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ROUGH HESITANT NEUTROSOPHIC SETS AND ITS APPLICATION IN MULTI CRITERIA DECISION MAKING

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Abstract: In this paper, rough hesitant neutrosophic sets are introduced. Also applying this set to multi criteria decision making problem. In addition an algorithm to handle decision making problem in online teaching company to select staff's are studied. Finally, a numerical example is employed to demonstrate the validness of the proposed rough hesitant neutrosophic sets.

I. INTRODUCTION

In this chapter we define rough hesitant neutrosophic set. Some operations of rough hesitant neutrosophic set are established. Moreover arithmetic mean operators and geometric mean operators of rough hesitant neutrosophic set are defined. Properties of these operators are proved. Score and accuracy function of rough hesitant neutrosophic sets are introduced. We develop multi-criteria decision making method based on the proposed operators. Finally we solve a numerical example to illustrate the feasibility, applicability and efficiency of the proposed methods.

II. ROUGH HESITANT NEUTROSOPHIC SETS

In this section we have to introduce the rough hesitant Neutrosophic set.

Definition 2.1. Let U be the universal set and ξ be an equivalence relation on U. Let H be the hesitant Neutrosophic set of U. The lower and upper approximations of H in the approximation (U, ξ) denoted by $\underline{H}$ and $\overline{H}$ and defined as follows:

$$\underline{H} = \langle \left(\mathrm{h}, \underline{H_t}(h), \underline{H_i}(h), \underline{H_f}(h) \right), \mathrm{h} \in \mathrm{U} \rangle$$

$$\overline{\mathrm{H}} = \langle \left(\mathrm{h}, \overline{H_t}(h), \overline{H_i}(h), \overline{H_f}(h) \right), \mathrm{h} \in \mathrm{U} \rangle$$

Where

$$\underline{H_t}(h) = \wedge_{s \in [h]_\xi} H_t(s)$$

$$\underline{H_i}(h) = \vee_{s \in [h]_\xi} H_i(s)$$

$$\underline{H_f}(h) = \vee_{s \in [h]_\xi} H_f(s)$$

Also

$$\overline{H_t}(h) = \vee_{s \in [h]_\xi} H_t(s)$$

$$\overline{H_i}(h) = \wedge_{s \in [h]_\xi} H_i(s)$$

$$\overline{H_f}(h) = \wedge_{s \in [h]_\xi} H_f(s)$$

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Example 2.2. Let $U = \{a, b, c, d, e, f\}$ be the universal set. Let H be the hesitant Neutrosophic set defined by

a	(0.9,0.8,1)	(0.3,0.2,0)	(0.1,0.3,0)
b	(0.7,0.8,0.9)	(0.1,0.2,0.1)	(0.2,0.1,0.2)
c	(0.8,0.8,0.7)	(0.2,0.3,0.4)	(0.1,0.2,0.3)

Let ξ be a congruence relations on H such that congruence classes are the subsets are given by $\{\{a\}, \{b, c\}\}$. Then the lower and upper approximations of H are given by,

a	(0.9,0.8,1)	(0.3,0.2,0)	(0.1,0.3,0)
b	(0.7,0.8,0.9)	(0.1,0.3,0.4)	(0.1,0.2,0.3)
c	(0.7,0.8,0.9)	(0.1,0.3,0.4)	(0.1,0.2,0.3)

And

a	(0.9,0.8,1)	(0.3,0.2,0)	(0.1,0.3,0)
b	(0.8,0.8,0.7)	(0.2,0.2,0.1)	(0.2,0.1,0.2)
c	(0.8,0.8,0.7)	(0.2,0.2,0.1)	(0.2,0.1,0.2)

Definition 2.3. Let $\xi(H_1)$ and $\xi(H_2)$ be two rough hesitant Neutrosophic fuzzy sets. Then $\xi(H_1) \subseteq \xi(H_2)$ if and only if the following conditions holds:

$$\underline{H_{1t}}(h) \leq \underline{H_{2t}}(h)$$

$$\underline{H_{1i}}(h) \geq \underline{H_{2i}}(h)$$

$$\underline{H_{1f}}(h) \geq \underline{H_{2f}}(h)$$

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And

$$\overline{H_{1t}}(h) \leq \overline{H_{1t}}(h)$$

$$\overline{H_{1i}}(h) \geq \overline{H_{1i}}(h)$$

$$\overline{H_{1f}}(h) \geq \overline{H_{1f}}(h)$$

Definition 2.4. Let $\xi(H_1)$ and $\xi(H_2)$ be two rough hesitant Neutrosophic fuzzy sets. Then $\xi(H_1) \cup \xi(H_2)$ is defined as follows.

$$(\underline{H_{1t}} \cup \underline{H_{2t}})(h) = max\left\{\underline{H_{1t}}(h), \underline{H_{2t}}(h)\right\}$$

$$(\underline{H_{1i}} \cup \underline{H_{2i}})(h) = min\left\{\underline{H_{1i}}(h), \underline{H_{2i}}(h)\right\}$$

$$(\underline{H_{1f}} \cup \underline{H_{2f}})(h) = min\left\{\underline{H_{1f}}(h), \underline{H_{2f}}(h)\right\}$$

And
$$(\overline{H_{1t}} \cup \overline{H_{2t}})(h) = max\{\overline{H_{1t}}(h), \overline{H_{2t}}(h)\}$$

$$(\overline{H_{1i}} \cup \overline{H_{2i}})(h) = min\{\overline{H_{1i}}(h), \overline{H_{2i}}(h)\}$$

$$(\overline{H_{1f}} \cup \overline{H_{2f}})(h) = min\{\overline{H_{1f}}(h), \overline{H_{2f}}(h)\}$$

Example 2.5 Consider the rough hesitant Neutrosophic set in example 2.2 . Then the union is given by,

