



Reaching Law based Intelligent Sliding Mode Controller

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Abstract: This paper presents design of reaching law based intelligent Sliding Mode Controller to improve the control of 2 degree of freedom serial robotic joints. Sliding Mode Control is very prominent control structure for robust control of the nonlinear systems. In this study, Radial Basis Function based intelligent sliding mode control is designed. This novel Control structure has two important components. First one is reaching laws that ensured finite-time convergence of the system trajectories to the sliding surface while reducing the chattering and second part of the control structure is an adaptive Radial Basis Function Neural Network that addresses the unknown system dynamics and external disturbances. The combined approach improve the accuracy, stability, and robust under model variations. The systems closed loop stability analysis is guaranteed by Lyapunov stability and adaptive laws are derived for online updating of neural network parameters. The proposed intelligent structure is implemented to robotic joints and simulation results demonstrate the effectiveness of the proposed control structure by achieving more accurate tracking, low control effort, and better disturbance rejection compared to conventional sliding mode control.

Keywords: Sliding Mode Control, Reaching Law, Radial Basis Function, Robust Control.

I. INTRODUCTION

The control of nonlinear system subject to parameter variations, modeling uncertainties and external disturbances remains a prime challenge in the field of control engineering. Many techniques have been proposed to address these issues and among them sliding mode control technique gained popularity due to its inherent robustness, disturbance rejection ability and easy to implementation [1], [2]. Due to such advantages of Sliding mode control, It becomes very popular in many areas such as motion control, process control, robotics and electrical drives [3], [4], [5], [6]. In [3], authors presented a basic principal, mathematical aspects and design methodology of sliding mode control for electric drive. In [4], Authors presented a simple design scheme for robot manipulator using a sliding mode control. In [5], Authors introduced a variable structure control for unpredictable chaotic systems exhibiting input nonlinearity. A sliding mode control technique was applied in [6] to minimize the disparity between power generation and consumption of electric power system.

SMC has many advantages so it became popular but it has also disadvantage: chattering, that is due to discontinuous nature of control and caused high frequency vibrations [7]. Chattering can excite unmodeled dynamics that cause damage to actuators. To mitigate the chattering effect researchers proposed a concept of reaching laws in SMC [8]. Reaching Laws directs the system dynamics during reaching phased to the sliding surface and allows the designer to meticulously control the rate of convergence and reducing the chattering without affecting the robustness [9] and also provide an improvement in transient performance.

The efficacy of SMC is depending upon availability of plant information and precise mathematical modeling of the system. However, real-time plants have nonlinearity and uncertainty that are difficult to describe effectively, resulting in suboptimal control actions. To overcome this problem, intelligent estimation methods using Radial Basis Function Neural Network (RBFNN) employed into the sliding mode control structure. RBF have characteristics of universal approximation and able to learn complicated nonlinear functions in real time [10], [11].



RBF networks minimize the dependency on exact system mathematical modeling by approximating the unknown system dynamics and disturbances.

Many researchers have tried to combine the advantage of SMC and RBFNN for smooth and robust control. For example, in [12], authors applied adaptive-RBF based sliding mode control to estimate unknown model parameters and minimize uncertainties in nonlinear systems. On the other hand, reaching law based SMC is applied in many applications, including robotic manipulators [13], induction motor drives [14], and active suspension [15] demonstrating the reducing the chattering effect and optimizing transient performance. However, limited research has focused on integrating the systematic advantages of reaching law design with the adaptive estimation power of RBFNN in a single control structure.

In this paper, we propose a Reaching law based Radial Basis Function Neural Network Sliding Mode Control structure for 2 degree of freedom serial flexible robotic joints. The proposed control structure employs three different reaching laws - constant, exponential and power rate reaching laws to ensure finite time convergence to the sliding surface with reducing chattering. A structure is also integrated with adaptive RBFNN to approximate the unknown nonlinearities and eliminating external disturbances in real time. The stability of the system is analyzed using Lyapunov theory to ensure system is uniformly ultimately bounded. Adaptive laws are designed to update the weights of RBFNN.

The effectiveness of the proposed control strategy is validated through simulation studies on a 2 degree of freedom serial flexible joint robotic arm. The results demonstrate that the proposed RBFNN - Reaching Law based SMC outperforms conventional SMC in terms of tracking accuracy, robustness to disturbances, and chattering suppression.

