



Modal Analysis and Dispersion Characteristics of a Plasma-Loaded Planar Optical Waveguide

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Abstract: A theoretical study of a three-layer plasma-loaded planar optical waveguide is presented. The structure consists of a silica guiding layer, a plasma-filled guiding region, and an air region. The study examines the influence of plasma loading on the effective refractive index and modal dispersion of the planar waveguide. The analysis is based on the phase-matching condition for a three-layer asymmetric slab waveguide, with Transverse Electric (TE) and Transverse Magnetic (TM) polarization effects introduced through the reflection phase shifts at the interfaces. The plasma refractive index is described through the plasma frequency, and the source study considers the normalized plasma-frequency parameter $\omega_p/\omega = 0.4$. For the numerical investigation, the silica refractive index is taken as 1.40, the air refractive index as 1.00, and the plasma-region width as $2.5 \mu\text{m}$. The calculated dispersion curves show distinct mode-dependent variation of effective refractive index with wavelength and plasma-region width. The results demonstrate that plasma loading modifies the modal propagation characteristics and provides a tunable parameter for planar optical-waveguide design.

Keywords: Plasma-loaded waveguide; Planar optical waveguide; Slab waveguide; Effective refractive index; Dispersion; Plasma frequency; TE/TM modes; Propagation constant.

1. INTRODUCTION

Optical waveguides are fundamental photonic structures used for the controlled confinement and propagation of electromagnetic energy. Their propagation characteristics are governed by the refractive-index distribution, geometrical configuration, and electromagnetic boundary conditions at the interfaces between different media [1–4]. Depending on their geometry, optical waveguides can be broadly classified into planar, channel, strip, and fiber structures. Among these configurations, planar or slab waveguides provide a relatively simple platform for investigating fundamental waveguiding phenomena because optical confinement is primarily required in one transverse direction while propagation occurs along the longitudinal direction [1,3]. Such structures are therefore widely used as basic models for understanding modal propagation, dispersion, polarization, and confinement in more complicated integrated optical systems [5–7].

A conventional planar waveguide generally consists of a guiding layer positioned between two surrounding media having lower refractive indices. Under appropriate conditions, electromagnetic radiation can remain confined within the guiding region through total internal reflection at the interfaces [2,4]. The boundary conditions imposed at these interfaces permit only discrete field distributions and corresponding propagation constants, which are identified as the guided modes of the structure [1,5]. The effective refractive index is an important parameter for describing these modes because it provides a direct measure of the phase propagation of the guided optical field [6,7]. Its value depends on the material refractive indices, wavelength, guiding-layer thickness, and modal order. Consequently, controlled modification of the material or geometrical parameters provides an effective means of tailoring the propagation characteristics of optical waveguides [8,9]. Polarization also plays an important role in planar-waveguide propagation. The electromagnetic solutions are commonly separated into transverse-electric and transverse-magnetic modal families according to the orientation of the electric and magnetic fields with respect to the direction of propagation [2,5]. The two polarization states experience different phase changes at dielectric interfaces, leading to different modal conditions and dispersion characteristics [4,10]. Consequently, the effective refractive index, cutoff behavior, and field confinement can differ for TE and TM modes [6,11]. This polarization dependence becomes particularly significant when a frequency-dependent material is introduced into one of the guiding regions.

The dispersion characteristics of a waveguide are determined by the combined influence of material and geometrical parameters. In general, the effective refractive index changes with wavelength because of material dispersion and wavelength-dependent modal confinement [7,12]. The thickness of the guiding region also determines the transverse phase accumulation and influences the number of modes supported by the structure [3,8]. A sufficiently narrow guiding region may support only a fundamental mode, whereas increasing its width can permit higher-order modes to propagate



[5,13]. Therefore, simultaneous control of the waveguide dimensions and refractive-index distribution can provide an effective approach for modifying modal dispersion and guiding characteristics. In conventional dielectric waveguides, the refractive indices of the constituent materials are generally treated as material parameters. The incorporation of a frequency-dependent medium, however, provides an additional mechanism for controlling electromagnetic propagation [14,15]. Plasma is particularly interesting for this purpose because it contains free charged particles whose collective response to an electromagnetic field depends strongly on frequency and particle density [11,14]. The plasma frequency represents a characteristic frequency associated with the collective oscillation of the charged particles and plays an important role in determining the electromagnetic response of the plasma [12,15]. Consequently, the optical properties of a plasma differ from those of a conventional passive dielectric and can vary substantially with operating frequency.