A	(0.9,0.8,0.8)	(0.3,0.2,0)	(0.3,0.3,0)
B	(0.7,0.2,0.4)	(0.4,0.2,0.3)	(0.8,0.2,0.3)
C	(0.7,0.8,0.8)	(0.1,0.3,0.4)	(0.3,0.2,0.3)

And

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a	(0.9,0.1,0.2)	(0.4,0.2,0)	(0.8,0.1,0)
b	(0.8,0.2,0.4)	(0.4,0.2,0.1)	(0.8,0.1,0.2)
c	(0.8,0.1,0.2)	(0.4,0.2,0.1)	(0.8,0.1,0.2)

Definition 2.5. Let $\xi(H_1)$ and $\xi(H_2)$ be two rough hesitant Neutrosophic fuzzy sets. Then $\xi(H_1) \cap \xi(H_2)$ is defined as follows.

$$(\underline{H_{1t}} \cap \underline{H_{2t}})(h) = min\left\{\underline{H_{1t}}(h), \underline{H_{2t}}(h)\right\}$$

$$(\underline{H_{1i}} \cap \underline{H_{2i}})(h) = max\left\{\underline{H_{1i}}(h), \underline{H_{2i}}(h)\right\}$$

$$(\underline{H_{1f}} \cap \underline{H_{2f}})(h) = max\left\{\underline{H_{1f}}(h), \underline{H_{2f}}(h)\right\}$$

And

$$(\overline{H_{1t}} \cap \overline{H_{2t}})(h) = min\{\overline{H_{1t}}(h), \overline{H_{2t}}(h)\}$$

$$(\overline{H_{1i}} \cap \overline{H_{2i}})(h) = max\{\overline{H_{1i}}(h), \overline{H_{2i}}(h)\}$$

$$(\overline{H_{1f}} \cap \overline{H_{2f}})(h) = max\{\overline{H_{1f}}(h), \overline{H_{2f}}(h)\}$$

Example 2.7 Consider the rough hesitant Neutrosophic set in example 2.2 . Then the intersection is given by,

a	(0,0.9,1)	(0.1,0.6,0.5)	(0.1,0.7,0.9)
b	(0.1,0.8,0.9)	(0.1,0.3,0.4)	(0.1,0.7,0.6)
c	(0,0.9,0.9)	(0.1,0.6,0.5)	(0.1,0.7,0.9)

And

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a	(0.7,0.8,0.2)	(0.3,0.2,0.2)	(0.1,0.3,0.2)
b	(0.1,0.8,0.7)	(0.2,0.2,0.3)	(0.2,0.7,0.6)
c	(0.7,0.8,0.7)	(0.2,0.2,0.2)	(0.2,0.1,0.2)

Definition 2.8. Let H be rough hesitant Neutrosophic fuzzy set . Then the complement of H, H^c is defined as follows:

$$\underline{H^c}(h) = \left\{\underline{H_f}(h), 1 - \underline{H_i}(h), \underline{H_t}(h)\right\}$$

And
$$\overline{H^c}(h) = \{\overline{H_f}(h), 1 - \overline{H_i}(h), \overline{H_t}(h)\}$$

For all $h \in H$.

Definition 2.9. If H_1 and H_2 be two rough hesitant Neutrosophic fuzzy sets. Then we define the following

1. $H_1 = H_2$ if and only if $\underline{H_1} = \underline{H_2}$ and $\overline{H_1} = \overline{H_2}$.
2. $H_1 \subseteq H_2$ if and only if $\underline{H_1} \subseteq \underline{H_2}$ and $\overline{H_1} \subseteq \overline{H_2}$.
3. $H_1 \cup H_2$ if and only if $\underline{H_1} \cup \underline{H_2}$ and $\overline{H_1} \cup \overline{H_2}$.
4. $H_1 \cap H_2$ if and only if $\underline{H_1} \cap \underline{H_2}$ and $\overline{H_1} \cap \overline{H_2}$.
5. $H_1 + H_2$ if and only if $\underline{H_1} + \underline{H_2}$ and $\overline{H_1} + \overline{H_2}$.
6. $H_1 \circ H_1$ if and only if $\underline{H_1} \circ \underline{H_2}$ and $\overline{H_1} \circ \overline{H_2}$.

Definition 2.10. Let H_1 and H_2 be two rough hesitant Neutrosophic fuzzy sets. Then $H_1 \oplus H_2$ is defined as follows:

$$\underline{H_{1t}}(h) \oplus \underline{H_{2t}}(h) = \underline{H_{1t}}(h) + \underline{H_{2t}}(h) - \underline{H_{1t}}(h)\,\underline{H_{2t}}(h)$$

$$\underline{H_{1i}}(h) \oplus \underline{H_{2i}}(h) = \underline{H_{1i}}(h)\,\underline{H_{2i}}(h)$$

$$\underline{H_{1f}}(h) \oplus \underline{H_{2f}}(h) = \underline{H_{1f}}(h)\,\underline{H_{2f}}(h)$$

And

$$\overline{H_{1t}}(h) \oplus \overline{H_{1t}}(h) = \overline{H_{1t}}(h) + \overline{H_{2t}}(h) - \overline{H_{1t}}(h)\,\overline{H_{2t}}(h)$$

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$$\overline{H_{1i}}(h) \oplus \overline{H_{2i}}(h) = \overline{H_{1i}}(h)\,\overline{H_{2i}}(h)$$

$$\overline{H_{1f}}(h) \oplus \overline{H_{2f}}(h) = \overline{H_{1f}}(h)\,\overline{H_{2f}}(h)$$

