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Time-Frequency Analysis of the Original and Resampled Square Wave Using Continuous Wavelet Transform and comparison with FFT using MATLAB

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Abstract: Square waves play a crucial role in signal processing, digital electronics, control systems, and communication protocols. However, resampling these signals to meet different hardware specifications can introduce spectral distortions and aliasing effects. This study uses the Continuous Wavelet Transform (CWT) to examine the impact of resampling on square wave characteristics, offering superior time-frequency resolution compared to traditional Fourier-based methods. A square wave is generated using harmonic summation and initially sampled at 100 Hz before being resampled to 200 Hz for hardware compatibility. The resampled signal is then analysed using CWT scalograms, Power Spectral Density (PSD) analysis, and waveform comparisons to assess spectral distortions. Results show that CWT provides a detailed understanding of the transient and frequency variations caused by resampling, ensuring optimal signal fidelity. This research highlights the importance of advanced time-frequency analysis techniques in maintaining signal integrity across different sampling rates, with applications in real-time signal processing and embedded system design.

Keywords: MATLAB, Fourier Series, Scalogram, Odd Harmonics, Continuous Wavelet Transform (CWT), FFT

I. INTRODUCTION

Square wave signals are essential in fields such as digital signal processing, communications, and control systems. They can be synthesized by combining harmonic components, utilizing Fourier series expansion to create approximation. Nonetheless, signal fidelity can deteriorate due to issues such as aliasing and inadequate harmonic representation. Additionally, different hardware systems operate at diverse sampling rates, requiring accurate resampling techniques to maintain signal integrity. This study employs Continuous Wavelet Transform (CWT) to analyse the impact of resampling on square wave characteristics. Wavelet analysis is increasingly utilized for examining localized power fluctuations within a time series. By breaking down the time series into time-frequency space, it becomes possible to identify the dominant variability patterns and observe how these patterns change over time [1]. By leveraging MATLAB's advanced signal processing toolset, we aim to generate, resample, and analyse square wave signals with a focus on maintaining spectral integrity across different sampling rates. Square waves are particularly useful for such analyses because their harmonic-rich nature provides a wide frequency spectrum to work with. The presence of odd harmonics makes them an excellent candidate for examining how different analytical tools, like the Continuous Wavelet Transform (CWT) or power spectrum, handle time-frequency localization and spectral analysis. Wavelet representation finds diverse applications in computer vision and signal processing. Specifically, it is instrumental in tasks such as signal matching, data compression, edge detection, texture discrimination, and fractal analysis. Its versatility stems from its ability to analyse signals in both time and frequency domains, making it a powerful tool for extracting meaningful features and patterns. Let me know if you'd like to explore any of these applications in detail [2]. Their sharp transitions also highlight the strengths and limitations of various methods, making square waves a valuable benchmark for testing signal processing techniques. The CWT offers a localised visual of time frequency analysis and FFT offers a globalised.

A square wave is generated by summing its odd harmonics:

$$S(t) = \frac{4}{\pi} \times \sum_{k=1,3,5..2n+1}^{N} \frac{A}{K} \times \sin\left(2 \times \pi \times f_0 \times t\right)$$

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Where:

- A = amplitude of the square wave
- f_o = fundamental frequency
- k = odd harmonic number (1, 3, 5, ..., N)
- N = highest odd harmonic used in the approximation
- t = time, Syntax : t = (0:1/fs: T-1/fs)

The Continuous Wavelet Transform (CWT) is a versatile mathematical method for examining signals whose frequency components change over time. Unlike the Fourier Transform, which is suited for stationary signals by offering only frequency information, the CWT enables time-frequency analysis, making it ideal for non-stationary signals[3]. It operates by applying wavelet functions—adjusted in scale and position—to the signal, allowing the identification of both high-frequency and low-frequency elements at different time intervals[3].

$$CWT(a,b) = \frac{1}{\sqrt{a}} \times \int_{-\infty}^{+\infty} \psi^* \times x(t) \times \frac{(t-b)}{a}$$

Where,

- x(t) is the signal,
- ψ is the mother wavelet,
- a is the scale factor that controls the width of the wavelet,
- b is the translation factor (shifting the wavelet over time),
- ψ^* is the complex conjugate of the wavelet

II. METHODOLOGY

The methodology for this research involves generating a square wave using odd harmonics, resampling the signal, and analysing it through Continuous Wavelet Transform (CWT) and spectral analysis. The following steps were carried out in MATLAB to meet the following objectives:

- Signal Generation

The original sampling frequency (fs) was set to 100 Hz, and the duration (T) of the signal was defined as 3 seconds. A time vector t was created to represent the discrete time intervals for the signal and was transposed to obtain the column vector.

