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A Mathematical Study of Multi-Component Redundant Systems under Preventive Maintenance and Replacement Strategies

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Abstract: This paper presents a mathematical model that simulates multi-component redundant systems with various types of preventive maintenance and replacement. The components of each group have different properties and follow Weibull distributions. These groups operate in series and provide the required level of redundancy. Planned corrective repairs as well as planned replacements can be applied on the individual components. The reliability, availability, and long-term expected average maintenance costs of such redundant systems are determined by applying a regenerative method. We present a numerical example to demonstrate the effectiveness of their developed mathematical model and compare the reliability and costs for several scenarios with respect to the frequency of the maintenance tasks (different maintenance cycles) and the time at which these maintenance tasks take place (different replacement times). As one could expect from an optimization problem there exists a tradeoff between the reliability of the system and its operating costs. While frequent maintenance operations improve the reliability of the system they lead to higher costs due to lost production. In contrast delayed maintenance leads to lower costs because less money will be spent on maintenance but it will increase the probability of failures. Therefore, an optimal policy has to be found that minimizes the expected average lifetime maintenance costs, i.e., it has to satisfy both the reliability and availability constraints.

Keywords: Multi-component repairable system, Redundant systems, Reliability analysis, Continuous-time Markov chain, Preventive replacement policy.

I. INTRODUCTION

Reliability is an important factor that affects the overall design of today's engineered systems. Modern systems that require reliability typically include multiple components (redundant) to enhance system operation and safety. In time, all components will degrade. The rate at which they fail will increase over time. Although preventive maintenance can help reduce unplanned component failures, it may be necessary to replace some of these components before their expected failure. Therefore, as part of the maintenance plan, we need to find a good trade-off between the reliability of the system and its costs. Mathematical models can assist us when making this decision. There has been extensive research on optimizing maintenance for repairable systems. However, most of the previous work assumes either that the components are identical or that maintenance is performed perfectly. Additionally, most prior work does not account for varying levels of redundancy among the different components. Both of these assumptions limit the practical applications of these models. This manuscript presents a new mathematical model that can be used to optimize maintenance for heterogeneous multi-component redundant systems. The model includes both preventive maintenance and corrective repair and replacement. The degradation of each component is modeled by a Weibull failure distribution. The imperfect nature of maintenance is accounted for through the use of the "effective age" of a component. Simultaneously, reliability, availability and average cost over time are optimized. A numerical example was developed to determine the best maintenance strategy. As such, the proposed method can provide useful direction for the maintenance management of large, complex engineered systems.

The reliability behavior of consecutive k-out-of-n systems having dependent components has been studied by Eryılmaz (2009). Arbitrary dependence between component failure events was considered. He formulated various reliability expressions for the system configurations. His study demonstrates how dependent components affect the reliability of the system. It provides a mathematical foundation for evaluating redundant systems. Park and Pham (2016) developed cost models to determine the optimal replacement time of failed units for age-replacement and block-replacement