This paper is structured as follows. In section II presents the Preliminary of SMC and reaching laws. In section III introduction to RBFNN structure is presented. Section IV is dedicated to design of reaching law based SMC-RBFNN control structure with Lyapunov stability along with adaptive mechanism for online updating of RBFNN weights. In section V Simulations are performed and results are compared of different reaching laws with RBFNN. Finally, section 6 having the conclusions.

II. SLIDING MODE CONTROL AND REACHING LAWS

Sliding Mode Control is a nonlinear control strategy that has advantages of robustness, parameter insensitivity, precision, and easy to implement. The design of an SMC has two stages. In the initial phase, we design the sliding surface, followed by the design of the control input in the subsequent phase. Consider a second order system

$$\ddot{x} = f(x) + g(x).u + dt \quad (1)$$

Let us define the error, difference between desired trajectory and actual trajectory as

$$e = (x_{des} - x) \quad (2)$$

Sliding surface S can be designed as

$$S = \left(\frac{d}{dt} + K\right)^{n-1} e \quad (3)$$

Where, n = relative degree between Input and Output.

K = Constant value, e is an error between output signal y and desired signal y_{des} . For second order dynamical system, sliding surface σ is

$$S = \left(\frac{d}{dt} + K\right)^1 e \quad (4)$$

$$S = Ke + \dot{e}$$

$$\dot{S} = K(\dot{e}) + (\ddot{e}) \quad (5)$$



$$\dot{S} = K(\dot{e}) + (\ddot{x}_{des} - \ddot{x})$$

$$\dot{S} = K(\dot{e}) + (\ddot{x}_{des} - f(x) - g(x).u - dt) \tag{6}$$

Following the construction of the sliding surface, a comprehensive control law is established that can reduce S to zero in finite time with minimal chattering. The control structure of SMC comprises two components: reaching mode and sliding mode.

If f(x) and g(x) are known in the above equation we can write a Sliding Mode control u as a

$$u = \frac{1}{g(x)}(\ddot{x}_{des} + K(\dot{e}) - f(x) + u_{swi}). \tag{7}$$

Three basic reaching law based switching control function can be written as

- 1) Constant rate.

$$u_{swi} = \alpha \operatorname{sgn}(S) \tag{8}$$

- 2) Constant plus proportional.

$$u_{swi} = \alpha \operatorname{sgn}(S) + k S \tag{9}$$

- 3) Power rate reaching law.

$$u_{swi} = \alpha S^\lambda \operatorname{sgn}(S) \tag{10}$$

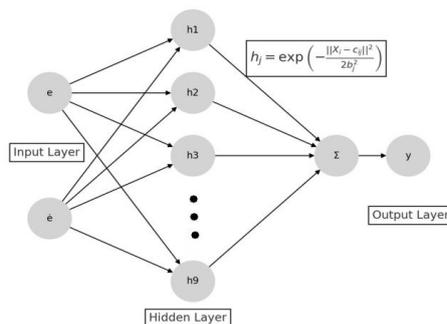
The control signal (7) along with any of three reaching law can be implemented only when there is a complete information of f(x) and g(x). However, generally f(x) is not known or cannot be easily calculated so it should be estimated for effective control of dynamical system.

III. RADIAL BASIS FUNCTION NEURAL NETWORK

Figure 1 shows the architecture of the Radial Basis Function Neural Network and flow of information to each layer. The RBF has 1st Input layer that receive an Input information, 2nd layer a that contain a Radial Basis Function activation function in each neuron and Output layer having a Linear activation function. The information goes from input layer ton Hidden layer along with associated weights. Hidden Layer receives the information and computes the output using Gaussian Function as

$$h_j = \exp\left(-\frac{|x_i - c_{ij}|^2}{2b_j^2}\right), \tag{11}$$

Where b and c are center and width of the neuron.



