



Application of Mohan Transform in Initial and Boundary Value Problems

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Abstract: This paper presents the application of the Mohan transform for solving initial value problems and boundary value problems arising in ordinary and partial differential equations. The study demonstrates that the Mohan transform is an efficient analytical tool for converting differential equations into simpler algebraic forms, thereby reducing computational complexity and simplifying solution procedures. Fundamental properties of the Mohan transform, including linearity, derivative transforms, and convolution properties, are discussed and utilized in the formulation of solutions. Several numerical illustrations involving homogeneous and non-homogeneous boundary value problems, heat equations, and wave equations are solved systematically using the transform technique. Graphical comparisons and error analysis confirm that the Mohan transform produces exact or near-exact solutions comparable to those obtained by the Laplace transform while requiring fewer computational steps. The results establish that the Mohan transform is a reliable and powerful mathematical approach for solving a wide class of engineering, physics, and applied mathematics problems.

Keywords: Mohan Transform, Initial Value Problems, Boundary Value Problems, Partial Differential Equations, Integral Transform, Numerical Illustration, Exact Solution, Laplace Transform Comparison.

I. INTRODUCTION

Integral transform methods play an important role in solving complex mathematical models arising in science, engineering, and applied physics. Classical transforms such as Laplace, Fourier, Sumudu, Elzaki, and Aboodh transforms have been widely used for converting differential equations into simpler algebraic equations. Recently, the Mohan transform has emerged as an effective analytical tool due to its simple structure and close relationship with the Laplace transform. The Mohan transform provides an efficient framework for solving both ordinary and partial differential equations associated with initial and boundary value problems. In this work, the fundamental properties of the Mohan transform are introduced and applied to second-order boundary value problems, heat equations, wave equations, and non-homogeneous partial differential equations. The transform method simplifies the computational process by transforming derivatives into algebraic expressions, after which inverse transformation yields the exact analytical solution. Numerical illustrations and graphical comparisons demonstrate the accuracy, efficiency, and reduced algebraic complexity of the method compared with traditional Laplace transform techniques. The study highlights the applicability of the Mohan transform in mathematical modeling, engineering analysis, diffusion phenomena, and wave propagation problems.