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policies. They used these models to compare the costs associated with each type of replacement, both under warranty and without warranty. The objective is to find a replacement policy which minimizes the long term cost of maintaining a unit. Their results show that determining when to replace a failed unit optimally will increase the overall performance of a system. Park and Pham emphasize the need to consider economics when developing a maintenance plan. Hajipour and Taghipour (2016) suggested a non-periodic inspection schedule for multi-component and k-out-of-m systems. They determined the best times at which to inspect the systems based on some reliability characteristics. The proposed inspection schedule eliminated unnecessary inspections. Their study shows improvement in system availability and reduction in maintenance costs due to inspection optimization. They indicate the importance of optimizing inspections in complex systems. Alaswad and Xiang (2017) reviewed studies that provide optimization models for Condition-Based Maintenance (CBM) of deteriorating systems. In particular, they discuss the use of stochastic degradation models as well as different approaches to making CBM decisions. They identified areas where additional research is needed regarding CBM optimization. They recommend combining reliability models with modern optimization methodologies. Hong et al. (2018) have studied the effects of dynamic operating conditions on system reliability. Dynamic operating conditions include changes in environmental or operational parameters over time. They presented a single, comprehensive methodology to evaluate system reliability. The new methodology allows for more accurate predictions of system failures. They demonstrated that changes in operating conditions can affect the reliability of a system. Qiu et al. (2019) have evaluated the effect of neglecting downtime during inspection periods on system availability and inspection optimization for repairable systems. They incorporated downtime during inspection into a previously developed model. Based on this model, they develop an optimal inspection policy. The results of their study show that neglecting downtime during inspections reduces the availability of a system. The study highlights the importance of including downtime during inspections when planning maintenance. A unified framework for evaluating system reliability under dynamic operating conditions is proposed by Xu et al. (2019). The proposed model includes time-dependent operating environments. Using the proposed model, reliability evaluations are more realistic than those performed using previous models. The proposed model also predicts system failures more accurately than existing models. The study supports maintenance decisions with its ability to perform more reliable evaluations of system failures. Hamdan et al. (2021) have studied an optimization problem related to preventive maintenance for repairable weighted k-out-of-n systems with components having differing degrees of importance. They formulated an optimization problem to minimize maintenance costs based on the degree of importance of each component. Optimal preventive maintenance policies are found through solving the optimization problem. The results of their study show significant increases in system reliability when comparing to un-weighted k-out-of-n systems. The study demonstrates the benefits of weighted redundancy modeling. Arabzadeh Jamali and Pham (2022) have developed an opportunistic maintenance model for load-sharing K-Out-Of-N systems with perfect PM and minimal CRS. They utilize maintenance opportunities effectively to reduce maintenance cost. The proposed strategy increases overall system reliability and demonstrates advantages in terms of cost reduction. The study presents successful implementation of reliability theory in practical engineering applications. Yang et al. (2022) have developed a condition-based maintenance strategy for redundant systems using reinforcement learning; i.e., arbitrary system structures may be utilized within this methodology. Their algorithm learns from experience to optimize maintenance decisions. Reinforcement learning resulted in increased maintenance efficiency, greater system reliability, and decreased maintenance cost relative to traditional maintenance strategies. Lu et al. (2023) have jointly evaluated the reliability of consecutive k-out-of-n systems with arbitrary dependency among components. They have developed mathematical formulations for evaluating dependent reliabilities for such systems. Their model was used in evaluating smart street lighting deployments and it shows that redundant configurations improve system reliability and functionality. This application emphasizes practical engineering uses for reliability theory. Ogunfowora and Najjaran (2023) have provided a literature review on intelligent maintenance decision-making using reinforcement learning (RL), and deep RL (DRL); e.g., the comparison of many maintenance scheduling strategies, as well as intelligent decision-making techniques, were discussed in their article. Their review indicates the growing number of ways in which AI is being employed in managing maintenance operations, while identifying potential areas for future research. Liu et al. (2024) have introduced a condition-based maintenance policy with non-periodic inspections for k-out-of-n : G systems with non-periodic inspections whose inspection intervals are adjusted according to system condition. Their study has shown that applying this maintenance policy leads to fewer unnecessary maintenance activities; furthermore, it is likely to lead to higher system reliability and increased efficiency in performing maintenance tasks. Their study provides evidence that employing flexible scheduling of inspections may be beneficial in reducing unnecessary maintenance activities and improving system performance, respectively. Wang et al. (2025) have investigated a joint optimization problem involving cooperative maintenance and inventory control for multiple k-out-of-n: F systems with interchanging components and common spares. Their model incorporates both cooperative maintenance and inventory control into one decision-making process. The simultaneous decision-making process minimizes total operating cost for the systems modeled by their study demonstrates that implementing coordinated inventory control and cooperative maintenance can decrease total operating cost for multiple k-out-of-n: F systems. Hu et al. (2026) have developed a condition-based maintenance

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planning model for K-out-of-N systems under partial observability; specifically, their model considers uncertainty about state information available during operation of the systems modeled by them. Decision-making concerning maintenance is made possible through utilization of condition monitoring data collected on the systems modeled by them. Results of their study indicate that utilizing condition monitoring data in planning maintenance will result in more effective execution of planned maintenance tasks; i.e., more reliable functioning of the systems, lower maintenance cost, etc., particularly under uncertain operating conditions.

Existing research provides valuable insights in many areas including reliability models, inspection planning for optimal conditions, predictive maintenance and replacement policy design for redundant systems. Most of the current studies focus solely on one type of maintenance method (preventive, corrective) or a particular design or configuration of the system. Also, few researchers have addressed heterogeneity in components with simultaneous application of multiple types of maintenance methods (preventure, corrective, imperfect) along with planned replacements. In addition, the literature is somewhat sparse regarding a holistic approach toward optimizing both reliability/availability as well as total life cycle costs when there are Weibull failure distributions. Therefore, this paper seeks to fill some of those voids by providing an integrated analytical model for multi-component redundant systems which incorporates both preventive maintenance and replacement strategies in order to provide optimized system performance as well as minimized long term maintenance costs.

II. MATHEMATICAL FORMULATION

Consider a repairable system made up of m groups of components that are heterogeneous. Group i contains ni statistically identical and interchangeable components arranged in k_i out of n_i redundancy. So group i remains operational as long as at least k_i of its components are working. Groups are connected in series at system level so failure of any group results in system failure.

Let $X_i(t) \in \{0,1,\dots,n_i\}$ denote the number of failed components in group i at time t. The complete component-state vector is

$$\boldsymbol{X}(t) = \left(X_1(t), X_2(t), \dots, X_m(t)\right) \tag{1}$$

The i^{th} group is operational if $X_i(t) \le n_i - k_i$. (2)

Hence, the operational state space of the system is

$$S_U = \{x = (x_1, x_2, \dots, x_m): 0 \le x_i \le n_i - k_i, i = 1,2,\dots,m\} \tag{3}$$

whereas the failed-state space is

$$S_F = \{x_i > n_i - k_i \; for\ at\ least\ one\ i\} \tag{4}$$

The system-state indicator is defined as

$$U(x) = \prod_{i=1}^{m} I(x_i \le n_i - k_i) \tag{5}$$

where $I(\cdot)$ is the indicator function. Therefore,

$$U(x) = \begin{Bmatrix} 1 & x \in S_U \\ 0 & x \in S_F \end{Bmatrix} \tag{6}$$

Let $A_i(t)$ denote the effective age of the components in group i. Between maintenance interventions,

$$\frac{dA_i(t)}{dt} = 1, i = 1,2,\dots,m \tag{7}$$

The failure behaviour of each functioning component in group i is described by the age-dependent hazard rate

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$$\lambda_i(a) = \lim_{\Delta t \to 0} \frac{P(a < T_i \leq a + \Delta t | T_i > a)}{\Delta t} \quad (8)$$

where T_i is the lifetime of a component belonging to group i. A general Weibull failure rate may be adopted as

$$\lambda_i(a) = \frac{\beta_i}{\eta_i}\left(\frac{a}{\eta_i}\right)^{\beta_i - 1} \quad (9)$$

where $\beta_i > 0$ and $\eta_i > 0$ are, respectively, the shape and scale parameters. The case $\beta_i > 1$ represents deteriorating components with increasing failure rates, while case $\beta_i = 1$ reduces the model to the exponential failure case.

