



Circle Generating Algorithms

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Abstract: The basic principle of this algorithm is to select the optimum raster location to represent a circular curve. To accomplish this, the algorithm always increments x. Increment of y is depends upon the distance between the actual circle location and the nearest pixel. This distance is called decision variable D.

Keywords: Pixel, Decision Variable.

I. INTRODUCTION

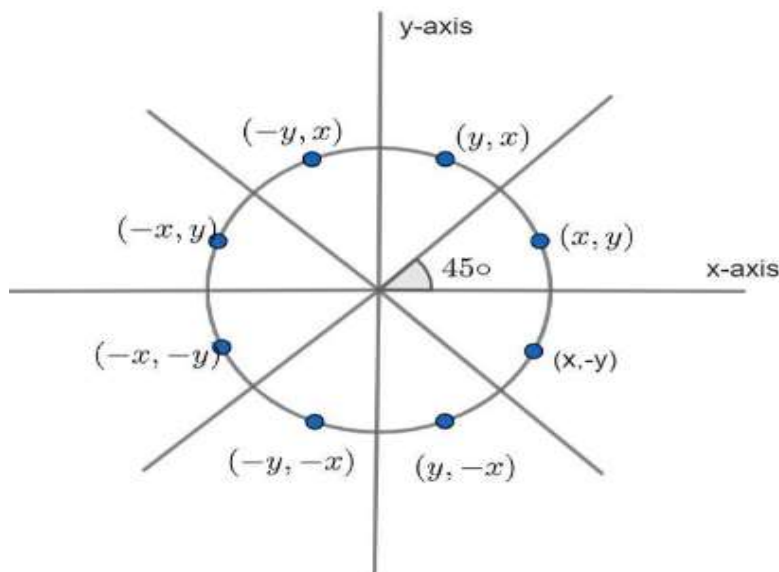
In computer graphics, a circle drawing algorithm is an algorithm for approximating a circular curve on discrete graphical media, such as pixel-based displays and printers. On such media, circle drawing requires an approximation (in nontrivial cases). Basic algorithms rasterize circle in one colour.

There are two algorithms which can be used to draw a circle in computer graphics.

1. DDA circle drawing algorithm
2. Bresenham's circle drawing algorithm.

II. EIGHT WAY SYMMETRY OF A CIRCLE

The circle drawing algorithm uses eight way symmetry of a circle. To generate the entire circle the algorithm rasterize the pixel positions of only 1/8 curve of the circle and replicate this pixels in other 7/8 part of the circle to generate entire circle.



III. DDA CIRCLE DRAWING ALGORITHM

The DDA circle drawing algorithm uses the basic characteristic of the circle to generate formulas which rasterizes the 1/8 part of the circle.



A. Working

- We know that, the equation of circle is
 $x^2+y^2=r^2$
- Differentiating the above equation we get
 $2x dx + 2y dy = 0$...r is constant
 $x dx + y dy = 0$
 $y dy = -x dx$
 $dy/dx = -x / y$
- From above equation, we can construct the circle by incrementing x value as $dx = \epsilon y$ and increment y value as $dy = -\epsilon x$
- We can calculate value of ϵ as
 $2^{n-1} \leq r < 2^n$
 $\epsilon = 2 - n$
- Applying these increments value we have
 $x_{n+1} = x_n + \epsilon y_n$
 $y_{n+1} = y_n - \epsilon x_n$
- This equations creates spiral instead of circle so changing the above equation as
 $x_{n+1} = x_n + \epsilon y_n$
 $y_{n+1} = y_n - \epsilon x_{n+1}$

B. Algorithm

Step 1: Read the radius (r), of the circle and calculate value of ϵ .