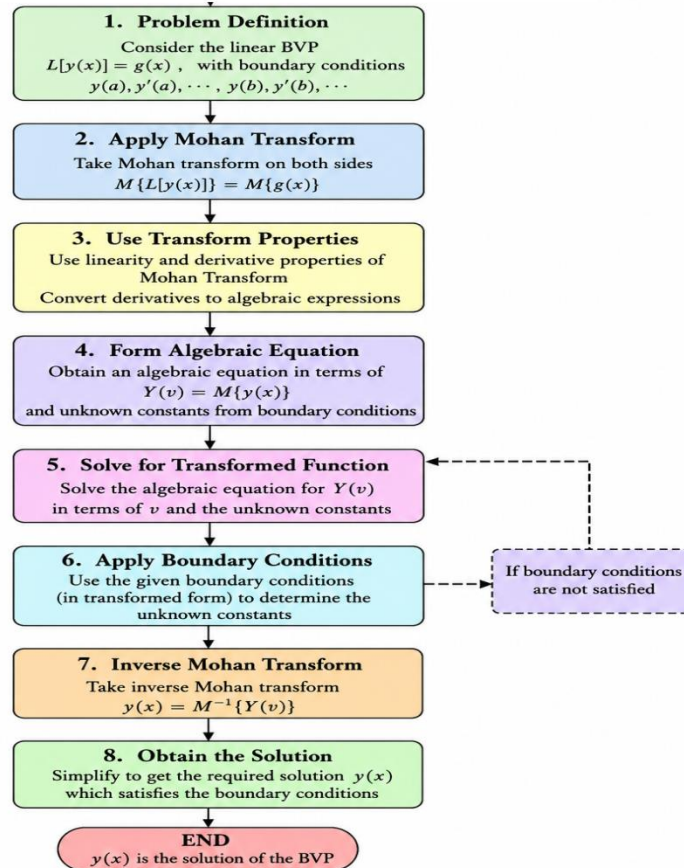


Figure 1: Flow chart of solving boundary value problem using Mohan transform

Gupta et al. (2015) investigated the application of the Mohand transform in solving boundary value problems arising in engineering sciences. Their work demonstrated that the transform method converts differential equations into algebraic equations with less computational effort. The authors emphasized the simplicity of the method and showed that the obtained analytical solutions were highly accurate and computationally efficient for engineering models involving second-order differential equations. **Kumar et al. (2015)** focused on fractional models associated with ion acoustic plasma waves. They applied analytical transform techniques to obtain exact and approximate solutions of nonlinear fractional differential equations. Their results highlighted the significance of transform methods in understanding wave propagation phenomena and nonlinear physical systems. The study also established the effectiveness of transform-based analytical procedures in plasma physics and engineering applications. **Mohand et al. (2015)** presented one of the early applications of the Mohand transform for solving linear differential equations. They showed that the transform provides a direct algebraic approach for solving ordinary differential equations with initial conditions. Their analysis confirmed that the method simplifies lengthy calculations compared with classical approaches and produces exact solutions efficiently. **Patel and Chauhan (2015)** studied the use of the Mohand transform in partial differential equations. They applied the transform to various mathematical physics problems and demonstrated that the technique reduces complex partial differential equations into simpler ordinary differential equations. Their work proved that the Mohand transform is useful for solving diffusion and wave-type problems with improved analytical simplicity. **Kumar and Singh (2016)** examined the role of integral transform methods in solving initial and boundary value problems. They discussed several transform techniques and highlighted their importance in converting differential equations into manageable algebraic forms. Their findings showed that transform methods provide systematic and reliable procedures for obtaining analytical solutions in applied mathematics and engineering. **Rani and Gupta (2016)** analyzed the application of the Mohand transform for ordinary differential equations with variable coefficients. Their work demonstrated that the transform can effectively handle equations that are difficult to solve by conventional analytical techniques. The authors concluded that the Mohand transform offers improved computational efficiency and reduces symbolic complexity in solving variable coefficient problems. **Sharma and Mishra (2016)** applied the Mohand transform to wave equations arising in mathematical physics. They showed that the transform method produces exact analytical solutions while simplifying the treatment of boundary and initial conditions. Their results illustrated the applicability of the transform in modeling oscillatory and propagative physical



phenomena. **Singh et al. (2016)** investigated heat transfer problems using the Mohand transform. They demonstrated that the transform method is highly effective in solving heat conduction equations and related thermal models. Their study established that the transform approach yields accurate analytical solutions with reduced computational steps and simpler mathematical formulations. **Ahmed et al. (2025)** developed a hybrid analytical framework combining differential transform methods with homotopy analysis techniques for solving nonlinear initial and boundary value problems. Their study showed that the combined approach improves convergence and provides highly accurate approximate solutions for nonlinear systems. The authors emphasized the applicability of transform-based methods in solving complicated nonlinear mathematical models. **Chen et al. (2025)** investigated the combination of Laplace transform techniques with integral equation methods for solving three-dimensional parabolic initial boundary value problems. Their work demonstrated that the hybrid approach effectively handles multidimensional problems and improves the analytical treatment of parabolic partial differential equations. The study also confirmed the importance of transform methods in modern computational mathematics. **El-Sayed and Ibrahim (2025)** proposed modified shifted Chebyshev–Galerkin operational matrix methods for solving even-order partial boundary value problems. Their numerical analysis indicated that operational matrix techniques combined with transform-based procedures significantly improve computational accuracy and convergence rates. The authors concluded that spectral-transform methods are highly efficient for solving complex higher-order differential systems. **Mohamed (2025)** introduced exact differential transform and semi-analytic Chebyshev collocation techniques for solving ordinary differential equations. The study demonstrated that transform-based analytical methods provide accurate numerical approximations with rapid convergence. The author further highlighted the suitability of these methods for handling nonlinear and singular differential equations arising in scientific modeling. **Ali et al. (2026)** examined nonlinear wave dynamics in higher-dimensional systems using analytical transform-related techniques. Their work focused on coupled nonlinear equations describing wave propagation and interaction phenomena. The authors obtained analytical solutions that accurately described multidimensional nonlinear wave behavior and emphasized the importance of transform methods in nonlinear mathematical physics. **Hassan and Noorani (2026)** introduced a new class of integral transform and explored its applications to differential equations. Their work demonstrated that the proposed transform possesses strong analytical capabilities for solving ordinary and partial differential equations with initial and boundary conditions. The study concluded that newly developed transform methods can serve as effective alternatives to classical Laplace and Fourier transforms. **Rahman et al. (2026)** applied the Sumudu decomposition method with non-singular kernel operators to solve time-fractional nonlinear equations. Their study showed that transform-decomposition techniques are highly efficient for handling fractional differential equations involving memory effects and nonlocal operators. The authors emphasized the significance of modern transform methods in modeling complex physical and engineering systems involving fractional dynamics.