When the system is in state x, the number of functioning components in group i is $n_i - x_i$. Therefore, the total failure-transition intensity from state $\boldsymbol{x}$ to state $\boldsymbol{x} + e_i$ is

$$q_{x,x+e_i} = (n_i - x_i)\lambda_i(a_i), x_i < n_i \quad (10)$$

where e_i is the i^{th} unit vector. Corrective repair of failed components is assumed to be performed independently. If the repair time of a failed component in group i is exponentially distributed with parameter μ_i, the repair-transition intensity from x to $x - e_i$ is

$$q_{x,x-e_i} = x_i \mu_i, x_i > 0 \quad (11)$$

The infinitesimal transition probabilities over a sufficiently small interval Δt are consequently given by

$$P\{X(t + \Delta t) = (x + e_i | X(t) = x)\} = (n_i - x_i)\lambda_i(a_i)\Delta t + o(\Delta t) \quad (12)$$

And

$$P\{X(t + \Delta t) = (x - e_i | X(t) = x)\} = x_i \mu_i \Delta t + o(\Delta t) \quad (13)$$

And

$$P\{X(t + \Delta t) = x | X(t) = x\} = 1 - \sum_{i=1}^{m} [(n_i - x_i)\lambda_i(a_i) + x_i \mu_i]\, \Delta t + o(\Delta t) \quad (14)$$

Let $p_x(a,t) = P\{X(t) = x, A(t) \in da\}/da$ denote the joint probability density of the component-state vector and effective-age vector

$$A(t) = (A_1(t), A_2(t), \ldots, A_m(t)) \quad (15)$$

For states not corresponding to maintenance or replacement epochs, the state probabilities satisfy the supplementary-variable forward equation

$$\frac{\partial p_x(a,t)}{\partial t} + \sum_{i=1}^{m,} \frac{\partial p_x(a.t)}{\partial a_i} = -\sum_{i=1}^{m} [(n_i - x_i)\lambda_i(a_i) + x_i \mu_i]\, p_x(a,t) + \sum_{i=1}^{m} (n_i - x_i + 1)\lambda_i(a_i)\, p_{x-e_i}(a,t) + \sum_{i=1}^{m} (x_i + 1)\mu_i p_{x+e_i}(a,t) \quad (16)$$

where terms involving states outside the admissible state space are taken as zero.

Preventive maintenance is carried out at scheduled intervals of length T, provided that the replacement condition has not already been reached. Maintenance is assumed to be imperfect. Immediately before and after the jth preventive-maintenance action, the effective ages satisfy

$$A_i(jT^+) = \rho_i A_i(jT^-), 0 \leq \rho_i \leq 1 \quad (17)$$

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where ρ_i is the age-reduction factor for group i. The special cases are

$$\rho_i = \begin{Bmatrix} 0 & \text{perfect preventive maintenance} \\ 1 & \text{minimal preventive maintenance} \\ 0 < \rho_i < 1 & \text{imperfect preventive maintenance} \end{Bmatrix} \quad (18)$$

Preventive maintenance restores all components detected as failed at the maintenance epoch. Thus, $X(jT^+) = 0$ and the post-maintenance system state is

$$\big(X(jT^+), A_i(jT^+)\big) = \big(0, \rho \circ A_i(jT^-)\big) \quad (19)$$

Where $\rho = (\rho_1, \rho_2, \dots, \rho_m)$ and $\circ$ denotes element-wise multiplication.

A replacement action is initiated when the deterioration level of any group reaches a prescribed threshold. Let r_i denote the replacement threshold for group i, where $1 \le r_i \le n_i$.

The state-based replacement time is

$$\tau_R^{(x)} = inf\{t > 0 : X_i(t) \ge r_i \text{ for at least one i}\} \quad (20)$$

An age-based replacement limit Li may also be imposed on each component group. The corresponding age-based replacement time is

$$\tau_R^{(a)} = inf\{t > 0 : A_i(t) \ge L_i \text{ for at least one i}\} \quad (21)$$

The actual replacement time is therefore $\tau_R = min\left\{\tau_R^{(x)}, \tau_R^{(a)}\right\}$ (22)

When system failure itself triggers immediate replacement, the system failure time is defined as

$\tau_F = inf\{t > 0 : X(t) \in S_F\}$ and the replacement time becomes

$$\tau_R = min\left\{\tau_R^{(x)}, \tau_R^{(a)}, \tau_F\right\} \quad (23)$$

Following replacement, all failed and functioning components are replaced by new components, so that $X(\tau_R^+) = 0$ and $A(\tau_R^+) = 0$ (24)

Thus, replacement epochs constitute regeneration points, and the stochastic process restarts from the new-system condition $(X(0), A(0)) = (0,0)$ (25)