The interaction of electromagnetic waves with plasma has been investigated extensively in plasma physics and electromagnetic-wave propagation [14–16]. Plasma can support distinctive wave-propagation phenomena because its dielectric response is strongly frequency-dependent. The use of plasma as a guiding or guiding-modifying medium has consequently attracted attention for controlling electromagnetic propagation and optical confinement [17]. Plasma-based guiding structures can provide spatially controllable refractive-index distributions, allowing the propagation characteristics of electromagnetic radiation to be modified through changes in plasma density and operating frequency [18,19]. Such properties make plasma an interesting candidate for tunable waveguide configurations. The integration of plasma with a planar optical waveguide provides a convenient configuration for investigating the combined influence of dielectric confinement and plasma dispersion. In a plasma-loaded waveguide, the modal properties are determined not only by the geometry and refractive indices of the dielectric regions but also by the frequency-dependent optical response of the plasma [17,20]. Changes in the plasma parameters can consequently modify the effective refractive index, propagation constant, and dispersion characteristics of the guided modes. The width of the plasma-loaded region is also expected to influence modal confinement because it changes the spatial distribution of the guided field and the phase accumulated across the guiding region [8,20].

A three-layer plasma-loaded planar waveguide therefore provides a relatively simple theoretical structure for studying tunable modal propagation. The structure considered in the present study consists of silica, a plasma-filled guiding region, and air. The silica and air refractive indices are taken as 1.40 and 1.00, respectively, while the plasma-region width is considered to be 2.5 μm . The plasma response is characterized using a normalized plasma-frequency parameter of 0.4, consistent with the supplied study. The effective refractive index is subsequently determined for different wavelengths and modal orders.

The original numerical investigation considered wavelengths in the range of approximately 400–800 nm and examined the variation of effective refractive index with wavelength and plasma-waveguide width. The calculated results demonstrated distinct modal behavior, with the effective refractive index varying according to both wavelength and mode order [22]. These results indicate that plasma loading can significantly influence the propagation characteristics of a planar optical waveguide. The present work is therefore focused on the theoretical investigation of modal dispersion in a three-layer plasma-loaded planar optical waveguide. Particular attention is given to the variation of effective refractive index with wavelength, plasma-region width, and modal order. The study provides a systematic basis for understanding the role of plasma as a frequency-dependent guiding medium and demonstrates its potential for modifying the modal propagation characteristics of planar optical-waveguide structures [17–22].

2. THEORETICAL BACKGROUND

2.1 Planar waveguide propagation

For a waveguide uniform along the z -direction, the electromagnetic fields can be represented in the form

$$E(x,y,z,t) = E(x,y) \exp(-i\beta z + i\omega t)$$

$$H(x,y,z,t) = H(x,y) \exp(-i\beta z + i\omega t)$$

where β is the propagation constant. The transverse field distribution remains invariant during propagation, apart from the phase accumulation associated with β . Guided modes occur only for discrete values of β that satisfy the electromagnetic boundary conditions.

In the ray-optics interpretation used in the source, the guided wave undergoes repeated total internal reflection between the interfaces. The horizontal component of the wave vector determines propagation along the waveguide, whereas the transverse component determines the standing-wave pattern across the guiding layer. The source gives

$$\beta = k n_1 \sin\theta_1$$



and distinguishes TE and TM modes through their different reflection phase shifts. The phase-matching condition for a guided mode is

$$2k n_1 W \cos\theta_1 - 2\phi_{10} - 2\phi_{12} = 2m\pi, \quad m = 0, 1, 2, \dots$$

2.2 Plasma as an optical medium

Plasma is an ionized medium containing electrons and ions. Because of its mobile charge carriers, its electromagnetic response differs from that of a neutral dielectric. The source defines the plasma frequency as

$$\omega_p^2 = ne^2/(\epsilon_0 m)$$

where n is the charged-particle number density, e is the particle charge, m is the charge-particle mass, and ϵ_0 is the permittivity of free space. For a collisionless plasma, the refractive index used in the study is represented by the frequency-dependent relation

$$n_p = [1 - (\omega_p/\omega)^2]^{1/2}$$

The dissertation specifies $\omega_p/\omega = 0.4$ for the numerical waveguide calculation. This parameter fixes the plasma loading for the reported dispersion calculations.