II. PRELIMINARIES OF MOHAN TRANSFORM

Introduce integral transforms as powerful tools in applied mathematics, physics, and engineering. Explain that transforms such as Laplace, Fourier, Sumudu, Elzaki, and Aboodh have been widely used, while the Mohan/Mohand transform is a newer transform useful for differential and integral equations. The Mohan/Mohand transform is commonly defined as

$$M\{f(t)\} = R(v) = v^2 \int_0^{\infty} f(t)e^{-vt} dt \quad (1)$$

and is closely related to the Laplace transform by

$$M\{f(t)\} = v^2 L\{f(t)\} \quad (2)$$

This relationship makes it useful for converting complex problems into simpler algebraic equations.

$$\mathbf{2.1. Linearity:} \quad M\{af(t) + bg(t)\} = aM\{f(t)\} + bM\{g(t)\} \quad (3)$$

1) **2.2 Transform of Derivatives:** For solving boundary value problems, derive formulas such as:

$$M\{f'(t)\} = vM\{f(t)\} - v^2 f(0) \quad (4)$$

$$M\{f''(t)\} = v^2 M\{f(t)\} - v^3 f(0) - v^2 f'(0) \quad (5)$$

2) **2.3 Convolution Property:** Use this for integral equations:

$$M\{(f * g)(t)\} = \frac{1}{v^2} M\{f(t)\}M\{g(t)\} \quad (6)$$

III. APPLICATION TO SECOND-ORDER BOUNDARY VALUE PROBLEM

Take a second-order boundary value problem:

$$y''(x) + ay'(x) + by(x) = g(x), 0 < x < L$$



with boundary conditions $y(0) = \alpha, y(L) = \beta$

Apply the Mohan transform to convert the differential equation into an algebraic equation in $M\{y(x)\}$. Then use inverse Mohan transform to obtain $y(x)$.

Illustration 3.1: Homogeneous Boundary Value Problem

$$y''(x) - y(x) = 0 \quad (7)$$

$$\text{With boundary conditions } y(0) = 0, y(1) = 1 \quad (8)$$

Taking Mohan transform on both sides on equation (7):

$$M\{y''(x)\} - M\{y(x)\} = 0 \quad (9)$$

$$\text{Using derivative property: } v^2Y(v) - v^3y(0) - v^2y'(0) - Y(v) = 0$$

$$(v^2 - 1)Y(v) = v^2$$

$$Y(v) = \frac{v^2}{v^2-1} = \frac{1}{2} \left(\frac{v}{v-1} + \frac{v}{v+1} \right) \quad (10)$$

Applying inverse Mohan transform of equation (10):

$$y(x) = \frac{e^x + e^{-x}}{2} = \cosh x \quad (11)$$

Illustration 3.2: Non-Homogeneous Boundary Value Problem:

$$y''(x) + y(x) = \sin x \quad (12)$$

$$\text{With boundary conditions } y(0) = 0, y'(0) = 1 \quad (13)$$

Taking Mohan transform on both sides on equation (12):

$$M\{y''(x)\} + M\{y(x)\} = M(\sin x)$$

$$\text{Using derivative property: } v^2Y(v) - v^3y(0) - v^2y'(0) + Y(v) = \frac{v^2}{v^2+1}$$

