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Noble Poly Shapes: An AI-Assisted Interactive Interface for the Enumeration of Noble Polyhedra

Akshat Wahal[1], Anupam Andrew Maxton[2]

Jumna Christian Junior High School, Uttar Pradesh, India[1,2]

Abstract: We describe Noble Poly Shapes, a browser-based graphical interface for exploring, visualizing, and extracting the complete set of noble polyhedra enumerated by Hill [1]. Noble polyhedra are those polyhedra that are simultaneously vertex-transitive and face-transitive; Hill's computer-assisted proof establishes that, aside from the two known infinite families, exactly 146 such polyhedra exist. The accompanying data library, noble-tools-revised [2], contains the enumeration code and a full set of exported models. This document summarizes the relevant definitions and results of that work and describes the respects in which Noble Poly Shapes differs from the original repository: namely, in providing browser-native visualization, interactive exploration of the catalog, and direct extraction of shape and face data, without requiring the user to run the original enumeration software.

1 INTRODUCTION

The regular polyhedra, and in particular the five convex Platonic solids, have been studied since antiquity for their exceptional symmetry: their symmetry groups act transitively on their vertices, edges, and faces simultaneously. A weaker and considerably richer condition is nobility: a polyhedron is noble if its symmetry group acts transitively on its vertices and on its faces, without any requirement on the edges. All regular polyhedra are noble, but the converse is far from true.

Beginning with Hess in 1875 [3], and continued by Brückner [4, 5], Grünbaum [6], and later Webb and Mikloweit using the software Stella [7], a steady but unsystematic stream of noble polyhedra was discovered over more than a century, with no proof that the search was ever complete. Hill [1] resolved this question by relating the existence of noble polyhedra to the roots of certain univariate and bivariate cubic polynomials arising from a parametrization of point-group orbits, giving both a proof of finiteness (Theorem 3.1) and a full computer enumeration (Theorem 3.2).

The present document has two purposes. First, in Sections 2 and 3, it summarizes the definitions and results of [1] that are relevant to understanding what is being visualized. Second, in Section 4, it describes Noble Poly Shapes, an interactive tool built on top of the noble-tools-revised data library [2], and states precisely what it adds relative to that repository.

2 DEFINITIONS

We restate the relevant definitions from [1] without proof.

Definition 2.1. A symmetry of a polyhedron P is an automorphism of its abstract structure whose induced permutation of vertices extends to an isometry of $\mathbb{R}^3$. The symmetry group $\Gamma(P)$ is the group generated by all such symmetries under composition.

Definition 2.2. A polyhedron is vertex-transitive if $\Gamma(P)$ acts transitively on its vertex set, and face-transitive if $\Gamma(P)$ acts transitively on its faces. A polyhedron is noble if it is both vertex-transitive and face-transitive.

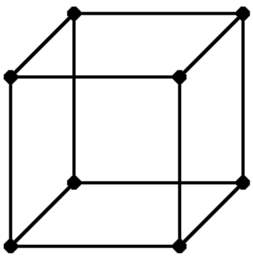

Figure 1: The cube: a familiar example of a regular, and therefore noble, polyhedron. Its symmetry group acts transitively on its eight vertices and, separately, on its six faces.

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Definition 2.3. A polyhedron is nondegenerate if it is finite, its realization is faithful (injective, with no two edges or faces sharing an identical vertex set), every face is planar, and no two faces sharing an edge are coplanar.

Among convex polyhedra, the only noble examples besides the regular polyhedra are the disphenoids—tetrahedra with four congruent, not-necessarily-equilateral triangular faces. Figure 1 illustrates the simplest kind of example: the cube's eight vertices form a single symmetry orbit, and its six faces form another, so the cube is both vertex- and face-transitive. Allowing nonconvexity yields a far larger set, including the infinite family of stephanoids (crown polyhedra) discovered by Hess.

Definition 2.4. Two polyhedra are facetings of each other if they share a vertex set. A noble polyhedron's vertex set is necessarily an orbit of a three-dimensional point group, since vertex-transitivity forces all vertices to lie at the same distance from a common fixed point.

3 SUMMARY OF HILL'S ENUMERATION

3.1 Method

Hill's method proceeds by parametrizing the orbits of each three-dimensional point group and identifying, for a given orbit type T, the polynomial conditions under which subsets of points in an orbit become coplanar—precisely the condition required for a set of points to form a face. This leads to a notion of criticality of orbits and an equivalence =c, under which noble facetings are shown to be equivalent within a class. Because the number of equivalence classes of any orbit type is finite, this reduces the search for noble polyhedra from an infinite family of candidate vertex sets to a finite number of test cases.