- Square Wave Generation

A square wave was synthesized by summing odd harmonics of a fundamental frequency (f0 = 5 Hz). The amplitude of each harmonic was scaled inversely proportional to its harmonic order (A/k), where k represents the harmonic number. The number of harmonics (N) was set to 10 to approximate the square wave.

- Resampling

The original signal was resampled to a higher sampling frequency (fsup = 200 Hz) to study the effects of resampling on signal integrity. The resample function in MATLAB was used, with the resampling ratio (p/q) calculated using the rat function to ensure precise resampling. The syntaxes used in the code are below:

```
fsup = 200; % New Sampling Frequency
[p, q] = rat(fsup/fs);
squareup = resample(y1, p, q); % Resampling
```

The resampling of the square wave signal aimed to enhance its temporal resolution, offering a more detailed representation in the time domain while effectively capturing any high-frequency components at the updated sampling rate. This process enabled a comparative time-frequency analysis between the original and resampled signals, shedding light on the impact of the altered sampling rate on the analysis's accuracy and precision.

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- Continuous Wavelet Transform (CWT) Analysis

The CWT was applied to both the original and resampled signals to analyse their time-frequency characteristics. The cwt function in MATLAB was used to compute and visualize the wavelet coefficients, providing insights into the signal's frequency components over time.

```
cwt(y1, fs) % CWT for original square wave
cwt(squareup, fsup) % CWT for resampled square wave
```

- Spectral Analysis

Power spectral density (PSD) analysis was performed using the pspectrum function to compare the frequency content of the original and resampled signals. This analysis helped identify any distortions or aliasing introduced during resampling. The PSD provides a quantitative measure of the signal's power distribution across different frequencies.

- Time-Frequency Analysis using FFT

To gain deeper insight into the spectral features of the square wave pre- and post-resampling, the Fast Fourier Transform (FFT) was applied to both the initial and resampled signals. This technique offers frequency-domain details, enabling a clear comparison of harmonic content, spectral leakage, and aliasing phenomena. Later, the FFT results will be compared to the CWT for both the original and resampled square wave.

III. RESULTS

The results of the analysis are mainly presented in two parts: Time-Frequency analysis using the Continuous Wavelet Transform (CWT), which produces a scalogram, and spectral analysis using the pspectrum function in MATLAB. The second part of the results displays the FFT of the original and the resampled square wave and collates it with the CWT.

CWT Analysis of the Original Signal

In the Continuous Wavelet Transform (CWT) plot for the original square wave (Figure 1), the signal's time-frequency localization is depicted, where the x-axis represents time in seconds, and the y-axis corresponds to frequency in hertz (Hz). The magnitude of wavelet coefficients is visualized using color intensity, with brighter areas indicating higher magnitudes.

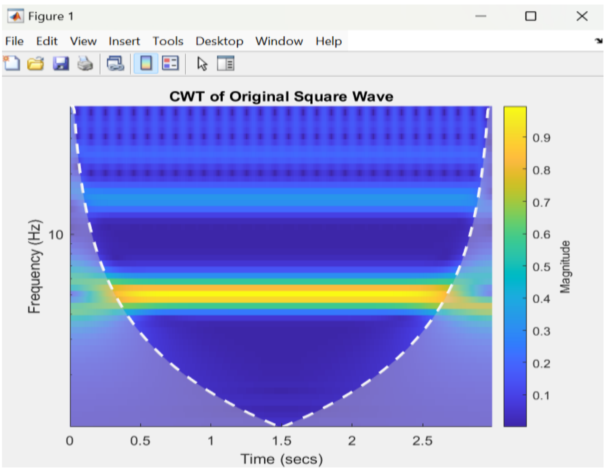

Fig .1 Scalogram of the Original Square Wave

- The vertical lines in the scalogram illustrate the abrupt transitions of the square wave, standing out as bright patterns at regular intervals, which align with the square wave's periodic nature.

- The resolution in the time domain is influenced by the original sampling rate (100 Hz), leading to slightly softened or blurred transitions in the scalogram.