3. PLASMA-LOADED PLANAR WAVEGUIDE MODEL

The investigated structure is an asymmetric three-layer planar optical waveguide. The source describes the waveguide as consisting of silica, a plasma-filled guiding region, and air. The plasma region has a width $W = 2.5 \mu\text{m}$. The refractive indices specified for the numerical study are $n_s = 1.40$ for silica and $n_a = 1.00$ for air. The plasma refractive index is determined from the plasma-frequency relation.

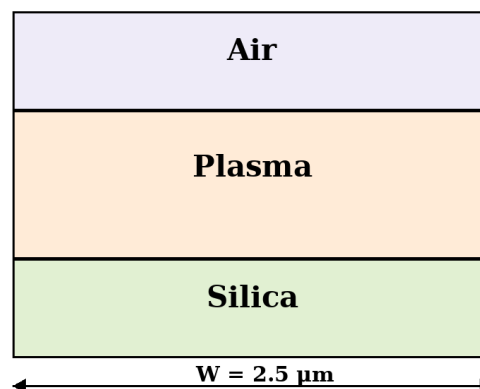


Fig. 1. Three-layer plasma-loaded planar waveguide considered in the present paper: silica/plasma/ air. The plasma-region width is $W = 2.5 \mu\text{m}$.

For a guided mode, the effective refractive index n_e lies between the relevant material indices in the guided region. The transverse propagation constants in the guiding and surrounding regions are determined by n_e , n_s , n_p , and n_a . The mode order is denoted by $m = 0, 1, 2, \dots$.

3.1 Dispersion relation

The numerical program included in the dissertation evaluates the following characteristic relation for the effective refractive index:

$$k_0 W (n_s^2 - n_e^2)^{1/2} - \tan^{-1} \{ (n_e^2 - n_p^2)^{1/2} / (n_s^2 - n_e^2)^{1/2} \} - \tan^{-1} \{ (n_e^2 - n_a^2)^{1/2} / (n_s^2 - n_e^2)^{1/2} \} - m\pi = 0$$

where $k_0 = 2\pi/\lambda$, W is the plasma-region width, n_s is the silica refractive index, n_p is the plasma refractive index, n_a is the air refractive index, n_e is the effective refractive index, and m is the modal order. This form is obtained directly from the zero-finding function used in the source MATLAB program.

4. NUMERICAL METHOD

The effective refractive index is obtained by solving the characteristic equation numerically for each wavelength and mode order. The source MATLAB program varies the wavelength from 400 nm to 800 nm in 50-nm increments and fixes the plasma-region width at $W = 2.5 \mu\text{m}$. A numerical zero-finding procedure is then used to determine n_e . The



calculated wavelength and effective-index pairs are written to a data file and subsequently plotted to obtain the dispersion curve.

The source program uses $n_s = 1.4$ and $n_a = 1.0$ and sets the mode index initially to $m = 0$ in the displayed function. The dissertation text also presents results for several mode orders, which are discussed below. The paper therefore retains the source's reported parameterization rather than introducing additional assumptions about unreported numerical cases.

5. RESULTS AND DISCUSSION

5.1 Effective refractive index versus wavelength

Figure 2 shows the calculated dispersion curves for mode orders $m = 0, 1, 2$, and 3 as the wavelength varies from approximately 400 to 800 nm. The effective refractive index decreases with increasing wavelength for all plotted modes. The fundamental mode ($m = 0$) remains closest to the silica refractive index and exhibits the weakest wavelength dependence. Higher-order modes show progressively stronger variation of n_e with wavelength.