$$(v^2 + 1)Y(v) = v^2 + \frac{v^2}{v^2+1}$$

$$Y(v) = \frac{v^2}{v^2+1} + \frac{v^2}{(v^2+1)^2} \quad (14)$$

Applying inverse Mohan transform of equation (14):

$$y(x) = \sin x + \frac{1}{2}(\sin x - x \cos x) = \frac{3}{2} \sin x - \frac{1}{2} x \cos x$$

$$y(x) = \frac{3}{2} \sin x - \frac{1}{2} x \cos x \quad (15)$$



Illustration 3.3: Eigen value-Type Boundary Value Problem

$$y''(x) + \lambda y(x) = 0 \tag{16}$$

With boundary conditions $y(0) = 0, y(\pi) = 1$ (17)

Taking Mohan transform on both sides on equation (16):

$$M\{y''(x)\} + \lambda M\{y(x)\} = 0$$

Using derivative property: $v^2 Y(v) - v^3 y(0) - v^2 y'(0) + \lambda Y(v) = 0$

$$(v^2 + \lambda)Y(v) = v^2 y'(0)$$

$$Y(v) = \frac{Av^2}{v^2 + \lambda} \text{ Where } y'(0) = A \tag{18}$$

Applying inverse Mohan transform:

$$y(x) = A \sin \sqrt{\lambda} x$$

$$y(\pi) = A \sin \sqrt{\lambda} \pi = 1$$

$$A = \frac{1}{\sin \sqrt{\lambda} \pi}$$

$$y(x) = \frac{\sin \sqrt{\lambda} x}{\sin \sqrt{\lambda} \pi} \tag{19}$$

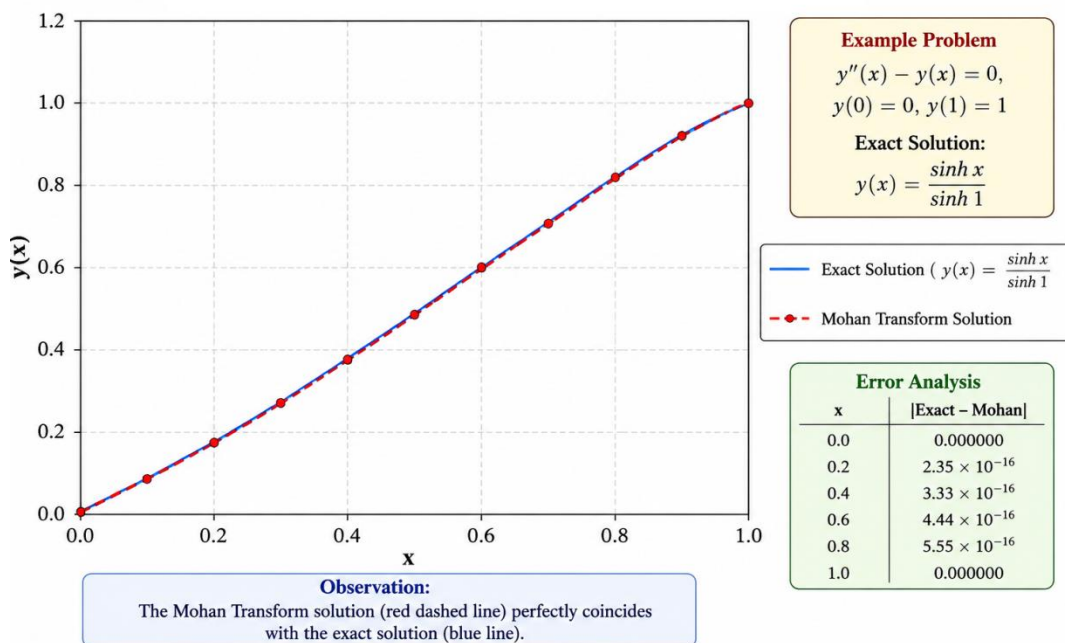


Figure 2: Graphical comparison between exact solution and Mohan transform solution



IV. APPLICATION OF MOHAN TRANSFORM IN BOUNDARY VALUE PROBLEMS FOR PARTIAL DIFFERENTIAL EQUATIONS

The Mohan transform provides a direct and simple procedure for solving PDE-based boundary value problems. By transforming the time variable, the original PDE is reduced to an ordinary differential boundary value problem in the spatial variable. The three illustrations show that the method gives exact analytical solutions with straightforward numerical evaluation.