The reliability of the system at time t is the probability that the system has not entered the failed-state space during $[0, T]$, namely, $R(t) = P(\tau_F > t)$ (26)

Equivalently, when failed states are treated as absorbing states,

$$R(t) = \sum_{x \in S_U} \int_{R_+^m} p_x(a, t)\, da \quad (27)$$

The system failure distribution is $F(t) = 1 - R(t)$ (28)

and the instantaneous system failure rate is $h_S(t) = -\frac{d}{dt} lnR(t) = -\frac{R'(t)}{R(t)}$ (29)

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The mean time to system failure is $MTTF = E(\tau_F) = \int_0^\infty R(t)dt$ (30)

For a repairable system, the point availability is defined as

$$R(t) = P\{X(t) \in S_U\} = \sum_{x \in S_U} \int_{R_+^m} p_x(a,t)\, da \quad (31)$$

The steady-state availability is $A_\infty = \lim_{t \to \infty} A(t)$ provided that the limit exists. Using the regenerative-cycle representation, the steady-state availability can also be written as

$$A_\infty = \frac{E(U_C)}{E(C_L)} \quad (32)$$

where U_C is the total operating time during a replacement cycle and C_L is the total cycle length, including maintenance and replacement downtime.

Let $N_i^F(\tau_R)$ denote the number of component failures in group i during a replacement cycle, $N_{PM}(\tau_R)$ the number of preventive-maintenance actions, and $D(\tau_R)$ the total system downtime. The total cost incurred during a replacement cycle is formulated as

$$C_{cycle} = \sum_{i=1}^m C_i^F N_i^F(\tau_R) + C_{PM} N_{PM}(\tau_R) + C_R + C_D D(\tau_R) + \sum_{i=1}^m C_i^I N_i^I(\tau_R) \quad (33)$$

where C_i^F is the corrective-repair cost per failed component of group i, CPM is the cost of one preventive-maintenance action, C_R is the complete replacement cost, C_D is the downtime cost per unit time, and C_i^I is the inspection cost for group i.

The expected long-run cost rate, obtained from the renewal-reward theorem, is

$$J(T,r,L) = \frac{E[C_{cycle}]}{E(C_L)} \quad (34)$$

Where $r = (r_1, r_2, \dots, r_m)$ and $L = (L_1, L_2, \dots, L_m)$

The maintenance and replacement optimization problem is $\min_{T,r,L} J(T,r,L)$ (35)

subject to $T > 0, 1 \le r_i \le n_i, r_i \in \mathbb{Z}, L_i > 0, i = 1,2, \dots, m$ (36)

and the reliability and availability constraints $R(t_0) \ge R_{min}$ and $A_\infty \ge A_{min}$ (37)

where t_0 is a specified mission duration, R_{min} is the minimum acceptable mission reliability, and A_{min} is the minimum acceptable steady-state availability. The optimal maintenance-replacement policy is therefore

$$(T^*, r^*, L^*) = \arg\min_{T,r,L} J(T,r,L) \quad (38)$$

III. NUMERICAL SIMULATION

Consider a redundant system with repairability consisting of two groups of heterogeneous components connected in series. For the system to work, both groups need to meet their redundancy requirements. Group one follows a configuration of 2 out of 3 while group two follows a 1 out of 2 configuration.

Thus, $m = 2, (n_1, k_1) = (3,2), (n_2, k_2) = (2,1)$
The system is operational when $X_1(t) \le 1$ and $X_2(t) \le 1$

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Therefore, the operational-state space is $S_U = \{(x_1, x_2): x_1 = 0,1; x_2 = 0,1\}$
or explicitly, $S_U = \{(0,1), (1,0), (0,1), (1,1)\}$
The system fails when either two components of the first group fail or both components of the second group fail. Hence,
$S_F = \{(x_1, x_2): x_1 \geq 2 \ or \ x_2 \geq 2\}$
The lifetime of each component is assumed to follow a Weibull distribution. The age-dependent failure rate of a component in group i is
$\lambda_i(a) = \frac{\beta_i}{\eta_i}\left(\frac{a}{\eta_i}\right)^{\beta_i - 1}$
The numerical values used in the simulation are given in Table 1.

Table 1. Component and maintenance parameters

Parameter	**Group 1**	**Group 2**
Number of components, n_i	3	2
Required functioning components, k_i	2	1
Weibull shape parameter, β_i	2.2	1.8
Weibull scale parameter, η_i	4.00 years	3.20 years
Imperfect-maintenance factor, ρ_i	0.35	0.45
Restoration cost per failed component, C_i^F	350	500

Since $\beta_1 > 1$ and $\beta_2 > 1$
The components of both groups have increasing failure rates. Group 1 experiences slightly stronger age-related deterioration than Group 2 because
$\beta_1 > \beta_2$

The corresponding component reliability functions are

$R_1(t) = exp\left[-\left(\frac{t}{4}\right)^{2.2}\right]$ and $R_2(t) = exp\left[-\left(\frac{t}{3.2}\right)^{1.8}\right]$

The probability that the 2-out-of-3 first group is operational at time t, in the absence of maintenance, is

$$R_{G_1}(t) = \binom{3}{2} R_1{}^2(t)[1 - R_1(t)] + R_1{}^3(t) = 3R_1{}^2(t) - 2R_1{}^3(t)$$