Definition 2.11. Let H_1 and H_2 be two rough hesitant neutrosophic fuzzy sets. Then $H_1 \otimes H_2$ is defined as follows:

$$\underline{H_{1t}}(h) \otimes \underline{H_{2t}}(h) = \underline{H_{1t}}(h)\,\underline{H_{2t}}(h)$$

$$\underline{H_{1i}}(h) \otimes \underline{H_{2i}}(h) = \underline{H_{1i}}(h) + \underline{H_{2i}}(h) - \underline{H_{1i}}(h)\,\underline{H_{2i}}(h)$$

$$\underline{H_{1f}}(h) \otimes \underline{H_{2f}}(h) = \underline{H_{1f}}(h) + \underline{H_{2f}}(h) - \underline{H_{1f}}(h)\,\underline{H_{2f}}(h)$$

And

$$\overline{H_{1t}}\,(h) \otimes \overline{H_{1t}}(h) = -\overline{H_{1t}}(h)\,\overline{H_{2t}}(h)$$

$$\overline{H_{1i}}(h) \otimes \overline{H_{2i}}(h) = \overline{H_{1i}}(h) + \overline{H_{2i}}(h) - \overline{H_{1i}}(h)\,\overline{H_{2i}}(h)$$

$$\overline{H_{1f}}(h) \otimes \overline{H_{2f}}(h) = \overline{H_{1f}}(h) + \overline{H_{2f}}(h) - \overline{H_{1f}}(h)\,\overline{H_{2f}}(h)$$

III. ROUGH HESITANT NEUTROSOPHIC ARITHMETIC MEAN OPERATORS

This section deals with the rough hesitant neutrosophic arithmetic mean operators.

Definition 3.1 Let $H_i = (\underline{H_i}\ ,\overline{H_i})$ in U be a set of rough hesitant neutrosophic fuzzy numbers. Then the rough hesitant Neutrosophic arithmetic mean operators (RHNAMO) is defined as follows:

$$RHNAMO(H_1\,,H_2\,,,,,H_n) = \langle \frac{1}{n}\ \oplus_{i=1}^{n} \underline{H_i}\,,\frac{1}{n}\ \oplus_{i=1}^{n} \overline{H_i}\rangle$$

Theorem 3.2 Let $H_i = (\underline{H_i}\ ,\overline{H_i})$ in U be a set of rough hesitant neutrosophic fuzzy numbers. Then the aggregated value $RHNAMO(H_1\,,H_2\,,,,,H_n)$ is also a rough hesitant Neutrosophic fuzzy number.

Proof: Since $\underline{H_i}$ and $\overline{H_i}$ are hesitant Neutrosophic fuzzy numbers. From definition 3.1 we see that $\frac{1}{n}\ \oplus_{i=1}^{n} \underline{H_i}$ and $\frac{1}{n}\ \oplus_{i=1}^{n} \overline{H_i}$ are hesitant Neutrosophic fuzzy numbers. Hence $RHNAMO(H_1\,,H_2\,,,,,H_n)$ is also a rough hesitant Neutrosophic fuzzy number.

Definition 3.3 Let $H_i = (\underline{H_i}\ ,\overline{H_i})$ in U be a set of rough hesitant neutrosophic fuzzy numbers and $(w_1\,,w_2\,,\ldots.w_n)$ be the weight structure of rough hesitant neutrosophic fuzzy numbers $(H_1\,,H_2\,,,,,H_n)$. Then the weighted rough hesitant neutrosophic arithmetic mean operators (WRHNAMO) is defined as follows:

$WRHNAMO(H_1\,,H_2\,,,,,H_n) = \langle \frac{1}{n}\ \oplus_{i=1}^{n} w_i\underline{H_i}\,,\frac{1}{n}\ \oplus_{i=1}^{n} w_i\overline{H_i}\rangle$ and $\sum_{i=1}^{n} w_i\ = 1$.

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Theorem 3.4 Let $H_i = (\underline{H_i}, \overline{H_i})$ in U be a set of rough hesitant neutrosophic fuzzy numbers. Then the aggregated value $WRHNAMO(H_1, H_2, ,, , H_n)$ is also a rough hesitant Neutrosophic fuzzy number.

Proof: Since $\underline{H_i}$ and $\overline{H_i}$ are hesitant neutrosophic fuzzy numbers. From definition 3.3 we see that $\frac{1}{n} \oplus_{i=1}^{n} w_i \underline{H_i}$ and $\frac{1}{n} \oplus_{i=1}^{n} w_i \overline{H_i}$ are hesitant neutrosophic fuzzy numbers and $\sum_{i=1}^{n} w_i = 1$. Hence $WRHNAMO(H_1, H_2, ,, , H_n)$ is also a rough hesitant neutrosophic fuzzy number.

IV. PROPERTIES OF ROUGH HESITANT NEUTROSOPHIC ARITHMETIC MEAN OPERATOR

In this section we discuss about properties of rough hesitant neutrosophic arithmetic mean operators.

Theorem 4.1 If $H_i = H$ (for i=1,2,,,,n) then $RHNAMO(H_1, H_2, ,, , H_n) = H$ and $WRHNAMO(H_1, H_2, ,, , H_n) = H$.

Proof: Since $H_i = H$ then $RHNAMO(H_1, H_2, ,, , H_n) = \langle \frac{1}{n} \oplus_{i=1}^{n} \underline{H_i}, \frac{1}{n} \oplus_{i=1}^{n} \overline{H_i} \rangle$

$$= \langle \underline{H}, \overline{\mathrm{H}} \rangle = H$$

Also $WRHNAMO(H_1, H_2, ,, , H_n) = \langle \frac{1}{n} \oplus_{i=1}^{n} w_i \underline{H_i}, \frac{1}{n} \oplus_{i=1}^{n} w_i \overline{H_i} \rangle$

$$= \langle H \oplus_{i=1}^{n} w_i, H \oplus_{i=1}^{n} w_i \rangle = \langle \underline{H}, \overline{\mathrm{H}} \rangle = H$$

And $\sum_{i=1}^{n} w_i = 1$.

Theorem 4.2 Both the operators are bounded.