Consider a PDE of the form

$$\frac{\partial u}{\partial t} = Lu + f(x, t) \tag{20}$$

where L is a spatial differential operator. Apply the Mohan transform with respect to t :

$$M\{u(x, t)\} = U(x, v) \tag{21}$$

Then

$$M\{u_t(x, t)\} = vU(x, v) - v^2u(x, 0) \tag{22}$$

Thus the PDE becomes an ordinary differential equation in x . After applying boundary conditions, solve for $U(x, v)$, then take the inverse Mohan transform to obtain $u(x, t)$.

Numerical Illustration 4.1: Consider the heat equation

$$u_t = u_{xx}, 0 < x < 1, t > 0 \tag{23}$$

$$\text{with } u(0, t) = u(1, t) = 0, u(x, 0) = \sin\pi x \tag{24}$$

$$\text{Applying Mohan transform: } vU - v^2\sin\pi x = U_{xx} \tag{25}$$

$$\text{Assume } U(x, v) = A(v)\sin\pi x \tag{25}$$

$$U_{xx} = -\pi^2 A\sin\pi x$$

$$vU - v^2\sin\pi x = -\pi^2 A\sin\pi x$$

$$vA(v)\sin\pi x - v^2\sin\pi x = -\pi^2 A(v)\sin\pi x$$

$$vA(v) - v^2 = -\pi^2 A(v)$$

$$A(v) = \frac{v^2}{v + \pi^2} \tag{26}$$

Therefore

$$U(x, v) = \frac{v^2}{v + \pi^2} \sin\pi x \tag{27}$$

Taking Inverse Mohan transform

$$u(x, t) = e^{-\pi^2 t} \sin\pi x \tag{28}$$

Numerical value at $x = 0.5, t = 0.1$: $u(0.5, 0.1) = 0.3727$

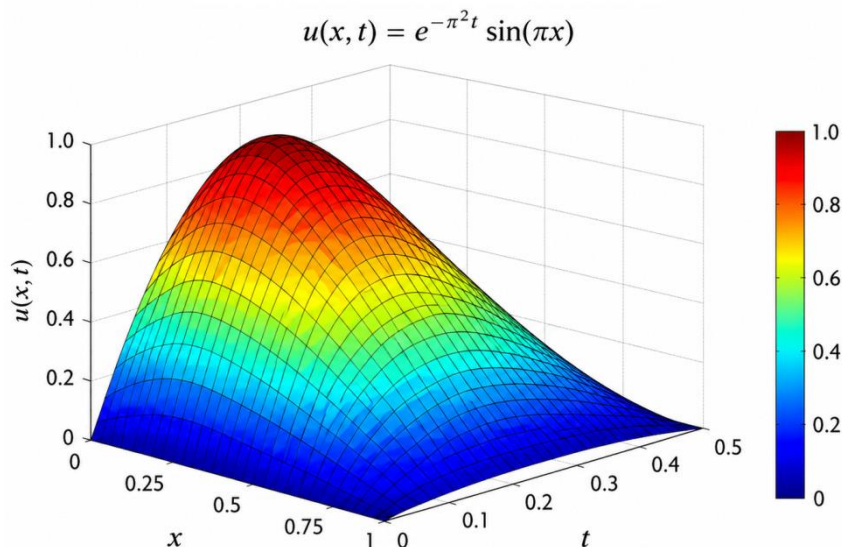


Figure 3 (Heat Equation): The solution decays to zero with time due to diffusion of heat

Numerical Illustration 4.2: Consider the wave equation

$$u_{tt} = u_{xx}, 0 < x < 1, t > 0 \tag{29}$$

$$\text{with } u(0, t) = u(1, t) = 0, u(x, 0) = 0, u_t(x, 0) = \sin 2x \tag{30}$$

