELNIKOV-SHANNON FORMULAE

Fig. #	Whittaker-Kotelni-kov-Shannon	Piranashvili
1	0.5348	1.0979
2	0.4825	0.9219
3(a)	0.3730	0.6526
3(b)	0.5540	1.1623
3(c)	0.5445	1.1287

We see that the SNR value is approximately 2 times higher for the generalized Piranashvili interpolation than for the Whittaker-Kotelnikov-Shannon interpolation. This means that the image interpolated by Piranashvili carries more useful information than the image interpolated by Whittaker-Kotelnikov-Shannon.

IV. CONCLUSION

This article examines the task of enlarging the size of a digital image using the Whittaker-Kotelnikov-Shannon interpolation formula and the generalized Piranashvili interpolation formulae. A comparison of the approximation accuracy and the quality of the resulting images in terms of the remainder term and the signal-to-noise ratio (SNR) shows that the generalized Piranashvili interpolation formulae provide high approximation accuracy and better image quality.

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