Proof: Let H_j (for j=1,2,,,,n) be a collection of rough hesitant neutrosophic numbers and let

$$H^- = (\min(\underline{H_{jt}}), \max(\underline{H_{ji}}), \max(\underline{H_{jf}}), \min(\overline{H_{jt}}), \max(\overline{H_{ji}}), \max(\overline{H_{jf}}))$$

$$H^+ = (\max(\underline{H_{jt}}), \min(\underline{H_{ji}}), \min(\underline{H_{jf}}), \max(\overline{H_{jt}}), \min(\overline{H_{ji}}), \min(\overline{H_{jf}}))$$

then $H^- \subseteq RHNAMO(H_1, H_2, ,, , H_n) \subseteq H^+$ and $H^- \subseteq WRHNAMO(H_1, H_2, ,, , H_n) \subseteq H^+$

Theorem 4.3. Monotonicity Property: If $H_j \subseteq H_j^*$ for j=1,2,,,,n then, $RHNAMO(H_1, H_2, ,, , H_n) \subseteq RHNAMO(H_1^*, H_2^*, ,, , H_n^*)$ and $WRHNAMO(H_1, H_2, ,, , H_n) \subseteq WRHNAMO(H_1^*, H_2^*, ,, , H_n^*)$.

Proof: Since $H_j \subseteq H_j^*$ for j=1,2,,,,n . Hence, $RHNAMO(H_1, H_2, ,, , H_n) \subseteq RHNAMO(H_1^*, H_2^*, ,, , H_n^*)$ and $WRHNAMO(H_1, H_2, ,, , H_n) \subseteq WRHNAMO(H_1^*, H_2^*, ,, , H_n^*)$.

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Theorem 4.4. Commutativity Property: If $(H_1^{\circ}, H_2^{\circ}, ,,, H_3^{\circ})$ is any permutation of $(H_1, H_2, ,,, H_n)$, then $RHNAMO(H_1, H_2, ,,, H_n) = RHNAMO(H_1^{\circ}, H_2^{\circ}, ,,, H_n^{\circ})$ and $WRHNAMO(H_1, H_2, ,,, H_n) = WRHNAMO(H_1^{\circ}, H_2^{\circ}, ,,, H_n^{\circ})$.

Proof: Since $(H_1^{\circ}, H_2^{\circ}, ,,, H_3^{\circ})$ is any permutation of $(H_1, H_2, ,,, H_n)$, then $RHNAMO(H_1, H_2, ,,, H_n) \cup RHNAMO(H_1^{\circ}, H_2^{\circ}, ,,, H_n^{\circ}) = RHNAMO(H_1, H_2, ,,, H_n) or RHNAMO(H_1^{\circ}, H_2^{\circ}, ,,, H_n^{\circ})$

Hence we have $(H_1, H_2, ,,, H_n) = RHNAMO(H_1^{\circ}, H_2^{\circ}, ,,, H_n^{\circ})$.

In similar way we can prove that $WRHNAMO(H_1, H_2, ,,, H_n) = WRHNAMO(H_1^{\circ}, H_2^{\circ}, ,,, H_n^{\circ})$.

V. ROUGH HESITANT NEUTROSOPHIC GEOMETRIC MEAN OPERATORS

This section deals with the rough hesitant neutrosophic geometric mean operators.

Definition 5.1 Let $H_i = (\underline{H_i}, \overline{H_i})$ in U be a set of rough hesitant neutrosophic fuzzy numbers. Then the rough hesitant Neutrosophic arithmetic mean operators (RHNGMO) is defined as follows: $RHNGMO(H_1, H_2, ,,, H_n) = \langle \otimes_{i=1}^{n} [\underline{H_i}]^{\frac{1}{n}}, \otimes_{i=1}^{n} [\overline{H_i}]^{\frac{1}{n}} \rangle$

Theorem 5.2 Let $H_i = (\underline{H_i}, \overline{H_i})$ in U be a set of rough hesitant neutrosophic fuzzy numbers. Then the aggregated value $RHNGMO(H_1, H_2, ,,, H_n)$ is also a rough hesitant Neutrosophic fuzzy number.

Proof: Since $\underline{H_i}$ and $\overline{H_i}$ are hesitant Neutrosophic fuzzy numbers. From definition 5.1 we see that $\otimes_{i=1}^{n} [\underline{H_i}]^{\frac{1}{n}}$ and $\otimes_{i=1}^{n} [\overline{H_i}]^{\frac{1}{n}}$ are hesitant Neutrosophic fuzzy numbers. Hence $RHNGMO(H_1, H_2, ,,, H_n)$ is also a rough hesitant Neutrosophic fuzzy number.

Definition 5.3 Let $H_i = (\underline{H_i}, \overline{H_i})$ in U be a set of rough hesitant neutrosophic fuzzy numbers and $(w_1, w_2, \ldots . w_n)$ be the weight structure of rough hesitant neutrosophic fuzzy numbers $(H_1, H_2, ,,, H_n)$. Then the weighted rough hesitant Neutrosophic arithmetic mean operators (WRHNAMO) is defined as follows:

$$WRHNGMO(H_1, H_2, ,,, H_n) = \langle \otimes_{i=1}^{n} [\underline{H_i}]^{w_i}, \otimes_{i=1}^{n} [\overline{H_i}]^{w_i} \rangle \text{ and } \sum_{i=1}^{n} w_i = 1.$$

Theorem 5.4 Let $H_i = (\underline{H_i}, \overline{H_i})$ in U be a set of rough hesitant neutrosophic fuzzy numbers. Then the aggregated value $WRHNGMO(H_1, H_2, ,,, H_n)$ is also a rough hesitant Neutrosophic fuzzy number.

Proof: Since $\underline{H_i}$ and $\overline{H_i}$ are hesitant neutrosophic fuzzy numbers. From definition 5.3 we see that $\otimes_{i=1}^{n} [\underline{H_i}]^{w_i}$ and $\otimes_{i=1}^{n} [\overline{H_i}]^{w_i}$ are hesitant neutrosophic fuzzy numbers and $\sum_{i=1}^{n} w_i = 1$. Hence $WRHNGMO(H_1, H_2, ,,, H_n)$ is also a rough hesitant neutrosophic fuzzy number.