Step 2 : start_x=0

start_y=r

Step 3: x1=start_x

y1=start_y

Step 4: do

{

x2=x1+ ϵ y1

y2=y1- ϵ x2

x1=x2

y1=y2

} while((y1-start_y) < ϵ or (start_x-x1) > ϵ)

Step 5: stop

C. Example

- Consider the circle with radius of 10. Generate a circle using DDA circle drawing algorithm.
- Solution:
r=10
Calculating value of ϵ
 $2^3 \leq 10 < 2^4$ 8<=10<16n=4
 $\epsilon = 2 - 4 = 1/16 = 0.0625$
Start_x=0
Start_y=10
X1=0
Y1=10



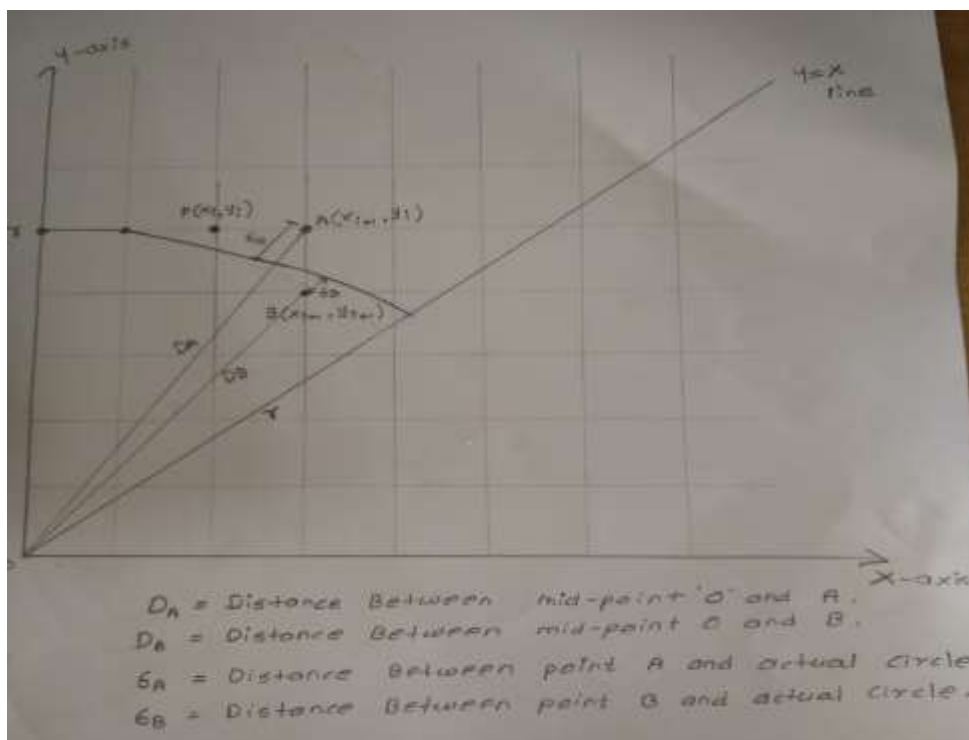
D. Pixel Positions

i	x	y	(x,y)	(y,x)	(x,-y)	(y,-x)	(-x,-y)	(-y,-x)	(-x,y)	(-y,x)
1	0	10	(0,10)	(10,0)	(0,-10)	(10,0)	(0,-10)	(10,0)	(0,10)	(-10,0)
2	0.63	9.96	(0,9)	(9,0)	(0,-9)	(9,0)	(0,-9)	(-9,0)	(0,9)	(-9,0)
3	1.25	9.88	(1,9)	(9,1)	(1,-9)	(9,-1)	(-1,-9)	(-9,-1)	(-1,9)	(-9,1)
4	1.87	9.76	(1,9)	(9,1)	(1,-9)	(9,-1)	(-1,-9)	(-9,-1)	(-1,9)	(-9,1)
5	2.48	9.61	(2,9)	(9,2)	(2,-9)	(9,-2)	(-2,-9)	(-9,-2)	(-2,9)	(-9,2)
6	3.08	9.42	(3,9)	(9,3)	(3,-9)	(9,-3)	(-3,-9)	(-9,-3)	(-3,9)	(-9,3)
7	3.67	9.19	(3,9)	(9,3)	(3,-9)	(9,-3)	(-3,-9)	(-9,-3)	(-3,9)	(-9,3)
8	4.24	8.93	(4,8)	(8,4)	(4,-8)	(8,-4)	(-4,-8)	(-8,-4)	(-4,8)	(-8,4)
9	4.80	8.63	(4,8)	(8,4)	(4,-8)	(8,-4)	(-4,-8)	(-8,-4)	(-4,8)	(-8,4)
10	5.34	8.30	(5,8)	(8,5)	(5,-8)	(8,-5)	(-5,-8)	(-8,-5)	(-5,8)	(-8,5)
11	5.86	7.93	(5,7)	(7,5)	(5,-7)	(7,-5)	(-5,-7)	(-7,-5)	(-5,7)	(-7,5)
12	6.36	7.53	(6,7)	(7,6)	(6,-7)	(7,-6)	(-6,-7)	(-7,-6)	(-6,7)	(-7,6)
13	6.83	7.10	(6,7)	(7,6)	(6,-7)	(7,-6)	(-6,-7)	(-7,-6)	(-6,7)	(-7,6)