Radial Basis Function (RBF) Network Schematic

Figure. 1 Radial Basis Function (RBF) Neural Network Structure



The output calculated by Radial Basis Function goes to the output layer with output weights. Then the final output is calculated as

$$Y = v_1h_1 + v_2h_2 + v_3h_3 + v_4h_4 + \dots + v_jh_j \tag{12}$$

IV. DESIGN OF REACHING LAW BASED RBFNN SLIDING MODE CONTROLLER

This section presents design of reaching law based radial basis function sliding mode controller. In the control equation 7 if $f(x)$ is not known then we can use radial basis function neural network for estimating the function $f(x)$. So, we can write the equating 7 as

$$u = \frac{1}{g(x)}(\ddot{x}_{des} + K(\dot{e}) - \hat{f}(x) + u_{swi}), \tag{13}$$

Where, $\hat{f}(x)$ is the function approximated by RBFNN.

Now submitting equation (13) in to equation (6)

$$\dot{S} = K(\dot{e}) + (\ddot{x}_{des} - f(x) - g(x)\left(\frac{1}{g(x)}(\ddot{x}_{des} + K(\dot{e}) - \hat{f}(x) + u_{swi})\right) - dt) \tag{14}$$

$$= K(\dot{e}) - f(x) - K(\dot{e}) + \hat{f}(x) - u_{swi} - dt \tag{15}$$

$$= -f(x) + \hat{f}(x) - u_{swi} - dt \tag{16}$$

$$= -\tilde{f}(x) - u_{swi} - dt \tag{17}$$

Where,

$$\tilde{f}(x) = \hat{f}(x) - f(x) = v^{*T}h(x) + \epsilon - \hat{v}^T h(x) = \tilde{v}^T h(x) + \epsilon$$

and

$$\tilde{v} = v^* - \hat{v}$$

Now define the Lyapunov function

$$V = \frac{1}{2}S^2 + \frac{1}{2}\eta \tilde{v}^T \tilde{v}, \text{ Where } \eta > 0. \tag{18}$$

Now, taking the derivative of V

$$\dot{V} = S \dot{S} + \eta \tilde{v}^T \dot{\tilde{v}} = S(-\tilde{f}(x) - u_{swi} - dt) - \eta \tilde{v}^T \dot{\hat{v}} \tag{19}$$

$$= S(-\tilde{v}^T h(x) - \epsilon - u_{swi} - dt) - \eta \tilde{v}^T \dot{\hat{v}} \tag{20}$$

$$= -\tilde{v}^T (S h(x) + \eta \dot{\hat{v}}) - S(\epsilon + dt + u_{swi})$$

Now, define the adaptive laws as

$$\dot{\hat{v}} = -\frac{1}{\eta} S h(x) \tag{21}$$

Then

$$\dot{V} = -S(\epsilon + dt + u_{swi}). \tag{23}$$

Now if we place switching law in above equation

For constant rate $u_{swi} = \alpha \operatorname{sgn}(S)$

$$\dot{V} = -S(\epsilon + dt + \alpha \operatorname{sgn}(S)) \tag{24}$$



$$= -S(\epsilon + dt) - \alpha |S| \quad (25)$$

If we choose $\alpha > \epsilon + dt$ $\dot{V} < 0$ and system will be asymptotically stable.

For constant plus proportional rate $u_{swi} = \alpha \operatorname{sgn}(S) + k S$

$$\dot{V} = -S(\epsilon + dt + \alpha \operatorname{sgn}(S) + k S) \quad (26)$$

$$= -S(\epsilon + dt) - \alpha |S| - S k S \quad (27)$$

If α and k are greater than $\epsilon + dt$, $\dot{V} < 0$ and system will be asymptotically stable.

For power reaching law $u_{swi} = \alpha S^\lambda \operatorname{sgn}(S)$

$$\dot{V} = -S(\epsilon + dt + \alpha |S|^\lambda \operatorname{sgn}(S)) \quad (28)$$

$$= -S(\epsilon + dt) - \alpha |S|^{\lambda+1} \quad (29)$$

If α is greater than $\epsilon + dt$, $\dot{V} < 0$ and system will be asymptotically stable.