VI. PROPERTIES OF ROUGH HESITANT NEUTROSOPHIC GEOMETRIC MEAN OPERATOR

In this section we discuss about properties of rough hesitant neutrosophic geometric mean operators.

Theorem 6.1 If $H_i = H$ (for i=1,2,,,,n) then $RHNGMO(H_1, H_2, ,,, H_n) = H$ and $WRHNGMO(H_1, H_2, ,,, H_n) = H$.

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Proof: Since $H_i = H$ then $RHNGMO(H_1, H_2, ,,, H_n) = \langle \otimes_{i=1}^{n} [\underline{H_i}]^{\frac{1}{n}}, \otimes_{i=1}^{n} [\overline{H_i}]^{\frac{1}{n}} \rangle$

$$= \langle \underline{H}, \overline{\mathrm{H}} \rangle = H$$

Also $WRHNGMO(H_1, H_2, ,,, H_n) = \langle \otimes_{i=1}^{n} [\underline{H_i}]^{w_i}, \otimes_{i=1}^{n} [\overline{H_i}]^{w_i} \rangle$

$$= \langle H \otimes_{i=1}^{n} w_i, H \otimes_{i=1}^{n} w_i \rangle = \langle \underline{H}, \overline{\mathrm{H}} \rangle = H$$

And $\sum_{i=1}^{n} w_i = 1$.

Theorem 6.2 Both the operators are bounded.

Proof: Let H_j (for j=1,2,,,,n) be a collection of rough hesitant neutrosophic numbers and let

$H^- = (\min(\underline{H_{jt}}), \max(\underline{H_{ji}}), \max(\underline{H_{jf}}), \min(\overline{H_{jt}}), \max(\overline{H_{ji}}), \max(\overline{H_{jf}}))$

$H^+ = (\max(\underline{H_{jt}}), \min(\underline{H_{ji}}), \min(\underline{H_{jf}}), \max(\overline{H_{jt}}), \min(\overline{H_{ji}}), \min(\overline{H_{jf}}))$

then $H^- \subseteq RHNGMO(H_1, H_2, ,,, H_n) \subseteq H^+$ and $H^- \subseteq WRHNGMO(H_1, H_2, ,,, H_n) \subseteq H^+$

Theorem 6.3 Monotonicity Property: If $H_j \subseteq H_j^*$ for j=1,2,,,,n then, $RHNGMO(H_1, H_2, ,,, H_n) \subseteq RHNGMO(H_1^*, H_2^*, ,,, H_n^*)$ and $WRHNGMO(H_1, H_2, ,,, H_n) \subseteq WRHNGMO(H_1^*, H_2^*, ,,, H_n^*)$.

Proof: Since $H_j \subseteq H_j^*$ for j=1,2,,,,n . Hence, $RHNGMO(H_1, H_2, ,,, H_n) \subseteq RHNGMO(H_1^*, H_2^*, ,,, H_n^*)$ and $WRHNGMO(H_1, H_2, ,,, H_n) \subseteq WRHNGMO(H_1^*, H_2^*, ,,, H_n^*)$.

Theorem 6.4. Commutativity Property: If $(H_1^\circ, H_2^\circ, ,,, H_3^\circ)$ is any permutation of $(H_1, H_2, ,,, H_n)$, then $RHNGMO(H_1, H_2, ,,, H_n) = RHNGMO(H_1^\circ, H_2^\circ, ,,, H_n^\circ)$ and $WRHNGMO(H_1, H_2, ,,, H_n) = WRHNGMO(H_1^\circ, H_2^\circ, ,,, H_n^\circ)$.

Proof: Since $(H_1^\circ, H_2^\circ, ,,, H_3^\circ)$ is any permutation of $(H_1, H_2, ,,, H_n)$, then $RHNGMO(H_1, H_2, ,,, H_n) \cup RHNGMO(H_1^\circ, H_2^\circ, ,,, H_n^\circ) = RHNGMO(H_1, H_2, ,,, H_n) or RHNGMO(H_1^\circ, H_2^\circ, ,,, H_n^\circ)$

Hence we have $(H_1, H_2, ,,, H_n) = RHNGMO(H_1^\circ, H_2^\circ, ,,, H_n^\circ)$.

In similar way we can prove that $WRHNGMO(H_1, H_2, ,,, H_n) = WRHNGMO(H_1^\circ, H_2^\circ, ,,, H_n^\circ)$.

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VII. SCORE AND ACCURACY FUNCTION OF ROUGH HESITANT NEUTROSOPHIC FUZZY ENVIRONMENT

Definition 7.1 Assume that $H = (\underline{H}\ , \overline{H})$ be a rough hesitant neutrosophic fuzzy number. The score and accuracy function of H are defined as follows:

$$S(H) = \frac{9 + \underline{H_t} + \overline{H_t} - \underline{H_i} - \overline{H_i} - \underline{H_f} - \overline{H_f}}{18}$$

And

$$A(H) = \frac{9 + \underline{H_t} + \overline{H_t} - \underline{H_f} - \overline{H_f}}{2}$$

Where $S(H) \in [0,1]$ and $A(H) \in [-1,1]$.

VIII. MULTI CRITERIA DECISION MAKING METHODS BASED ON ARITHMETIC MEAN OPERATORS

This section deals with multi criteria decision making method based on arithmetic mean operators of rough hesitant neutrosophic fuzzy sets. Moreover we introduced algorithms for rough hesitant neutrosophic arithmetic mean operators, weighted rough hesitant neutrosophic arithmetic mean operators, rough hesitant neutrosophic geometric mean operators and weighted rough hesitant neutrosophic arithmetic mean operators. The relation between alternatives and criteria in terms of rough hesitant neutrosophic numbers.