The second group follows a 1-out-of-2 configuration. Its reliability is

$$R_{G_2}(t) = 1 - [1 - R_2(t)]^2 = 2R_2(t) - R_2{}^2(t)$$

Because the two groups are connected in series, the reliability of the system without maintenance is

$$R_S(t) = R_{G_1}(t) R_{G_2}(t)$$

Accordingly,

$$R_S(t) = \left[3R_1{}^2(t) - 2R_1{}^3(t)\right]\left[2R_2(t) - R_2{}^2(t)\right]$$

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At $t = 1$, $R_1(1) = exp\left[-\left(\frac{1}{4}\right)^{2.2}\right] \approx 0.9537$

And

$$R_2(1) = exp\left[-\left(\frac{1}{3.2}\right)^{1.8}\right] \approx 0.8836$$

Hence, $R_{G_1}(t) = 3(0.9537)^2 - 2(0.9537)^3 \approx 0.9938$

$$R_{G_2}(t) = 2(0.8836) - (0.8836\,)^2 \approx 0.9865$$

The one-year reliability of the complete system without preventive maintenance is therefore

$$R_S(t) = 0.9938 \times 0.9865 \approx 0.9804$$

Preventive maintenance is performed after every T years. Maintenance is imperfect, and the effective ages immediately after maintenance are

$$A_1(jT^+) = 0.35A_1(jT^-)$$

and

$$A_2(jT^+) = 0.45A_2(jT^-)$$

Thus, preventive maintenance removes 65% of the accumulated effective age of Group 1 and 55% of the accumulated effective age of Group 2.

A system replacement is conducted at the planned replacement age L, or immediately following system failure, whichever occurs first. The replacement time is consequently

$$\tau_R = min\{\tau_F, L\}$$

The following economic parameters are considered.

Table 2. Cost and downtime parameters

Parameter	Value
Cost of one preventive-maintenance action, C_{PM}	1,200
Planned replacement cost, C_R^P	12,000
Failure replacement cost, C_R^F	18,000
Downtime cost per year, C_D	25,000
Duration of one preventive-maintenance action, d_{PM}	0.015 year
Planned replacement duration, d_R^P	0.060 year
Failure replacement duration, d_R^F	0.120 year

The total cost during one replacement cycle is evaluated from

$$C_{cycle} = C_{PM}N_{PM} + \sum_{i=1}^{2} C_i^F N_i^F + C_R^P I_p + C_R^F I_F + C_D[d_{PM}N_{PM} + d_R^P I_P + d_R^F I_F]$$

Where

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$$I_P = \begin{cases} 1 & \text{if planned replacement occurs} \\ 0 & Otherwise \end{cases}$$

$$I_F = \begin{cases} 1 & \text{if replacement is caused by system failure} \\ 0 & Otherwise \end{cases}$$

The expected long-run cost per unit time is

$$J(T, L) = \frac{E[C_{cycle}]}{E[C_L]}$$

where C_L is the total cycle length, including the operating period and maintenance or replacement downtime.

The steady-state availability is calculated as

$$A_\infty(T, L) = \frac{E[U_C]}{E[U_C]+E[D_C]}$$

where U_C and D_C denote the uptime and downtime in one replacement cycle.

A Monte Carlo simulation was performed for the candidate preventive-maintenance intervals

$T \in \{0.50, 0.75, 1.00, 1.25, 1.50\}$ years and planned replacement ages

$L \in \{3, 4, 5, 6\}$ years.

For every policy combination, 10,000 replacement cycles were simulated. Component failure times conditional on the current effective age were generated using the Weibull cumulative hazard function. For a component having effective age a, its residual lifetime Y_i was generated from

$$Y_i = \eta_i \left[\left(\frac{a}{\eta_i} \right)^{\beta_i} + E \right]^{1/\beta_i} - a$$

E∼Exponential(1).

At each maintenance epoch, failed components were restored, while the effective ages of surviving components were reduced according to the corresponding ρ_i. The principal simulation results for a planned replacement age of six years are shown in Table 3.

Table 3. Effect of preventive-maintenance interval for $L = 6$ years

(T) (years)	Expected cycle length (years)	Probability of failure replacement	Expected PM actions per cycle	Availability	Cost rate per year
0.5	5.82	0.068	10.659	0.964	5,241
0.75	5.565	0.171	6.467	0.969	4,528
1	5.286	0.29	4.364	0.969	**4,334**
1.25	4.941	0.402	3.187	0.966	4,428
1.5	4.641	0.546	2.246	0.959	4,597

The results indicate that highly frequent preventive maintenance substantially decreases the probability of failure-induced replacement. For example, reducing the maintenance interval from 1.50 years to 0.50 years decreases the probability of failure replacement from approximately 0.546 to 0.648

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However, the 0.50-year policy requires approximately 10.66 preventive-maintenance actions per cycle. The resulting maintenance expenditure causes its long-run cost rate to increase to approximately 5,241 per year.

Conversely, infrequent maintenance reduces the direct preventive-maintenance cost but increases the occurrence of costly system failures. The 1.50-year policy has only approximately 2.25 preventive-maintenance actions per cycle, but more than half of the cycles terminate because of system failure.

Among the investigated policies with L=6 years, the minimum cost rate is obtained at $T^* = 1.00$ year.