- The horizontal lines in the scalogram correspond to the fundamental frequency (5 Hz) and its odd harmonics (15 Hz, 25 Hz), appearing as bright bands at integer multiples of the fundamental frequency.

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- The harmonic amplitudes decrease as the frequency increases, demonstrated by the fading brightness of higher-frequency bands. This observation aligns with the theoretical structure of a square wave, where each harmonic's amplitude diminishes in inverse proportion to its harmonic number.

CWT analysis of the resampled Square Wave

The scalogram for the resampled square wave (Figure 2) shows similar time-frequency characteristics but with improved resolution due to the new higher sampling rate (200 Hz).

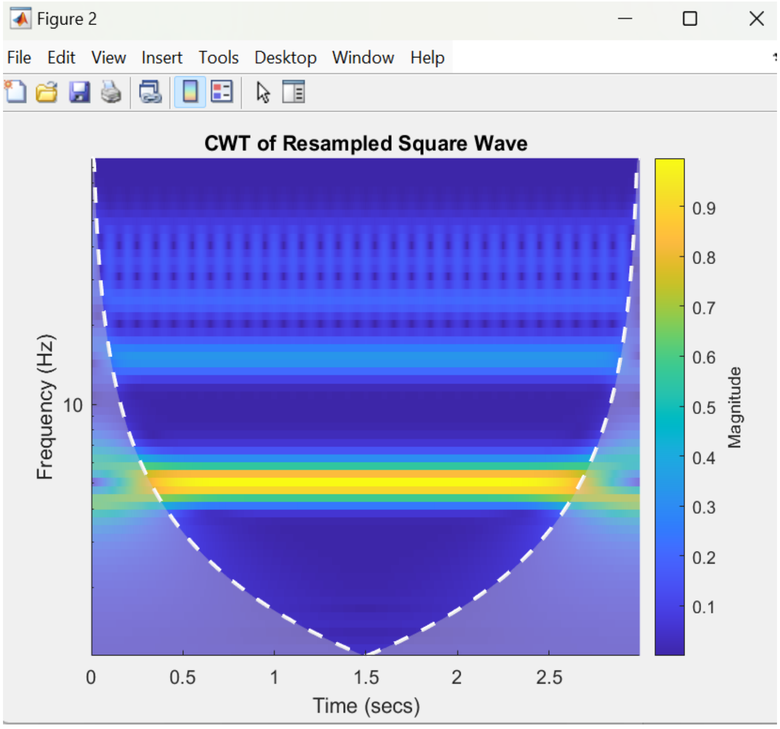

Fig.2 Scalogram of the Resampled Square Wave

- The square wave transitions in the scalogram appear sharper and more distinct for the resampled signal, compared to the original signal. This improvement in clarity stems from the increased sampling rate, which enhances the time-domain resolution.

- The periodic behaviour of the square wave remains evident, marked by bright vertical lines occurring at consistent intervals.

- The horizontal bands, which depict the fundamental frequency and its odd harmonics, continue to be present with identical frequency spacing as in the original signal. However, the higher sampling rate produces a more refined representation of these harmonic elements.

- The harmonic amplitudes reduce with rising frequencies, as anticipated, though the higher-frequency bands are better defined due to decreased spectral leakage

Power Spectrum Analysis of the original Square Wave

The power spectrum plot illustrates the power spectral density (PSD) of the original square wave signal, with the y-axis representing power in decibels (dB) and the x-axis denoting frequency in Hz. It shows how the signal's power is distributed among various frequencies, emphasizing the fundamental frequency and its harmonics.

The power spectrum plot displays the power spectral density (PSD) of the original square wave signal in decibels (dB) on the y-axis and frequency in Hz on the x-axis.