	C_1	C_2	…	C_n
H_1	$\langle(\underline{H_{11}}, \underline{H_{11}}, \underline{H_{11}}), (\overline{H_{11}}, \overline{H_{11}}, \overline{H_{11}})\rangle$	$\langle(\underline{H_{12}}, \underline{H_{12}}, \underline{H_{12}}), (\overline{H_{12}}, \overline{H_{12}}, \overline{H_{12}})\rangle$	….	$\langle(\underline{H_{1n}}, \underline{H_{1n}}, \underline{H_{1n}}), (\overline{H_{1n}}, \overline{H_{1n}}, \overline{H_{1n}})\rangle$
H_2	$\langle(\underline{H_{21}}, \underline{H_{21}}, \underline{H_{21}}), (\overline{H_{21}}, \overline{H_{21}}, \overline{H_{21}})\rangle$	$\langle(\underline{H_{22}}, \underline{H_{22}}, \underline{H_{22}}), (\overline{H_{22}}, \overline{H_{22}}, \overline{H_{22}})\rangle$	….	$\langle(\underline{H_{2n}}, \underline{H_{2n}}, \underline{H_{2n}}), (\overline{H_{2n}}, \overline{H_{2n}}, \overline{H_{2n}})\rangle$
…..	…..	……		…..
H_m	$\langle(\underline{H_{m1}}, \underline{H_{m1}}, \underline{H_{m1}}), (\overline{H_{m1}}, \overline{H_{m1}}, \overline{H_{m1}})\rangle$	$\langle(\underline{H_{m2}}, \underline{H_{m2}}, \underline{H_{m2}}), (\overline{H_{m2}}, \overline{H_{m2}}, \overline{H_{m2}})\rangle$	….	$\langle(\underline{H_{mn}}, \underline{H_{mn}}, \underline{H_{mn}}), (\overline{H_{mn}}, \overline{H_{mn}}, \overline{H_{mn}})\rangle$

8.1 ALGORITHM FOR ROUGH HESITANT NEUTROSOPHIC ARITHMETIC MEAN OPERATOR

1. Decision maker forms a rough hesitant neutrosophic number decision matrix. The relation between alternative $H_i(i = 1,2, \dots m)$ and criterion $C_j(j = 1,2, \dots n)$ is given in table. Here $\langle(\underline{H_{ij}}, \underline{H_{ij}}, \underline{H_{ij}}), (\overline{H_{ij}}, \overline{H_{ij}}, \overline{H_{ij}})\rangle$ is rough hesitant neutrosophic number relating value of the H_i with respect to the criterion C_j for decision maker.
2. By Definition 3.1 determine the aggregation values for the decision matrix.
3. By Definition 5.1 determine the score values and accuracy values.
4. All the score values are arranged in descending order. If tie occurs in score values, then the accuracy values are considered for making preference rank order. The alternative corresponding to the highest score value(accuracy value) corresponds the best choice.

8.2 ALGORITHM FOR WEIGHTED ROUGH HESITANT NEUTROSOPHIC ARITHMETIC MEAN OPERATOR

1. Decision maker forms a rough hesitant neutrosophic number decision matrix. The relation between alternative $H_i(i = 1,2, \dots m)$ and criterion $C_j(j = 1,2, \dots n)$ is given in table. Here $\langle(\underline{H_{ij}}, \underline{H_{ij}}, \underline{H_{ij}}), (\overline{H_{ij}}, \overline{H_{ij}}, \overline{H_{ij}})\rangle$ is

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rough hesitant neutrosophic number relating value of the H_i with respect to the criterion C_j for decision maker.
2. Determine the criteria weights.
3. By Definition 3.3 determine the aggregation values for the decision matrix.
4. By Definition 5.1 determine the score values and accuracy values.
5. All the score values are arranged in descending order. If tie occurs in score values, then the accuracy values are considered for making preference rank order. The alternative corresponding to the highest score value(accuracy value) corresponds the best choice.

8.3 ALGORITHM FOR ROUGH HESITANT NEUTROSOPHIC GEOMETRIC MEAN OPERATOR

1. Decision maker forms a rough hesitant neutrosophic number decision matrix. The relation between alternative $H_i(i = 1,2, \dots m)$ and criterion $C_j(j = 1,2, \dots n)$ is given in table. Here $\langle (\underline{H_{ij}}, \underline{H_{ij}}, \underline{H_{ij}}), (\overline{H_{ij}}, \overline{H_{ij}}, \overline{H_{ij}}) \rangle$ is rough hesitant neutrosophic number relating value of the H_i with respect to the criterion C_j for decision maker.
2. By Definition 3.1 determine the aggregation values for the decision matrix.
3. By Definition 5.1 determine the score values and accuracy values.
4. All the score values are arranged in descending order. If tie occurs in score values, then the accuracy values are considered for making preference rank order. The alternative corresponding to the highest score value(accuracy value) corresponds the best choice.

8.4 ALGORITHM FOR WEIGHTED ROUGH HESITANT NEUTROSOPHIC GEOMETRIC MEAN OPERATOR

1. Decision maker forms a rough hesitant neutrosophic number decision matrix. The relation between alternative $H_i(i = 1,2, \dots m)$ and criterion $C_j(j = 1,2, \dots n)$ is given in table. Here $\langle (\underline{H_{ij}}, \underline{H_{ij}}, \underline{H_{ij}}), (\overline{H_{ij}}, \overline{H_{ij}}, \overline{H_{ij}}) \rangle$ is rough hesitant neutrosophic number relating value of the H_i with respect to the criterion C_j for decision maker.
2. Determine the criteria weights.
3. By Definition 3.3 determine the aggregation values for the decision matrix.
4. By Definition 5.1 determine the score values and accuracy values.
5. All the score values are arranged in descending order. If tie occurs in score values, then the accuracy values are considered for making preference rank order. The alternative corresponding to the highest score value(accuracy value) corresponds the best choice.