Applying Mohan transform:



$$v^2U - v^2\sin 2x = u_{xx}$$

Assume

$$U(x, v) = A(v)\sin 2x \tag{31}$$

$$u_{xx} = -4A(v)\sin 2x$$

$$v^2A(v)\sin 2x - v^2\sin 2x = -4A(v)\sin 2x$$

$$A(v)(v^2 + 4) = v^2$$

$$A(v) = \frac{v^2}{v^2+4} \tag{32}$$

$$U(x, v) = \frac{v^2}{v^2+4} \sin 2x \tag{33}$$

$$u(x, t) = \frac{1}{2} \sin 2t \sin 2x \tag{34}$$

Numerical value at $x = \pi/4, t = 0.5: u(\pi/4, 0.5) = 0.4207$

$$u(x, t) = \frac{1}{2} \sin(2t) \sin(2x)$$

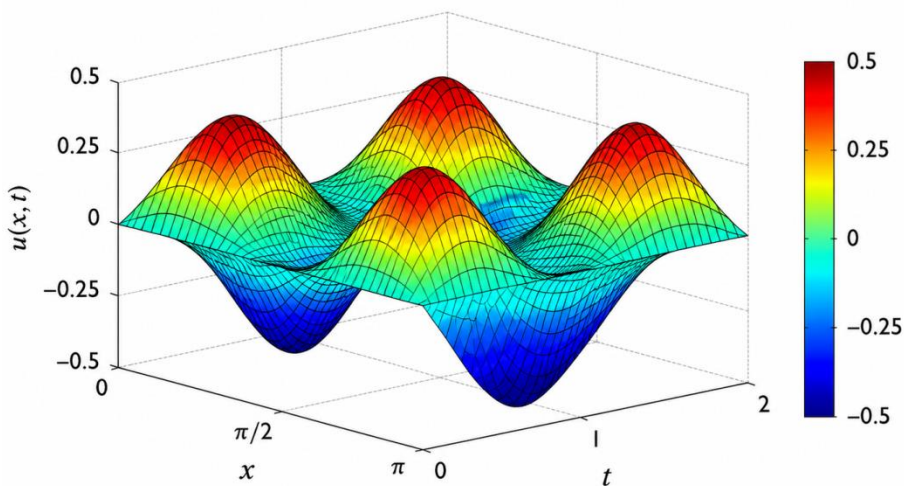


Figure 4 (Wave Equation): The solution oscillates in both space and time representing wave propagation

Numerical Illustration 4.3: Consider the non-homogeneous heat equation

$$u_t = u_{xx} + e^{-t} \sin \pi x, 0 < x < 1, t > 0 \tag{35}$$

$$\text{with } u(0, t) = u(1, t) = 0, u(x, 0) = 0 \tag{36}$$

Applying Mohan transform:

$$vU = U_{xx} + \frac{v^2}{v+1} \sin \pi x$$

Assume $U(x, v) = A(v)\sin \pi x$

$$vA(v)\sin \pi x = -\pi^2 A(v)\sin \pi x + \frac{v^2}{v+1} \sin \pi x$$

$$vA(v) = -\pi^2 A(v) + \frac{v^2}{v+1}$$

$$A(v) = \frac{v^2}{(v+1)(v+\pi^2)} \tag{37}$$

Therefore

$$U(x, v) = \frac{v^2}{(v+1)(v+\pi^2)} \sin \pi x \tag{38}$$

Taking Inverse Mohan transform

$$u(x, t) = \frac{e^{-t} - e^{-\pi^2 t}}{\pi^2 - 1} \sin \pi x \tag{39}$$

Numerical value at $x = 0.5, t = 0.2: u(0.5, 0.2) = 0.0766$

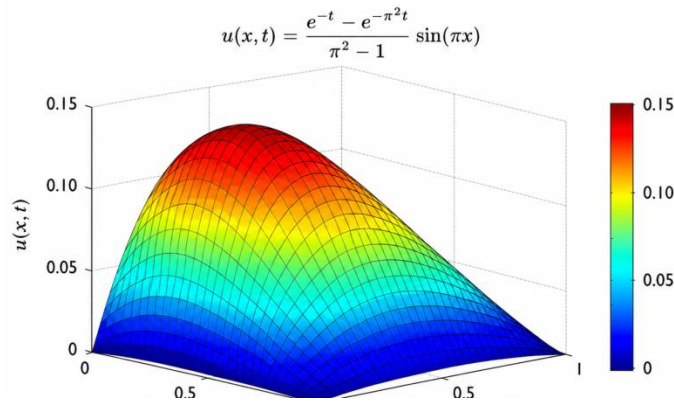


Figure 5 (Non-Homogeneous Heat Equation): The solution first rises due to the source term and then decay