V. SIMULATION AND RESULTS

To validate the effectiveness of the proposed Reaching Law-Based Radial Basis Function Sliding Mode Control (RBF-SMC) scheme, numerical simulations were conducted on a 2-degree-of-freedom (2-DOF) serial flexible joints robotic arm. The mathematical model of the plant is given as:

$$\begin{aligned} \ddot{x}_{11} &= -141.2872 x_{11} + 141.2872 x_{12} - 70.6434 \dot{x}_{11} + 140.0419 u_1 \\ \ddot{x}_{12} &= 39.0594 x_{11} - 39.0594 x_{12} - 70.6434 \dot{x}_{11} \\ \ddot{x}_{21} &= -1140.9 x_{21} + 1140.9 x_{22} - 142.6 \dot{x}_{21} + 248.1524 u_2 \\ \ddot{x}_{22} &= 373 x_{21} - 373 x_{22} - 141.2872 \dot{x}_{22} - 2.6 \dot{x}_{22} \end{aligned}$$

We used 2-5-1 RBF Neural Network structure to build the controller. We select the initial weight of RBF as 0.1, Gaussian function centre and width are selected as [-0.1 ; -0.050 0 0.05 0.1] and [5; 5; 5; 5; 5]. The desired trajectory is chosen as a square wave of 20 degree angular position with 0.1 Hz frequency. The simulation was performed in MATLAB/SIMULINK for period of 10 seconds with a sampling time of 0.001 s. The initial conditions of the system were set to [0; 0; 0; 0] and a bounded disturbance random signal with amplitude of 4 degree was added to evaluate robustness.

The controller parameter for reaching law is selected as mentioned in table 1.

TABLE I REACHING LAW CONTROLLER PARAMETER

Sr.No	Reaching Law	Parameters
1	constant rate	$\alpha = 35$
2	Constant plus Proportional rate	$\alpha = 35, k = 5$
3	power reaching law	$\alpha = 35, \lambda = 0.5$

Fig. 1 shows the position control of serial flexible joint robotic arm for the desired trajectory of 20 degree square wave and corresponding control signal for stage 1 and Fig. 2 shows for the stage 2 using constant reaching law based RBFNN sliding mod control. From Fig. 1 and Fig. 2 it can be seen that maximum overshoot for the stage 1 is 0.38 degree i.e. 1.9 % and steady state error is 0.02 while for stage 2 the maximum overshoot is 2.1 % and steady state error is 0.02 degree. From the figures it can be seen that heavy chattering is occurring in both the joints.

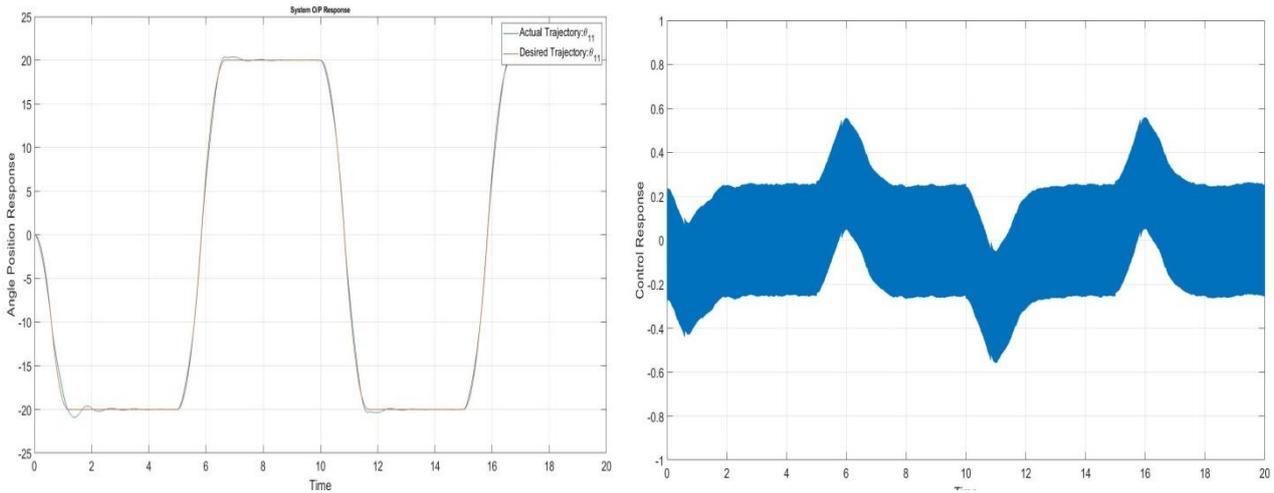


Fig.1 System output response and Control response for stage 1: Constant Reaching Law