IX. NUMERICAL EXAMPLE BASED ON PROPOSED METHODS

In this section, we present a numerical example for the applicability of the proposed methods. Suppose an online teaching organization wants to introduce excellent teachers to improve the level of teaching. There are three teachers who are selected by the teaching experts. Based on the priority level, the criteria of investigation is successively morality (C_1) , teaching capacity (C_2) and educational experience (C_3). Then the rough hesitant neutrosophic matrix is presented in the following table.

	C_1	C_2	C_3
H_1	$\langle$ [(0.9,0.80.1), (0.9,0.8,0.1)], [(0.3,0.2,0), (0.3,0.2,0)], [(0.1,0.3,0), (0.1,0.3,0)] $\rangle$	$\langle$ [(0,0.9,0.8), (0.7,0.1,0.2)], [(0.1,0.6,0.5), (0.4,0.2,0.2)], [(0.3,0.7,0.9), (0.8,0.1,0.2)] $\rangle$	$\langle$ [(0.1,0.8,0.7), (0.6,0,0.1)] [(0,0.5,0.4), (0.4,0.1,0.1)], [(0.2,0.6,0.8), (0.7,0,0.1)] $\rangle$
H_2	$\langle$ [(0.7,0.8,0.9), (0.8,0.8,0.7)], [(0.1,0.3,0.4), (0.2,0.2,0.1)], [(0.1,0.2,0.3), (0.2,0.1,0.2)] $\rangle$	$\langle$ [(0.1,0.2,0.4), (0.1,0.2,0.4)], [(0.4,0.2,0.3), (0.4,0.2,0.3)], [(0.8,0.7,0.6), (0.8,0.7,0.6)] $\rangle$	$\langle$ [(0,0.1,0.3), (0,0.1,0.3)] [(0.3,0.1,0.2), (0.3,0.1,0.2)], [(0.7,0.6,0.5), (0.7,0.6,0.5)] $\rangle$

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H_m	$\langle[(0.7,0.8,0.9),(0.8,0.8,0.7)],[(0.1,0.3,0.4),(0.2,0.2,0.1)],[(0.1,0.2,0.3),(0.2,0.1,0.2)]\rangle$	$\langle[(0,0.9,0.8),(0.7,0.1,0.2)],[(0.1,0.6,0.5),(0.4,0.2,0.2)],[(0.3,0.7,0.9),(0.8,0.1,0.2)]\rangle$	$\langle[(0.1,0.8,0.7),(0.6,0,0.1)],[(0,0.5,0.4),(0.4,0.1,0.1)],[(0.2,0.6,0.8),(0.7,0,0.1)]\rangle$

9.1 SOLUTION USING ROUGH HESITANT NEUTROSOPHIC ARITHMETIC MEAN OPERATORS

1. The relation between the alternatives and criteria are given by the table
2. The aggregation values for the decision matrix is

$$H_1 = \langle [(0.33,0.192,0.017),(0.607,0,0.001)], [(0.133,0.02,0),(0.350,0.001,0)], [(0.198,0.042,0),(0.575,0,0)] \rangle$$

$$H_2 = \langle [(0.267,0.005,0.036),(0.3,0.005,0.028)], [(0.263,0.002,0.008),(0.292,0.001,0.02)], [(0.515,0.028,0.03),(0.529,0.014,0.02)] \rangle$$

$$H_3 = \langle [(0.233,0.192,0.168),(0.588,0,0.009)], [(0.067,0.03,0.027),(0.292,0.001,0.0007)], [(0.198,0.028,0.072),(0.53,0.0003,0.001)] \rangle$$

3. The score values are

$$S(H_1) = 0.6066$$

$$S(H_2) = 0.6094$$

$$S(H_3) = 0.5721$$

Since all the score values are different, in this case there is no need to calculate the accuracy values.

4. All the score values are arranged in descending order, $S(H_2) \geq S(H_1) \geq S(H_3)$

Hence $S(H_2)$ is the best choice.

9.2 SOLUTION USING WEIGHTED ROUGH HESITANT NEUTROSOPHIC ARITHMETIC MEAN OPERATORS

1. The relation between the alternatives and criteria are given by the table
2. The aggregation values for the decision matrix is

$$H_1 = \langle [(0.3318,0.1911,0.019),(0.6045,0,0.007)], [(0.1361,0.0204,0),(0.3566,0.0014,0)], [(0.1957,0.0643,0),(0.5069,0,0)] \rangle$$

$$H_2 = \langle [(0.2654,0.0053,0.0358),(0.2986,0.0053,0.0279)], [(0.2678,0.0020,0.0082),(0.2978,0.0014,0.0204)], [(0.5069,0.0276,0.03),(0.5213,0.0138,0.0200)] \rangle$$

$$H_3 = \langle [(0.2323,0.1911,0.1672),(0.5853,0,0.0093)], [(0.0678,0.0306,0.0272),(0.2978,0.0014,0.0007)], [(0.1950,0.0276,0.071),(0.5213,0.0003,0.0013)] \rangle$$

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3. The score values are

$$S(H_1) = 0.6019$$

$$S(H_2) = 0.6089$$

$$S(H_3) = 0.5502$$

Since all the score values are different, in this case there is no need to calculate the accuracy values.

4. All the score values are arranged in descending order, $S(H_2) \geq S(H_1) \geq S(H_3)$

Hence $S(H_2)$ is the best choice.