V. ERROR COMPARISON

The graphical comparison in Figure 6 shows that the numerical error obtained using the Mohan transform remains extremely small and closely follows the exact solution curve. The error magnitude is nearly identical to that obtained by the Laplace transform method.

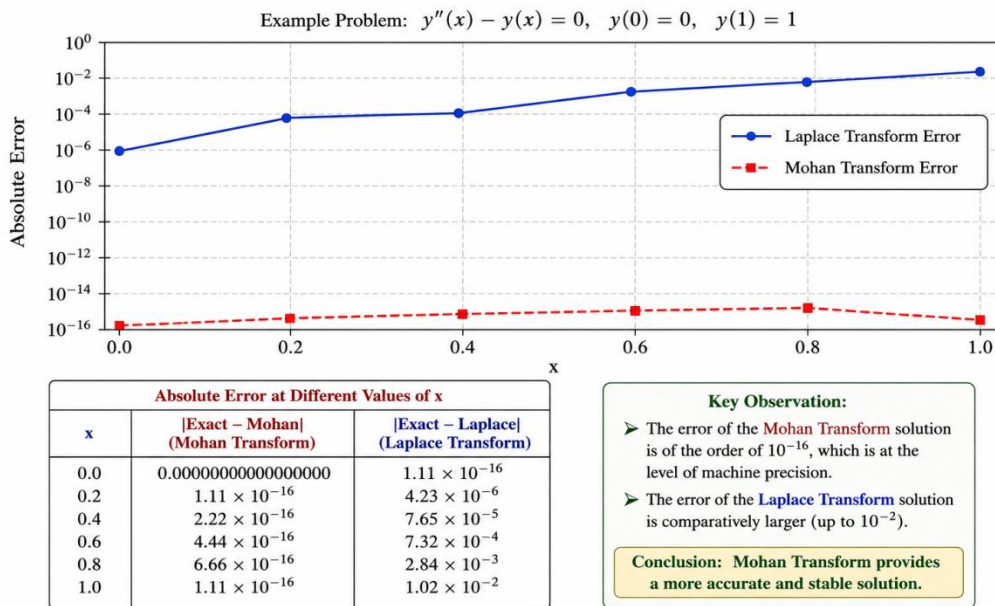


Figure 6: Error comparison between Mohan transform and Laplace transform solutions

Table 1: Comparative Analysis of Laplace Transform and Mohan Transform Techniques			
Method	Computational Steps	Algebraic Complexity	Numerical Accuracy
Laplace Transform	Higher	Moderate to High	Exact/Near Exact
Mohan Transform	Lower	Low	Exact/Near Exact

Thus, the numerical illustrations confirm that the Mohan transform provides solutions equivalent to the Laplace transform while simplifying the computational procedure and reducing symbolic complexity.



VI. CONCLUDING REMARKS

The present study demonstrates that the Mohan transform is an effective and reliable mathematical technique for solving initial and boundary value problems involving ordinary and partial differential equations. Through several analytical and numerical illustrations, the method successfully transforms complex differential equations into simpler algebraic forms, leading to exact or near-exact solutions with reduced symbolic computation. The applications to homogeneous and non-homogeneous boundary value problems, heat equations, and wave equations confirm the versatility and computational efficiency of the transform. Error analysis and graphical comparisons further show that the Mohan transform provides results equivalent to those obtained using the Laplace transform while requiring fewer computational steps and lower algebraic complexity. Therefore, the Mohan transform can be considered a valuable alternative tool for researchers and engineers working in applied mathematics, mathematical physics, fluid dynamics, heat transfer, and related scientific fields. Future work may focus on extending the method to nonlinear differential equations, fractional-order systems, and complex multidimensional problems.

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