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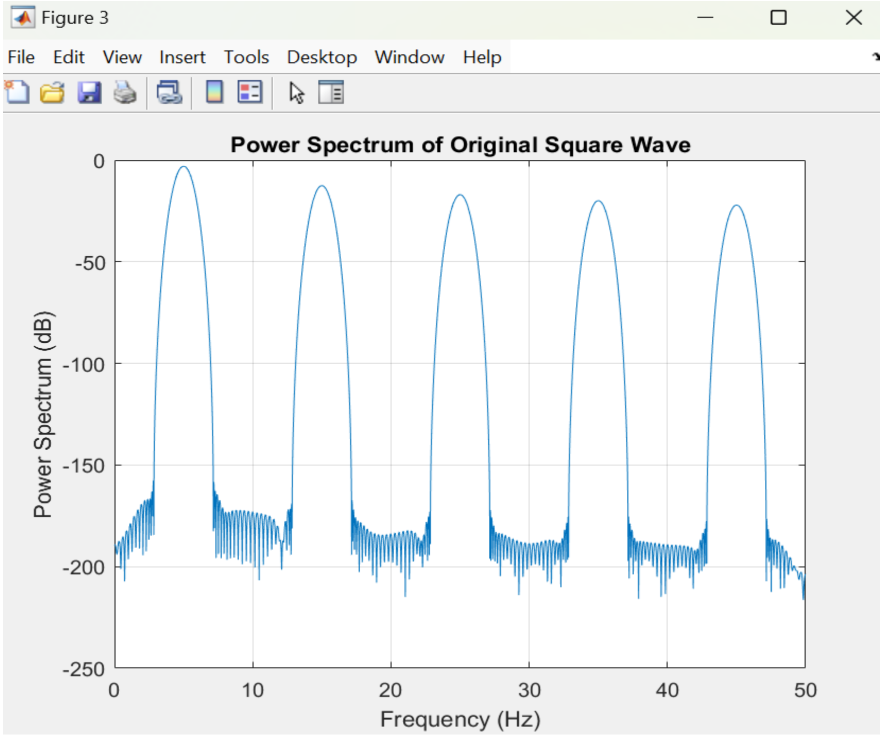

Fig.3 Power Spectrum of the Original Square Wave

- The power spectrum reveals a prominent peak at 5 Hz, representing the fundamental frequency of the square wave and its dominant frequency component. This peak has the highest amplitude, signifying that the fundamental frequency holds the greatest power within the signal.

- Additionally, distinct peaks are evident at 15 Hz, 25 Hz, 35 Hz, and 45 Hz, corresponding to the odd harmonics of the fundamental frequency. These peaks align with the theoretical structure of a square wave, which is formed by the summation of its odd harmonics.

- As expected, the amplitude of these harmonic peaks diminishes with increasing frequency. For instance, the peak at 15 Hz is smaller than the one at 5 Hz, while the 25 Hz peak is smaller than the 15 Hz peak. This trend follows the theoretical principle that each harmonic's amplitude is inversely proportional to its harmonic number

Power Spectrum Analysis of the Resampled Wave

The power spectrum of the resampled square wave provides a comprehensive view of the frequency components after the signal has been resampled at a higher rate. The higher sampling rate enhances frequency resolution, making the harmonic components more distinct and easier to identify. The power spectrum of the resampled square wave provides a comprehensive view of the frequency components after the signal has been resampled at a higher rate.

The power spectrum of the resampled square wave provides a comprehensive view of the frequency components after the signal has been resampled at a higher rate. The higher sampling rate enhances frequency resolution, making the harmonic components more distinct and easier to identify. The fundamental frequency remains dominant, with its peak clearly visible, alongside well-defined peaks for the odd harmonics, which align with the theoretical composition of a square wave.

Additionally, the higher sampling rate reduces spectral leakage, resulting in sharper representations of higher-frequency harmonics. The expected trend of decreasing harmonic amplitudes with increasing frequency is still evident, consistent with the theoretical behavior of a square wave. The power spectrum plot displays the power spectral density (PSD) of the resampled square wave signal in decibels (dB) on the y-axis and frequency in Hz on the x-axis. The harmonics are evenly spaced at intervals of 10 Hz, which is twice the fundamental frequency (5 Hz). This spacing is consistent with the presence of only odd harmonics in a square wave.