The corresponding cost rate is $J(1,6) \approx 4{,}334$ per year

The expected replacement-cycle length is $E[C_L] \approx 5.286$ years while the probability that a replacement cycle terminates due to system failure is $P[\tau_F < L] \approx 0.290$. The effect of the planned replacement age is shown in Table 4.

Table 4. Selected maintenance-replacement policies

T (years)	L (years)	Expected cycle length	Failure-replacement probability	Expected PM actions	Cost rate per year
0.75	5	4.726	0.13	5.629	4,960
0.75	6	5.565	0.171	6.467	4,528
1	4	3.733	0.184	2.782	5,157
1	5	4.529	0.242	3.595	4,673
1	6	5.286	0.29	4.364	**4,334**
1.25	5	4.323	0.355	2.555	4,704
1.25	6	4.941	0.402	3.187	4,428
1.5	5	4.116	0.42	2.251	4,955
1.5	6	4.641	0.546	2.246	4,597

A comparatively short replacement age, such as $L = 4\, years$ decreases the exposure of the system to advanced deterioration. Nevertheless, frequent planned replacement increases the average annual cost. For example,

$J(1,4) \approx 5{,}157$ whereas $J(1,6) \approx 4{,}334$

The cost reduction obtained by increasing the replacement age from four to six years is

$$\frac{5157-4334}{5157} \times 100 \approx 15.96\ \%$$

51575157−4334×100≈15.96%.
However, the probability of failure replacement increases from 0.184 to 0.290. Thus, the decision maker must balance economic performance against the risk of unplanned system failure. The mission reliability obtained under selected preventive-maintenance policies is given in Table 5.

Table 5. Estimated mission reliability under preventive maintenance

Maintenance interval (T)	R(1)	R(3)
0.50 year	0.994	0.969
0.75 year	0.991	0.929
1.00 year	0.981	0.871
1.25 years	0.981	0.842
1.50 years	0.981	0.746

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For a short one-year mission, the reliability differences are relatively small because the components are initially new. The effect of preventive maintenance becomes much more pronounced over a three-year mission. The three-year reliability decreases from approximately 0.969 for T=0.50 year to 0.746 for T=1.50 years.

Suppose that the decision maker requires $A_\infty \geq 0.96$ and $R(3) \geq 0.85$.

The policy $(T, L) = (1.50{,}6)$ does not satisfy the reliability constraint because

$R(3) = 0.746 < 0.85$

Similarly, $(T, L) = (1.50{,}6)$ gives $R(3) = 0.842 < 0.85$ and is therefore marginally infeasible. The policy $(T, L) = (1.00{,}6)$ satisfies both requirements because $R(3) = 0.871 < 0.85$ and

$A_\infty \approx 0.969 > 0.96$

Consequently, the constrained optimal policy among the investigated alternatives is

$T^* = 1.00\ year, L^* = 6.00\ years$ with $J(T^*, L^*) \approx 4{,}334\ per\ years$ and $A_\infty(T^*, L^*) \approx 0.969$

Numerical analysis shows there is an economic tradeoff between how often preventive maintenance is done and risk of failures needing replacement. Shorter maintenance intervals improve reliability but also lead to very high costs and downtime. Longer intervals reduce planned maintenance costs but components deteriorate more and there is higher risk of emergencies and replacement costs. For assumed system and economic parameters annual preventive maintenance along with planned six year replacements works out best in balancing cost, availability and reliability.

IV. RESULTS AND DISCUSSION

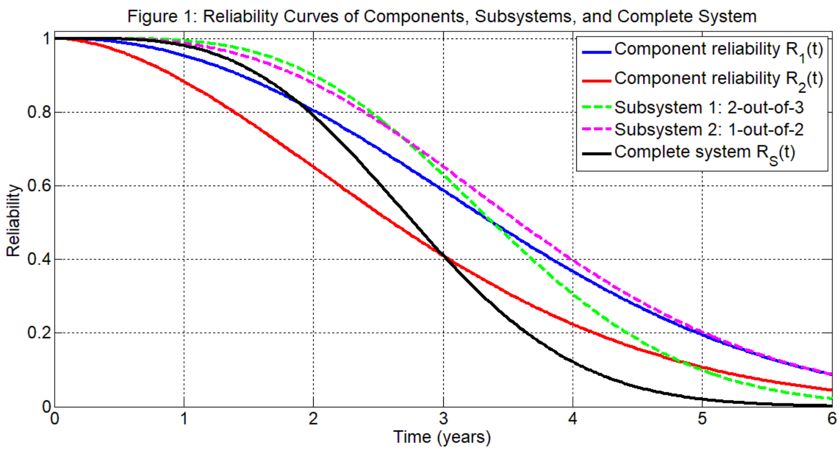

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Figure 2:Age-Dependent Weibull Failure-Rate Curves