IV. BRESENHAMS CIRCLE DRAWING ALGORITHM

Bresenham's circle drawing algorithm is also called as mid-point circle drawing algorithm as it assumes the origin as the centre of the circle. This algorithm working principle of the algorithm is to find the nearest pixel from the actual circle line.

A. Working





- The distance of pixel A and B from the origin are given as:

$$DA = \sqrt{(x_i+1)^2 + (y_i)^2}$$

$$DB = \sqrt{(x_i+1)^2 + (y_i-1)^2}$$
- Now the distance between the acute circle and pixel A & B are given as:

$$\square A = DA - r$$

$$\square B = DB - r$$
- To avoid square root term

$$\square A = DA^2 - r^2$$

$$\square B = DB^2 - r^2$$
- This implies

$$\square A = (x_i+1)^2 + (y_i)^2 - r^2$$

$$\square B = (x_i+1)^2 + (y_i-1)^2 - r^2$$
- We can define a decision variable d_i as

$$d_i = \square A + \square B$$
 - $\square A$ is always going to be positive (A is outside the true circle)
 - $\square B$ is always going to be negative (B is inside the true circle)
- Using the above equations we can derive the decision parameter D as:
 1. Initial value of d is $3-2r$
 2. When $d_i < 0$, we have $\square A < \square B$ and pixel A is chosen. And when pixel A is chosen, increment $x=x+1$ and $y=y$ and value of d changes to $d=d+4x+6$
 3. When $d_i > 0$, we have $\square A > \square B$ and pixel B is chosen. And when pixel B is chosen, increment $x=x+1$ and $y=y-1$ and value of d changes to $d=d+4(x-y)+10$

B. Algorithm

Step 1: Read the radius (r) of the circle.

Step 2: initialize the decision variable $d=3-2r$

Step 3: initialize starting points as $x=0$ & $y=r$

Step 4: do

```

{
    plot(x, y)
    if(d<=0)
    {
        d=d+4x+6
    }
    else
    {
        d=d+4(x-y)+10
        y=y-1
    }
    x=x+1
}while(x<y)

```

Step 5: stop.

C. Example

- Consider the circle with radius of 10. Generate a circle using bresenhams circle drawing algorithm.
- Solution:

$$r=10$$
 Calculating initial value of $d=3-2r$ $d=3-2*10$ $d=-17$
 Initial value of $x=0$ & $y=10$



D. Pixel Positions

d	x	y	(x,y)	(y,x)	(-x,y)	(-y,x)	(-x,-y)	(-y,-x)	(x,-y)	(y,-x)
-17	0	10	(0,10)	(10,0)	(0,10)	(-10,0)	(0,-10)	(-10,0)	(0,-10)	(10,0)
-11	1	10	(1,10)	(10,1)	(-1,10)	(-10,1)	(-1,-10)	(-10,-1)	(1,-10)	(10,-1)
-1	2	10	(2,10)	(10,2)	(-2,10)	(-10,2)	(-2,-10)	(-10,-2)	(2,-10)	(10,-2)
13	3	10	(3,10)	(10,3)	(-3,10)	(-10,3)	(-3,-10)	(-10,-3)	(3,-10)	(10,-3)
-5	4	9	(4,9)	(9,4)	(-4,9)	(-9,4)	(-4,-9)	(-9,-4)	(4,-9)	(9,-4)
17	5	9	(5,9)	(9,5)	(-5,9)	(-9,5)	(-5,-9)	(-9,-5)	(5,-9)	(9,-5)
11	6	8	(6,8)	(8,6)	(-6,8)	(-8,6)	(-6,-8)	(-8,-6)	(6,-8)	(8,-6)
13	7	7	(7,7)	(7,7)	(-7,7)	(-7,7)	(-7,-7)	(-7,-7)	(7,-7)	(7,-7)

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