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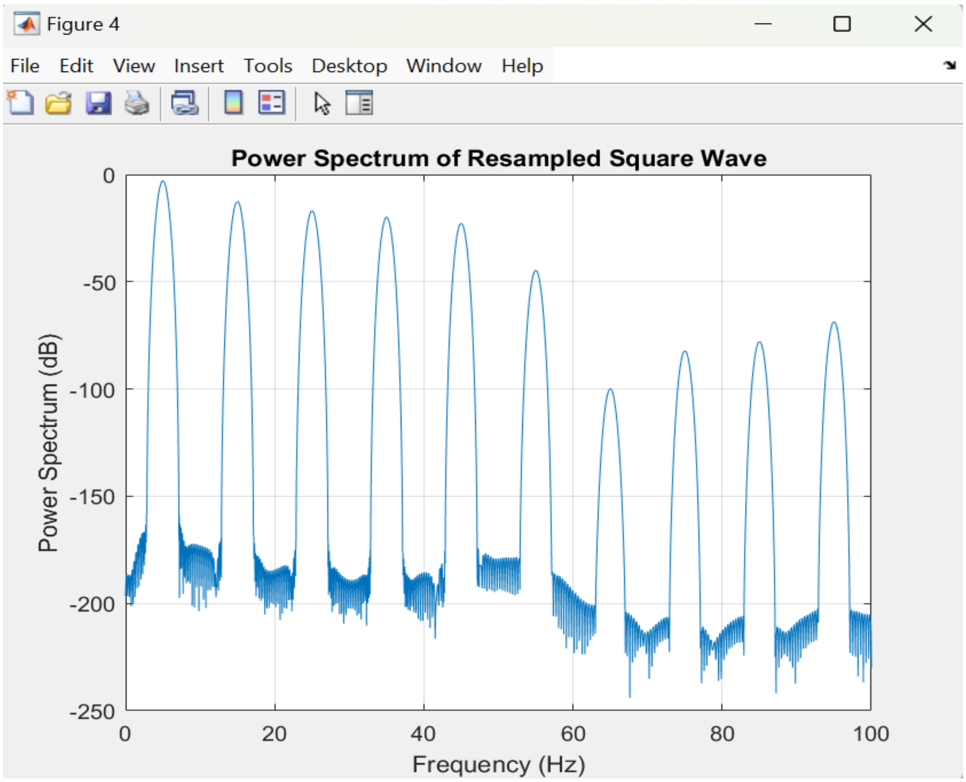

Fig.4 Power Spectrum of the Resampled Square Wave

- The power spectrum of the resampled square wave reveals a prominent peak at 5 Hz, corresponding to the fundamental frequency and confirming its role as the dominant frequency component in the signal, as seen in the original.

- This peak, with the highest amplitude, demonstrates that the fundamental frequency retains the greatest power in the resampled signal. Additionally, distinct peaks appear at 15 Hz, 25 Hz, 35 Hz, and 45 Hz, representing the odd harmonics of the fundamental frequency

- These peaks verify that the harmonic structure of the square wave remains intact after resampling. Furthermore, the amplitude of the harmonic peaks decreases with increasing frequency, following the theoretical expectation for a square wave.

- For instance, the peak at 15 Hz has a lower amplitude than the one at 5 Hz, and the peak at 25 Hz is smaller than the peak at 15 Hz.

FFT analysis and Comparison with the CWT

- The FFT plots in figure 5 and figure 6 reveal the spectral characteristics of both the original and resampled square wave signals respectively. The original signal displays distinct peaks at the anticipated harmonic frequencies (5 Hz, 15 Hz, ..., 45 Hz), confirming precise harmonic synthesis.

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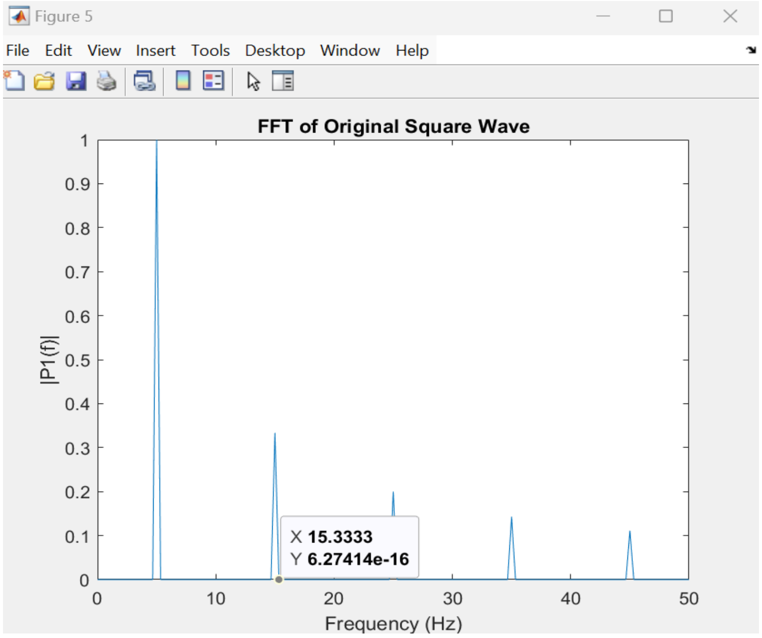