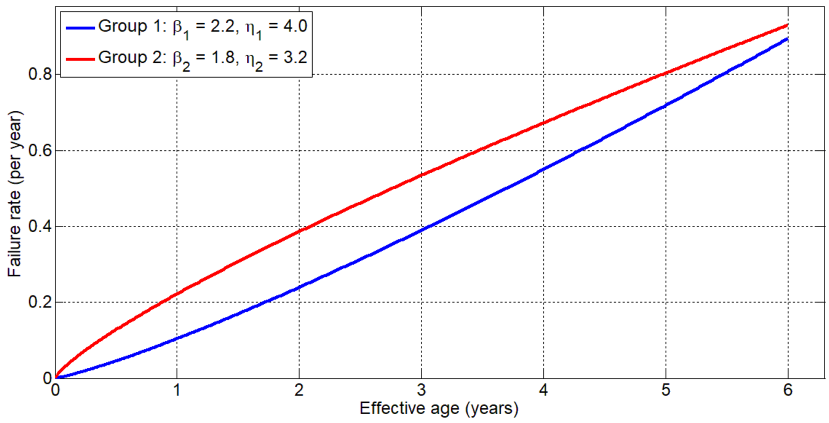

Figure 3: Cost-Rate Curves for Alternative Replacement Ages

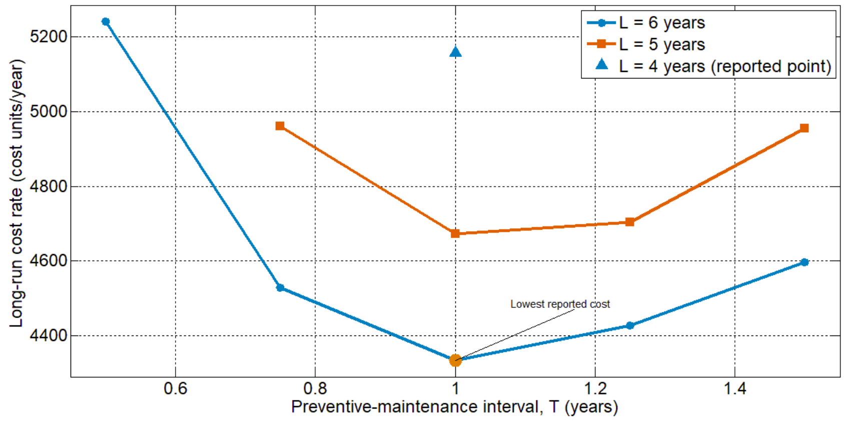

Figure 4:Reliability and Availability under Alternative Maintenance Intervals

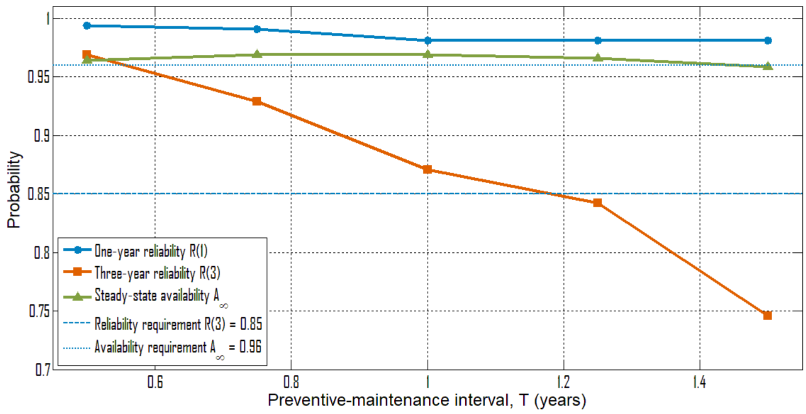

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Figure 5:Effective-Age Reduction under Annual Imperfect Preventive Maintenance

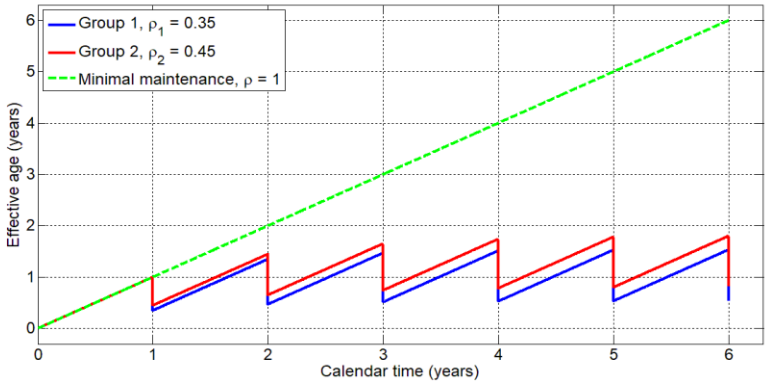

Figure 1 plots reliability curves for two component groups, two redundant subsystems and the complete system over mission time. At time zero, when the system is brand new, reliability is 100%. Reliability of components decreases continuously with time due to aging. Group 2 loses reliability faster than Group 1; this suggests Group 2 is more prone to failure. Curves for redundant subsystems stay above corresponding component curves. This shows benefit of redundancy. Configurations 2 out of 3 have higher reliability compared to single components. Configuration 1 out of 2 also improves survival. Redundant components compensate for individual failures and slow degradation of subsystems. System reliability declines faster than subsystem reliability because subsystems are connected in series. Failure of either subsystem results in system failure overall so system reliability depends on the weakest subsystem. Throughout operation time, the system curve stays below all subsystem curves. Differences between system and subsystem curves grow larger as time progresses. This suggests aging reduces benefits of redundancy gradually. Near mission end reliability of the complete system approaches zero. Figure shows redundancy greatly improves performance of subsystems but series configuration limits reliability of the system overall. Results justify preventive maintenance and timely replacement to keep acceptable reliability over long periods.