9.3 SOLUTION USING ROUGH HESITANT NEUTROSOPHIC GEOMETRIC MEAN OPERATORS

1. The relation between the alternatives and criteria are given by the table
2. The aggregation values for the decision matrix is

$$H_1 = \left\langle \begin{matrix} [(0,1.1245,1.1541),(0.7254,0.965,0.738)], \\ [(0,1.0736,0.966),(0.3671,0.793,0.672)], \\ [(0.185,1.1366,0.191),(0.387,0.739,0.672)] \end{matrix} \right\rangle$$

$$H_2 = \left\langle \begin{matrix} [(0,1.027,1.141),(0,1.028,1.095)], \\ [(0.232,0.842,0.957),(0.292,0.793,0.842)], \\ [(0.386,1.1216,1.093),(0.486,1.106,1.074)] \end{matrix} \right\rangle$$

$$H_3 = \left\langle \begin{matrix} [(0,1.241,1.235),(0.698,0.966,1.023)], \\ [(0,1.093,1.068),(0.292,0.793,0.738)], \\ [(0.185,1.122,1.210),(0.485,0.671,0.0793)] \end{matrix} \right\rangle$$

3. The score values are

$$S(H_1) = 0.0380$$

$$S(H_2) = 0.0957$$

$$S(H_3) = 0.0718$$

Since all the score values are different, in this case there is no need to calculate the accuracy values.

4. All the score values are arranged in descending order, $S(H_2) \geq S(H_1) \geq S(H_3)$

Hence $S(H_2)$ is the best choice.

9.4 SOLUTION USING WEIGHTED ROUGH HESITANT NEUTROSOPHIC GEOMETRIC MEAN OPERATORS

1. The relation between the alternatives and criteria are given by the table. The weights of criteria are $w_1 = 0.3318$ $w_2 = 0.3399$ and $w_3 = 0.3283$
2. The aggregation values for the decision matrix is

$$H_1 = \left\langle \begin{matrix} [(0,1.247,1.153),(0.724,0.965,0.739)], \\ [(0,1.076,0.966),(0.365,0.788,0.674)], \\ [(0.1831,1.141,1.190),(0.384,0.732,0.674)] \end{matrix} \right\rangle$$

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$$H_2 = \left\langle \begin{array}{c} [(0,1.028,1.140),(0,1.028,1.094)], \\ [(0.230,0.0838,0.958),(0.290,0.788,0.143)], \\ [(0.384,1.126,1.093),(0.484,1.110,1.0732)] \end{array} \right\rangle$$

$$H_3 = \left\langle \begin{array}{c} [(0,1.249,1.233),(0.888,0.965,1.023)], \\ [(0,1.096,1.067),(0.290,0.788,0.739)], \\ [(0.183,1.126,1.209),(0.484,0.663,0.794)] \end{array} \right\rangle$$

3. The score values are

$$S(H_1) = 0.0374$$

$$S(H_2) = 0.0906$$

$$S(H_3) = 0.0732$$

Since all the score values are different, in this case there is no need to calculate the accuracy values.

4. All the score values are arranged in descending order, $S(H_2) \geq S(H_1) \geq S(H_3)$

Hence $S(H_2)$ is the best choice.

X. CONCLUSION

In this paper, we propose the model of rough hesitant neutrosophic sets. In addition an algorithm to handle decision making problem in online teaching company to select staff's are studied. Finally, a numerical example is employed to demonstrate the validness of the proposed rough hesitant neutrosophic sets.

REFERENCES

[1]. Atanassov K (1986), Intuitionistic fuzzy sets, Fuzzy Sets Syst, 20(1):87–96
[2]. Biswas P, Pramanik S, Giri BC (2016), GRA method of multiple attribute decision making with single valued neutrosophic hesitant fuzzy set information. In: Smarandache F, Pramanik S (eds) New trends in neutrosophic theory and applications. Pons Editions, Brussels, pp 55–63
[3]. Broumi S, Smarandache F (2014), Rough neutrosophic sets, Ital J Pure Appl Math ,32:493–502.
[4]. Liu PD, Teng F (2017),Some interval-valued Neutrosophic hesitant fuzzy uncertain linguistic Bonferroni mean aggregation operators and their application in multiple attribute decision making, Int J Uncertain Quantif, 7(6):525–572.
[5]. Liu PD, Zhang LL (2017), An extended multiple criteria decision-making method based on neutrosophic hesitant fuzzy information, J Intell Fuzzy Syst, 32(6):4403–4413.
[6]. Mahmood T, Ye J, Khan Q (2016), Vector similarity measures for simplified neutrosophic hesitant fuzzy set and their applications, J Inequal Spec Funct, 7(4):176–194.
[7]. Majumdar P, Samant SK (2014), On similarity and entropy of neutrosophic sets, J Intell Fuzzy Syst, 26(3):1245–1252.
[8]. Pawlak Z (1982), Rough sets, Int J Comput Inform Sci, 11:341–356.
[9]. Şahin R, Liu PD (2016), Correlation coefficient of single-valued neutrosophic hesitant fuzzy sets and its applications in decision making. Neural Comput Appl.
[10]. Salama AA, Broumi S (2014), Roughness of neutrosophic sets, Elixir Appl Math, 74:26833–26837.
[11]. Smarandache F (1998), Neutrosophy: neutrosophic probability, set, and logic. American Research Press, Rehoboth.
[12]. Smarandache F (1999) A unifying field in logics. neutrosophy: neutrosophic probability, set and logic. American Research Press, Rehoboth.
[13]. Smarandache F (2002), A unifying field in logics: neutrosophic logic, Int J Mult Valued Log 8(3): 385–438, ISSN: 1023–6627.
[14]. Torra V (2010), Hesitant fuzzy sets, Int J Intell Syst, 25:529–539.
[15]. Torra V, Narukawa Y (2009), on hesitant fuzzy sets and decision, In: The 18th IEEE international conference on fuzzy systems, Jeju Island, pp 1378–1382.
[16]. Xia MM, Xu ZS (2011), Hesitant fuzzy information aggregation in decision making, Int J Approx Reason, 52:395–407.
[17]. Ye J (2018),Multiple-attribute decision-making method using similarity measures of single-valued neutrosophic hesitant fuzzy sets based on least common multiple cardinality, J Intell Fuzzy Syst 34(6):4203–4211.