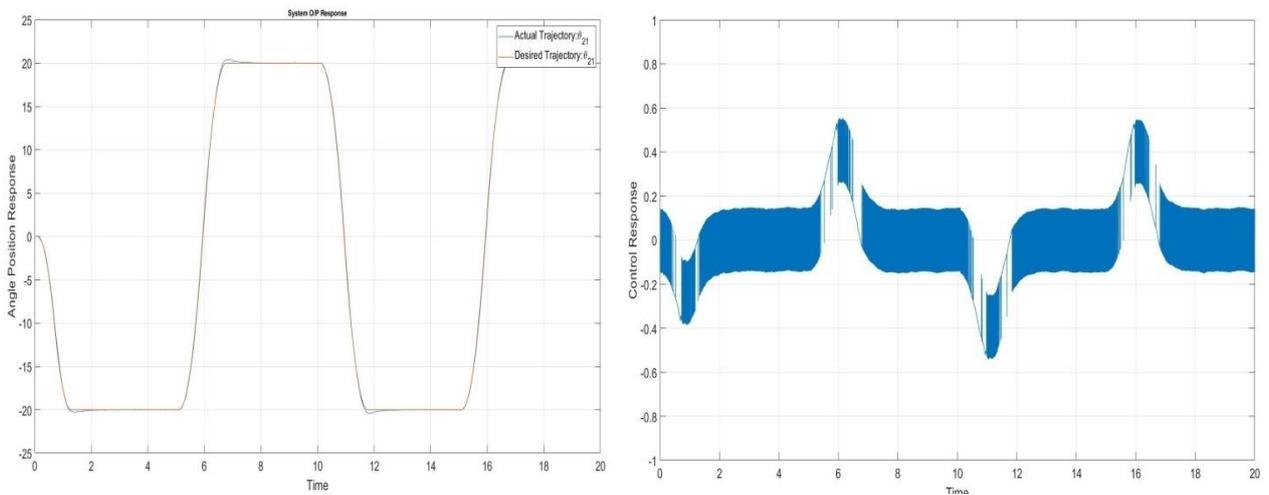


Fig.2 System output response and Control response for stage 2: Constant Reaching Law

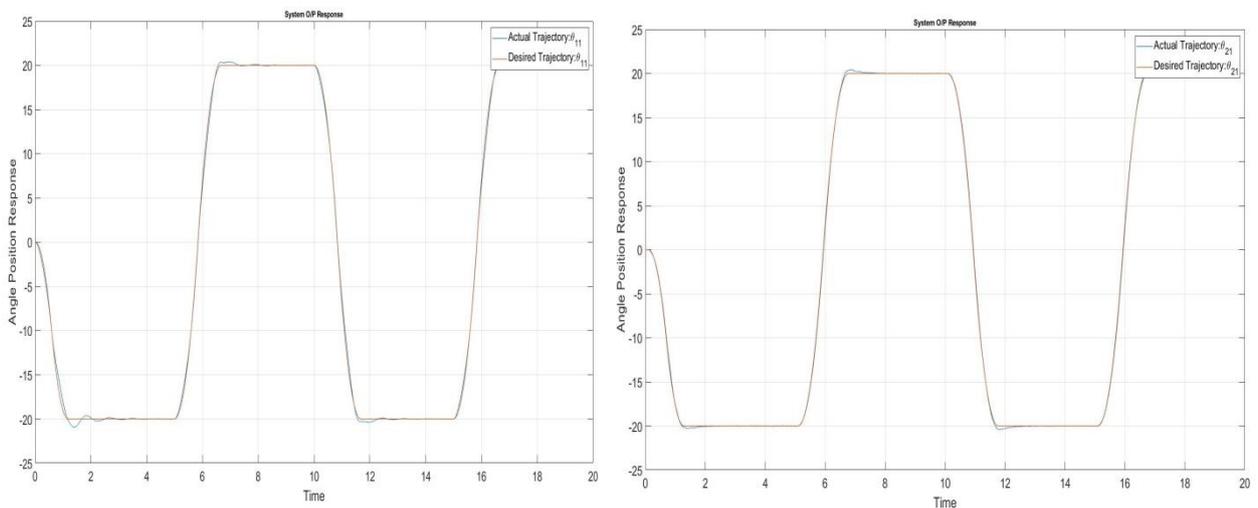


Fig.3 System output response and Control response for stage 1: Constant+Proportional Reaching Law