Figure 2 shows failure rate curves for two component groups that vary with effective age. Both curves increase continuously. This shows components deteriorate progressively over time. Failure rates are very low at the start of operation and steadily increase as components age. This is typical of Weibull distribution in wear out region. Throughout operation Group 2 has higher failure rates than Group 1. This means Group 2 deteriorates faster. Smaller scale parameter of Group 2 causes failures to happen sooner. So Group 2 becomes a more critical group. As effective age increases the gap between curves remains noticeable. This points to different aging characteristics of the groups. Increasing failure rates mean likelihood of failure rises with time. So relying only on corrective maintenance might not be enough during long operation. Maintenance becomes more important as components age. Prompt replacement also reduces risk of surprises failures. Figures support using strategies of preventive maintenance and replacement based on age to improve long reliability and performance of systems overall.

Figure 3 illustrates the long-run cost rate for different preventive-maintenance intervals and planned replacement ages. The cost curves exhibit a clear non-linear trend. The cost decreases initially as the maintenance interval increases. It then increases for larger maintenance intervals. This behaviour indicates the existence of an optimal maintenance policy. The replacement age of six years produces the lowest overall cost among the reported policies. The minimum cost occurs at a maintenance interval of approximately one year. At this point, the long-run cost rate is about 4334 cost units per year. Shorter maintenance intervals increase maintenance frequency. This results in higher maintenance costs. Longer maintenance intervals increase component deterioration. This leads to more corrective repairs and failure-induced replacements. The curve corresponding to a replacement age of five years follows a similar trend but remains above the six-year curve. This indicates a higher operating cost. Earlier replacement increases replacement expenditure. Consequently, the overall cost rate becomes larger. Only one reported result is available for the four-year replacement age. This point also lies above the minimum-cost solution. This suggests that very early replacement is not economically attractive. Overall, the figure demonstrates the trade-off between maintenance frequency and replacement cost. Excessively frequent maintenance is expensive. Excessively delayed maintenance also increases the total cost. An annual preventive-maintenance interval combined with a six-year replacement age provides the most economical policy among the investigated alternatives.

Figure 4 compares reliability for different maintenance intervals over one year and three years. Overall, reliability remains very high regardless of the maintenance interval. As maintenance intervals lengthen just a little bit, reliability

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drops only slightly. This suggests that performance of the system for short periods is only marginally affected by maintenance schedule. Reliability drops much more sharply for three years. Reliability declines continuously as maintenance intervals lengthen. Failure probability increases more as time goes on longer. Reliability falls below target level of 0.85 when maintenance intervals exceed roughly one year. Policies therefore do not meet the specified reliability requirement. Availability stays relatively stable over the range we look at. It stays close to target level of 0.96; there is a slight decrease for longer maintenance intervals; this is because repair time and frequency increase. Overall this figure shows how preventive maintenance scheduling influences system performance. Longer intervals between maintenance reduce long term reliability; availability is slightly reduced too. Shorter maintenance intervals improve reliability but they can increase maintenance costs. An interval of one year strikes a good balance among reliability, availability and efficiency of maintenance. It meets both reliability and availability requirements while avoiding too frequent maintenance.

Figure 5 demonstrates how annual imperfect preventive maintenance impacts the effective age of the system. Minimal maintenance is represented by the green dashed line which increases at an equal rate for every one year of calendar time. There is no reduction in effective age as a result of minimal maintenance. Blue and red lines are examples of imperfect preventive maintenance. Effective age increases from the last maintenance action until the next. Each year there is a large drop in effective age immediately after the maintenance action. The drop represents age reduction resulting from the maintenance performed. The blue line has a lower effective age due to a smaller restoration factor ($\rho_1 = 0.35$) while the red line maintains a slight higher effective age due to a larger restoration factor ($\rho_2 = 0.45$). Regardless of whether it is blue or red, both of these maintenance techniques will have an impact slowing down the aging of the system relative to minimal maintenance. This same pattern repeats itself each year and further validates that periodic imperfect maintenance does restore the system as well as slows down the build up of effective age. A lower restoration factor provides greater rejuvenation and therefore results in a younger operational condition of the system throughout its lifetime.

V. CONCLUDING REMARKS

We presented a stochastic reliability model to analyze multi-component redundant systems with multiple components as well as preventive and/or corrective maintenance strategies. It is assumed that there are different heterogeneous groups of components which are connected in series. Furthermore, it is assumed that the time to failure follows a Weibull distribution. To evaluate imperfect preventive maintenance, it is assumed that after each preventive maintenance action (i.e., repair or replacement) the effective age of the system is decreased. Therefore, three performance metrics have been defined in order to assess the system behavior: reliability, availability and long-term cost. These performance metrics have been computed using a regenerative point process approach. Finally, some numerical examples have been analyzed to show how the proposed methodology can be used in practice. As shown by the analysis of these examples, there exists a trade-off between maintenance frequency and operating costs. Indeed, frequent maintenance actions improve the reliability of the system but increase the total maintenance cost. On the other hand, longer maintenance intervals reduce the planned maintenance cost but increase the failure risk. From this analysis, it has been determined that when an annual preventive maintenance strategy is employed, where all components of a group are replaced at a planned replacement age equal to six years, the most satisfactory compromise is found among cost, reliability and availability for the chosen parameters. In conclusion, the proposed model allows engineers to develop efficient maintenance policies for complex redundant systems. Further studies could include extending the model to account for correlated failures; resource constraint on maintenance operations; scheduling of maintenance activities; and variable conditions of operation.

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