Theorem 3.1 (Finiteness, [1]). The only noble polyhedra with a prismatic symmetry group are the stephanoids and disphenoids. Other than these, the noble polyhedra fall into a finite number of equivalence classes, such that all members of a class share the same set of noble facetings.

The finite set of remaining cases is checked computationally: equivalence classes are symbolically identified using Python (numpy, sympy) and the Wolfram Language, and the exact vertex positions are recovered by approximating the positive real roots of the associated polynomials with a root-finding algorithm (Jenkins–Traub).

Theorem 3.2 (Enumeration, [1]). Discounting the stephanoids and disphenoids, there are exactly 146 noble polyhedra up to similarity.

3.2 Results

The 146 nonprismatic noble polyhedra are distributed across 16 orbit types: one each in the T, O, C, tO, tC, and rC types; 4 in I; 6 in ID; 7 in D; 17 in tI; 6 in tD; 19 in rD; 7 in sC; 3 in gC; 33 in sD; and 38 in gD. No noble polyhedra occur in the CO, tT, rT, rP, sT, gT, or gP orbit types. All but four of the 146 (gD-19.1, gD-28.1, D-4, and D-5) have a polyhedral dual; the remaining four have fissary duals, whose vertex figures are compound rather than simple polygons.

4 NOBLE POLY SHAPES

4.1 The source data library

Alongside the paper, Hill released the full codebase used for the enumeration as noble-tools-revised [2], an open-source repository (GPL-3.0) containing the Python and Wolfram Language scripts used to run the enumeration, a library directory of OFF model files for all 146 polyhedra together with the minimal polynomials describing their exact vertex positions, and an export.py pipeline for regenerating these files. This library is exact and complete, but using it requires installing and running Python and Wolfram Language scripts; viewing the resulting OFF files further requires separate third-party 3D-modeling software.

4.2 Interface and contribution

Noble Poly Shapes (https://noble-poly-shapes.ai.studio) is a browser-based graphical interface built directly on the openly licensed data in noble-tools-revised. To the author's knowledge, it is the first graphical interface built specifically for exploring the complete enumeration of the 146 noble polyhedra described in [1]. It makes no change to the underlying enumeration, proofs, or geometric data in [1, 2]; its contribution is confined to access and presentation. Concretely, it differs from the source repository as follows:

(i) No local installation. All 146 polyhedra are viewable directly in a web browser, without installing Python, numpy, sympy, Wolfram Language, or third-party OFF viewers.

(ii) Interactive visualization. Polyhedra are rendered directly in the GUI and can be rotated and viewed from different angles, in place of static exported model files. The model can also be shown in an exploded view, with faces separated out, and its vertices, edges, and faces can be examined individually. The interface is for exploring the fixed catalog of 146 already-enumerated polyhedra; it does not accept user-supplied input to construct new or arbitrary noble polyhedra.

(iii) Catalog browsing. The full set of 146 polyhedra can be searched and compared interactively, rather than read from the static tables of the paper's appendix.

(iv) Shape and face extraction. Individual face geometries and shape data for a selected polyhedron can be extracted directly through the interface, rather than parsed by hand from OFF and polynomial output files.

In summary, [1] proves that the set of noble polyhedra is complete and enumerates it exactly; noble-tools-revised [2] provides the exact underlying data; and Noble Poly Shapes provides an interactive means of viewing, exploring, and extracting that data without requiring the user to run the original research code.

ACKNOWLEDGMENTS

All mathematical results, the enumeration, and the source data library described in this document are the work of Connor Hill. This document and the accompanying interface together form an independent presentation layer built on top of that published research.

I would like to sincerely thank my brother, Aaditya Wahal, who helped adapt the original Python and Wolfram Language code for use on the website. Artificial intelligence tools were also used in the course of this project, including in the development of the website.

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BIOGRAPHY

Akshat Wahal is a student with a strong interest in computational mathematics and the methods used throughout this project, including symbolic computation, geometric enumeration, and interactive visualization. He is the developer of Noble Poly Shapes.

Anupam Andrew Maxton holds a B.Tech in Agriculture from the Sam Higginbottom Institute of Agriculture, Technology and Sciences, and an M.Tech in Remote Sensing and GIS from the same institute, where he graduated as a gold medalist. He has also worked on a NASA-funded plasma physics project at the University of Leicester, United Kingdom. His research interests include digital signal processing and VLSI-based video processing.

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