Fig. 3 shows the position control of serial flexible joint robotic arm for the desired trajectory of 20 degree square wave and corresponding control signal for stage 1 and Fig. 4 shows for the stage 2 for constant plus proportional reaching law based RBFNN sliding mode control. From Fig. 3 and Fig. 4 it can be seen that maximum overshoot for the stage 1 is 0.36 degree i.e. 1.8 % and steady state error is 0.015 while for stage 2 the maximum overshoot is 2.15 % and steady state error is 0.02 degree. In this case also heavy chattering is observed in both the joints.

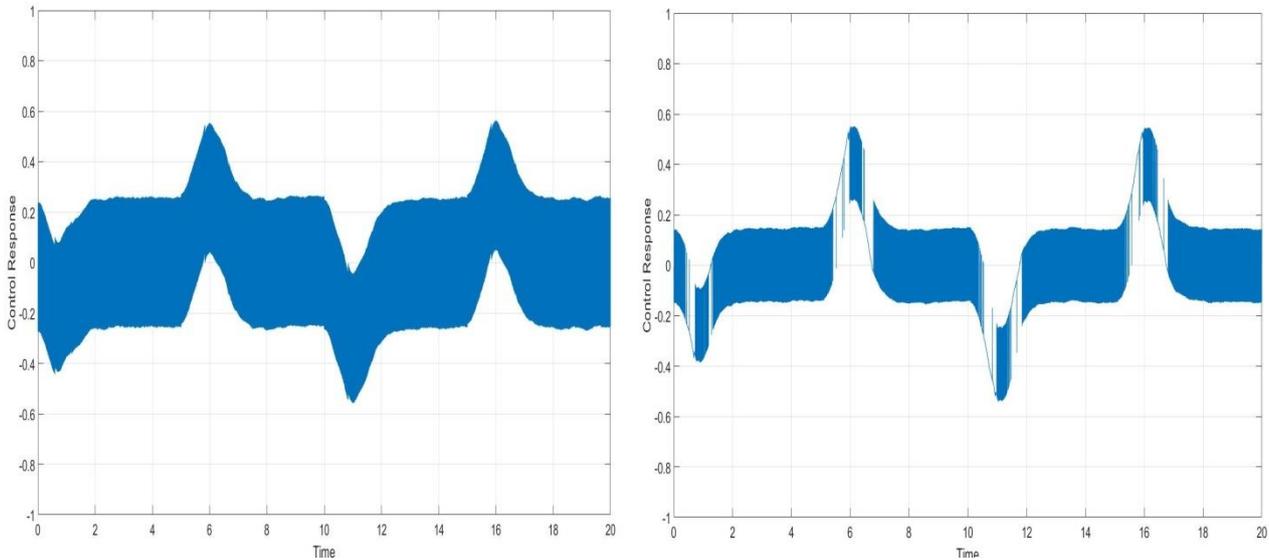


Fig.4 System output response and Control response for stage 1: Constant Proportional Reaching Law