Fig.5 FFT of the Original Square Wave

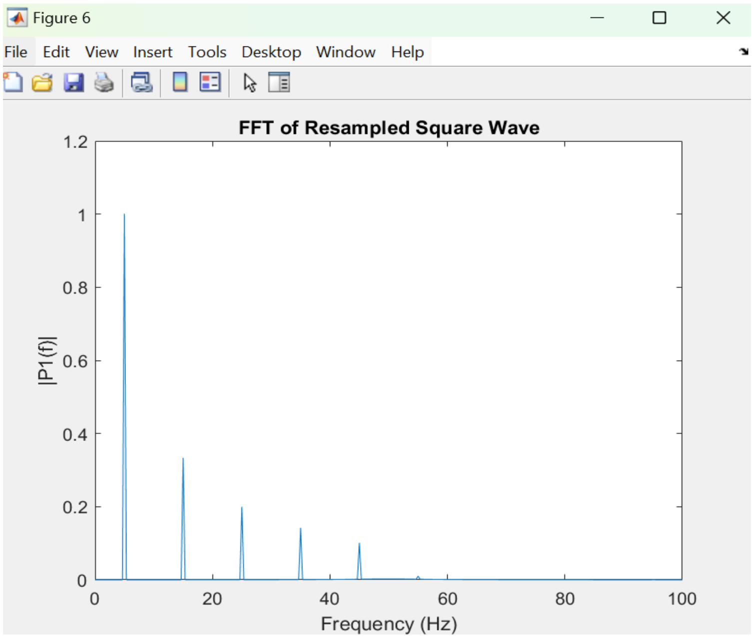

Fig.6 FFT of the Resampled Square Wave

- After resampling, while the fundamental frequency remains prominent, the amplitudes of higher-order harmonics diminish slightly, accompanied by minor spectral smearing—evidence of interpolation-induced artifacts. This aligns with findings from the CWT analysis and highlights the importance of time-frequency methods for evaluating transient signals like square waves.

- In conclusion, FFT complements CWT by offering high frequency resolution, while CWT excels in providing both time and frequency localization.

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IV. CONCLUSION

This research highlights how resampling square waves can affect both their spectral quality and waveform accuracy, particularly in scenarios where higher sampling rates are required due to hardware constraints. By constructing the square wave via its classic Fourier series of odd harmonics and resampling it from 100 Hz to 200 Hz, the study analyzed changes observed in the time and frequency domains. The application of the Continuous Wavelet Transform (CWT) showcased superior time-frequency localization over traditional approaches, uncovering transient features caused by resampling[4]. The findings indicate that, while resampling enhances temporal resolution, it may also lead to spectral leakage or aliasing issues if not carefully handled [5]. Additionally, the CWT proved more effective than Fourier Transform in studying non-stationary signal patterns like square waves, especially under conditions of dynamic sampling. These insights are vital for fields such as real-time digital signal processing, embedded systems, and communication protocols, where maintaining signal integrity throughout processing stages is critical. A comparative evaluation of FFT and CWT was performed to assess their efficiency in representing the signal's frequency characteristics. The FFT effectively illustrated dominant harmonics but fell short in providing time resolution, rendering it less effective for capturing localized variations from resampling. In contrast, the CWT offered a combined time-frequency representation, showcasing both spectral components and their temporal changes. Although CWT demands higher computational resources, its capability to uncover transient phenomena makes it particularly advantageous for analyzing non-stationary or dynamically sampled signals.

V. FUTURE SCOPE

This study establishes a foundation for several exciting research directions. Firstly, the approach could be applied to other waveforms like triangular, sawtooth, and chirp signals to examine how resampling affects their spectral and temporal properties, enhancing its relevance to areas such as audio processing, biomedical signal analysis, and telecommunications. Future investigations could delve into the effects of different harmonic counts, the implications of downsampling, and comparisons with other analytical techniques like the Short-Time Fourier Transform (STFT) or the Hilbert-Huang Transform (HHT) to expand the scope of analysis [6]. Thirdly, studying the influence of varying harmonic counts during square wave synthesis may identify the ideal number of harmonics to preserve signal fidelity under resampling, which is essential for digital signal processing and control applications. Lastly, analyzing the effects of downsampling on square waves could provide insights for optimizing bandwidth and storage in contexts like data compression and low-power embedded systems[7]. These future directions would deepen the understanding of resampling and support the development of more advanced signal processing methods across diverse fields.

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