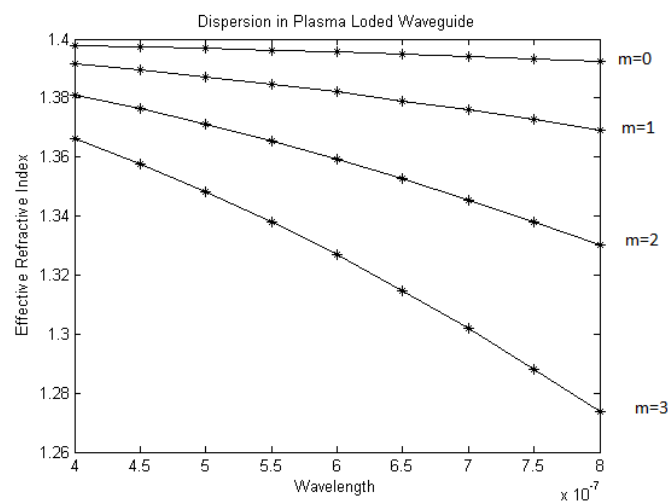


Fig. 2. Effective refractive index versus wavelength for different modal orders in the plasma-loaded waveguide.

5.2 Effective refractive index versus plasma-region width

Figure 3 presents the calculated effective refractive index as a function of the width of the plasma-loaded region. The plotted curves show the characteristic evolution of the modal effective index for $m = 0$ through $m = 4$. For each higher-order mode, a finite width is required before a guided solution develops away from the low-index limit. As the plasma-region width increases, the effective refractive index rises toward a mode-dependent limiting value.

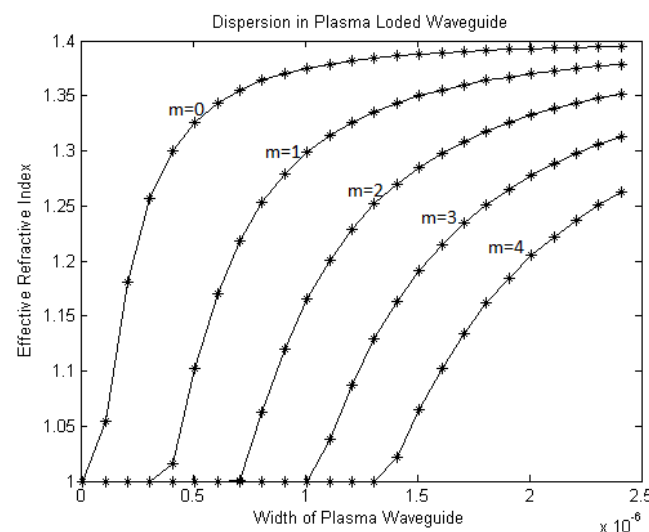


Fig. 3. Effective refractive index versus plasma-waveguide width for different modal orders.



5.3 Physical interpretation

The numerical results indicate that plasma loading substantially influences the modal propagation characteristics of the planar waveguide. The effect arises because the plasma refractive index depends on frequency through the plasma frequency. Consequently, both wavelength and plasma-region width alter the balance between field confinement in the guiding region and field penetration into the surrounding media. The modal dependence seen in Figs. 3 and 4 is consistent with the discrete mode structure of a three-layer planar waveguide.

The original dissertation concludes that propagation of electromagnetic waves is affected by filling the planar waveguide with plasma. The present reformulation makes the same conclusion more explicit: plasma loading acts as a frequency-dependent waveguide parameter and can therefore modify the effective refractive index and dispersion characteristics.

6. CONCLUSION

A theoretical and numerical study of a three-layer plasma-loaded planar optical waveguide has been reformulated as a research-paper presentation. The structure consists of silica, a plasma-filled region of width $2.5\ \mu\text{m}$, and air, with refractive indices 1.40 and 1.00 assigned to silica and air, respectively. The plasma response is represented through the plasma frequency, with the source study using $\omega_p/\omega = 0.4$. The effective refractive index is obtained from the three-layer slab-waveguide characteristic equation and evaluated numerically as a function of wavelength and plasma-region width.

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The reported dispersion curves show that the effective refractive index depends strongly on both wavelength and modal order. Higher-order modes exhibit stronger dispersion and require a sufficient plasma-region width for guided propagation. These findings demonstrate that plasma loading can be used to modify the modal propagation characteristics of planar optical waveguides and provide a basis for further investigation of tunable plasma-assisted optical devices.

The source study is primarily a theoretical/numerical investigation. A publication-grade extension should explicitly compare TE and TM dispersion curves, sweep the plasma-frequency ratio over a wider range, examine plasma collisions and losses, calculate propagation loss and field distributions, and compare numerical predictions with an independent full-wave solver or experimental measurements. These extensions are not claimed as results of the supplied dissertation.

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