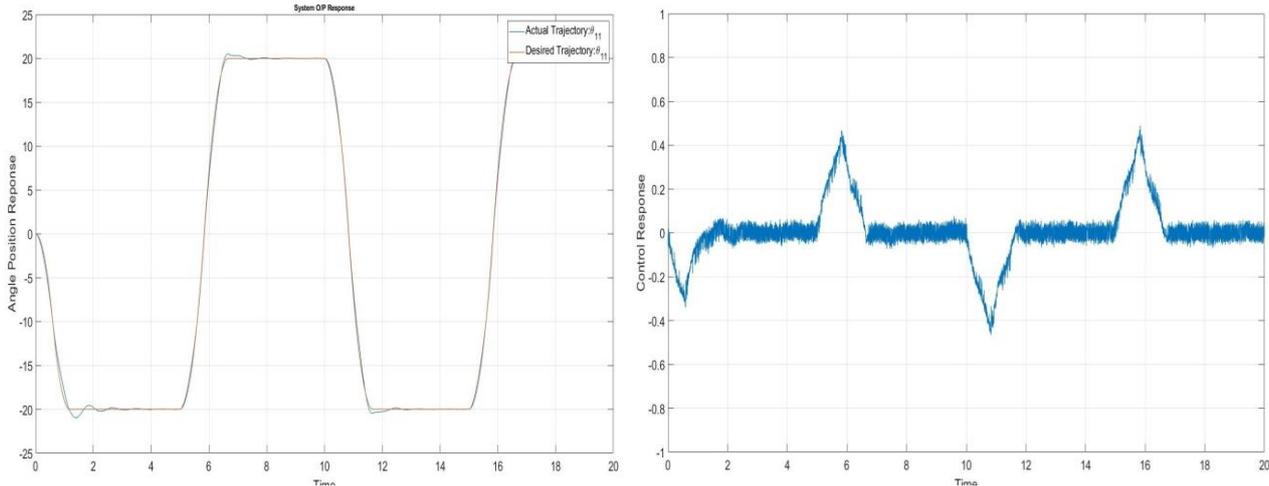


Fig.5 System output response and Control response for stage 1: Power Reaching Law

Fig. 5 and Fig. 6 show the tracking and control response for serial flexible joint robotic arm for the desired trajectory of 20 degree square wave and corresponding control signal for stage 1 and stage 2 for power reaching law based RBFNN sliding mode control. From Fig. 5 and Fig. 6 it can be seen that maximum overshoot for the stage 1 is 0.5 degree i.e. 2.5 % and steady state error is 0.02 while for stage 2 the maximum overshoot is 2.0 % and steady state error is 0.02 degree. However, it can be seen that chattering is very much minimized in the case of Power rate reaching law in comparison to the other two reaching laws. This law chattering can reduce the vibration and less wear and tear to the actuator.

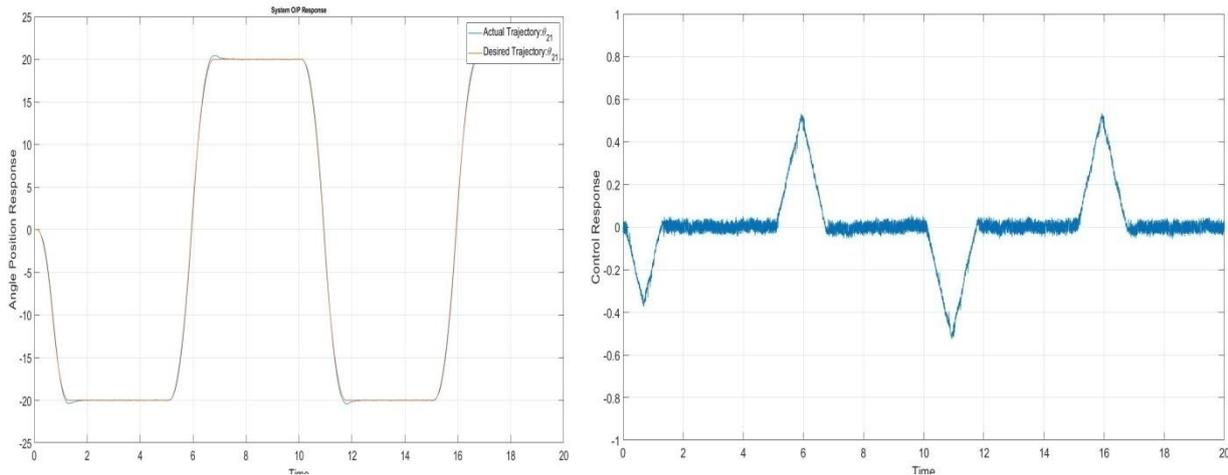


Fig.6 System output response and Control response for stage 2: Power Reaching Law

VI. CONCLUSION

This paper describe the method for designing of three reaching law based radial basis function NN sliding mode control for the serial joint robotic arm. The control design consists of reaching laws -constant, constant plus proportional and power rate reaching laws and function approximation using the radial basis function neural network to estimate the model uncertainties and variation of parameters. The stability of the controller is verified by the Lyapunov stability and finite time convergence stability is analysed. The simulation results for all three control structure presents and it can be seen that power rate reaching law based radial basis function NN is able to reduce the chattering effect in control signal along with good tracking accuracy and disturbance rejection.

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